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DYNAMIC STABILISATION OF DRIFT INSTABILITIES IN AN INHOMOGENEOUS PLASMA

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This paper demonstrates dynamic stabilisation of the drift instabilities in an inhomogeneous plasma by applying an oscillating electric field parallel to the magnetic field. Unlike the previous studies, this demonstration hinges on the parametric effect of the high-frequency electric field, and does not involve any requirement of an increase in the frequency of the drift waves in the presence of the high-frequency field.

1. Introduction

Stabilisation of micro-instabilities driven by spatial inhomogenities in the plasma is of interest in connection with the confinement of high-temperature plasmas. One way of achieving this stabilisation is to use a sheared magnetic field (Pearlstein and Berk[1], Antonsen[2], Shivamoggi[3]). Another method is the dynamic stabilisation by applying an oscillating electric field parallel to the magnetic field (Fainberg and Shapiro[4], Okamoto et al.[5]). Experimental demonstration of this result was given by Nishida et al.[8]. The theoretical demonstration of this dynamic stabilisation given by Fainberg and Shapiro[4] and Okamoto et al.[5] was based on a model that excluded any parametric action produced by the oscillating electric field, and hinged on an increase in the frequency of the drift waves in the presence of the oscillating electric field. Furthermore, the calculations given by Okamoto et

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al.[5] apparently have some errors. The purpose of this paper is to demonstrate the dynamic stabilisation of drift instabilities using a model that focusses attention instead on the parametric action produced by the oscillating electric field. This parametric stabilisation is found not to be hinged on any requirement of an increase in the frequency of the drift waves in the presence of the oscillating field.

2. *Electrostatic drift waves*

Consider an inhomogeneous, low-β (β being the ratio of the particle pressure to the magnetic pressure) plasma with $n_0 = n_0(x)$, (n being the number density of the particles and the subscript 0 refers to the equilibrium state), placed in a uniform magnetic field $\vec{B}_0 = B_0 \hat{i}_z$. An oscillating electric field $\vec{E}_0 = \hat{i}_z E_0 \cos \omega_0 t$ is applied in the direction of the magnetic field. The former is assumed to be uniform in the plasma (the »dipole approximation«). One may choose for the equilibrium distribution function of the particles (Fainberg and Shapiro[4]),

$$f_{s0} = n_0 \left(\frac{m_s}{2\pi K T_s}\right)^{3/2} \left[1 + \hat{\varepsilon}_s \left(x + \frac{u_y}{\Omega_s}\right)\right] e^{-\frac{m_s u^2}{2KT_s}} \tag{1}$$

where

$$\Omega_s = \frac{e_s B_0}{m_s c}, \quad \vec{u} = \vec{v} - \frac{e_s \vec{E}_0(t)}{m_s \omega_0}, \quad \vec{E}_0(t) = \hat{i}_z E_0 \sin \omega_0 t$$

$$\hat{\varepsilon}_s = \left(\frac{1}{n_0}\frac{dn_0}{dx}\right)_{x=0} \ll \frac{1}{a_s} \tag{2}$$

$$a_s = \frac{\sqrt{KT_s/m_s}}{\Omega_s}$$

v being the velocity, m the mass, e the charge, T the temperature, K the Boltzmann constant, c the velocity of light, a the Larmor radius, and the subscript s refering to the ions and electrons.

If the equilibrium specified by (1) is perturbed, the perturbations evolve according to Vlasov's equation and Maxwell's equations,

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla f_s + \frac{e_s}{m_s}\left[\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right] \cdot \nabla_v f_s = 0 \tag{3}$$

$$\nabla \cdot \vec{E} = 4\pi \sum_s e_s \int d\vec{v} f_s \tag{4}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} \sum_s e_s \int d\vec{v}\vec{v} f_s + \frac{1}{c}\frac{\partial E}{\partial t} \tag{5}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{6}$$

$$\nabla \cdot \vec{B} = 0. \tag{7}$$

Let us write all the relevant variables as

$$q = q_0 + q_1, \; |q_1| \ll |q_0| \tag{8}$$

where the subscript 1 refers to the perturbations. Upon linearising the perturbations with respect to the state of equilibrium, Eq. (3) becomes

$$\frac{\partial f_{s1}}{\partial t} + \vec{v} \cdot \nabla f_{s1} + \frac{e_s}{m_s} [\hat{i}_z E_0 \cos \omega_0 t + \frac{1}{c} \vec{v} \times \vec{B}_0] \cdot \nabla_v f_{s1} =$$

$$= -\frac{e_s}{m_s} \left[\vec{E}_1 + \frac{1}{c} \vec{v} \times \vec{B}_1\right] \cdot \nabla_v f_{s0}. \tag{9}$$

Let us transform $\vec{v}$ to $\vec{u}$ (defined in (2)), and put

$$f_{s1} (\vec{u}, \vec{x}, t) = F_{s1} (\vec{u}, \vec{x}, t)\, e^{\frac{i k_s e_s \omega E_0}{m_s \omega_0^2} \cos \omega_0 t} \tag{10}$$

$\vec{k}$ being the wave vector of the perturbations. Eq. (9) then becomes

$$\frac{\partial F_{s1}}{\partial t} + \vec{u} \cdot \nabla F_{s1} + \frac{e_s}{m_s}\left(\frac{1}{c} \vec{u} \times \vec{B}_0\right) \cdot \nabla_u F_{s1} =$$

$$= -\frac{e_s}{m_s}\left[\vec{E}_1 + \frac{1}{c}\left(\vec{u} + \frac{e_s \vec{E}_0}{m_s \omega_0}\right) \times \vec{B}_1\right] \nabla_u f_{s0}\, e^{-i\mu_s \cos \omega_0 t} \tag{11}$$

where,

$$\mu_s \equiv \frac{k_s e_s E_0}{m_s \omega_0^2}.$$

One may formally solve Eq. (11) by introducing the parameter t, and defining the transformation (Rosenbluth, Krall and Rostoker[6])

$$t, \vec{x}, \vec{u} \Rightarrow t', \vec{x}'(t), \vec{u}'(t)$$

by

$$\left.\begin{aligned} t' &= t \\ \frac{d\vec{x}'}{dt} &= \vec{u}'(t) \\ \frac{d\vec{u}'}{dt} &= \Omega_s(\vec{u}' \times \hat{i}_z) \end{aligned}\right\} \tag{12}$$

which describe the orbits of the charged particles moving in unperturbed force fields

$$\vec{F}_s = \frac{e_s}{c}(\vec{u} \times \vec{B}_0).$$

In terms of this transformation, Eq. (11) becomes

$$\frac{d}{dt'} F_{s1}(\vec{x}', \vec{u}', t') = -\frac{e_s}{m_s}\left[\vec{E}_1 + \frac{1}{c}\left(\vec{u}' + \frac{e_s \vec{E}_0}{m_s \omega_0^2}\right) \times \vec{B}_1\right] \cdot \nabla_{u'} f_{s0}\, e^{-i\mu_s \cos\omega_0 t'} \tag{13}$$

and choosing the initial condition

$$t' = t,\ \vec{x}' = \vec{x},\ \vec{u}' = \vec{u} \tag{14}$$

one obtains

$$F_{s1}(\vec{x}, \vec{u}, t) = -\frac{e_s}{m_s}\int_{-\infty}^{t} dt' \left[\vec{E}_1 + \frac{1}{c}\left(\vec{u}' + \frac{e_s \vec{E}_0}{m_s \omega_0^2}\right) \times \vec{B}_1\right] \cdot \nabla_{u'} f_{s0}\, e^{-i\mu_s \cos\omega_0 t'}. \tag{15}$$

One now assumes that all perturbed quantities have space-time dependence given by $e^{i(\vec{k}\cdot\vec{x} - \omega t)}$. (Now, because the equilibrium distribution function f_{s0} is a function of x, strictly speaking, it is not possible to express the fields and the perturbed distribution function exactly in terms of single harmonic dependence, as in the foregoing. But, such complications can be avoided by adopting a »local approximation«, Krall[7]). It will be assumed further that ω has a small positive imaginary part so that the perturbations vanish at $t = -\infty$.

The perturbed electromagnetic field can be taken to be electrostatic for waves for which

$$\frac{\omega}{k_z} \ll V_A = \frac{B_0}{\sqrt{4\pi n_0 m_i}}$$

(Krall[7]), and this is valid for low-β plasmas. Thus, putting

$$\vec{E}_1 = -\nabla\varphi_1 \tag{16}$$

the dispersion relation follows from Eq. (14)

$$\nabla^2 \varphi_1 = -4\pi \sum_s e_s \int d\vec{v} f_{s1} \tag{17}$$

on using (10) and (15).

Noting,

$$\left.\begin{aligned} \nabla\varphi_1 \cdot \nabla_u f_{s0} &= \left[-\frac{m_s}{KT_s}\vec{u}\cdot\nabla\varphi_1 + \frac{i\,k_y\,\hat{\varepsilon}_s}{\Omega_s}\varphi_1\right] f_{s0} \\ \frac{d\varphi_1}{dt} &= \frac{\partial\varphi_1}{\partial t} + \vec{u}\cdot\nabla\varphi_1 + il\,\omega_0\,\varphi_1 \\ e^{-i\mu_s \cos\omega_0 t} &= \sum_{l=-\infty}^{\infty} i^l J_l(-\mu_s)\, e^{-i\,l\,\omega_0 t} \end{aligned}\right\} \tag{18}$$

$J_n(x)$ being the Bessel function of order n, one obtains from (15),

$$F_{s1} = \sum_{l=-\infty}^{\infty} \frac{e_s}{m_s} f_{s0} \left[-\frac{m_s}{KT_s} + i\int_{-\infty}^{t}\left(-\frac{\omega\, m_s}{KT_s} + \frac{k_y\,\hat{\varepsilon}_s}{\Omega_s} - \frac{l\,\omega_0\, m_s}{KT_s}\right) \times \right.$$

$$\left. \times\, e^{i[k_y(y'-y) + k_z(z'-z) - (\omega+l\omega_0)(t'-t)]} dt'\right] i^l J_l(-\mu_s)\, \varphi_1(\vec{k}, \omega). \tag{19}$$

Noting from (12) and (14) that

$$\left.\begin{aligned} u'_y &= u_\perp \cos(\Omega_s \tau + \varphi) \\ u'_z &= u_z \end{aligned}\right\} \tag{20}$$

and

$$\left.\begin{aligned} y' - y &= \frac{u_\perp}{\Omega_s}[\sin(\Omega_s\tau + \varphi) - \sin\varphi] \\ z' - z &= u_z \tau \end{aligned}\right\} \tag{21}$$

where $\tau = t' - t$

(19) becomes

$$F_{s1} = \frac{n_0\, e_s}{m_s}\left[\sum_{l=-\infty}^{\infty} (-i)^l J_l(\mu_s)\left(\frac{m_s}{2\pi\, KT_s}\right)^{3/2}\left\{-\frac{m_s}{KT_s} - \left(\frac{\omega\, m_s}{KT_s} - \frac{\hat{\varepsilon}_s k_\perp}{\Omega_s}\right) \times \right.\right.$$

$$\left.\left. \times \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{J_m\left(\frac{k_\perp u_\perp}{\Omega_s}\right) J_n\left(\frac{k_\perp u_\perp}{\Omega_s}\right)}{-\omega + k_{\parallel} u_{\parallel} + n\Omega_s} e^{i(n-m)\varphi} \times e^{-\frac{m_s u^2}{2KT_s}}\right\} \varphi_1(\vec{k}, \omega - l\,\omega_0)\right] \tag{22}$$

where $k_{\parallel} \equiv k_z$, $k_\perp \equiv k_y$.

The perturbed charge density is then given by

$$\varrho_{s1} = e_s \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} du_{\parallel} \int_0^{\infty} du_{\perp}\, u_{\perp}\, F_{s1}. \tag{23}$$

Using (22), (23) gives

$$\varrho_{s1} = -\frac{n_0 e_s^2}{KT_s} \left\{ \sum_{l=-\infty}^{\infty} (-i)^l J_l(\mu_s)\, k^2 \lambda_{D_s}^2\, \chi_s(\vec{k}, \omega)\, \varphi_1(k, \omega - l\omega_0) \right\} \tag{24}$$

where,

$$\chi_s(\vec{k}, \omega) = \frac{1}{k^2 \lambda_{D_s}^2} \left[1 + \sum_{n=-\infty}^{\infty} I_n \left(\frac{k^2 KT_s}{m_s \Omega_s^2} \right) e^{-\frac{k^2 KT_s}{m_s \Omega_s^2}} \times \frac{\omega - \hat{\omega}_s}{k_{\parallel} V_{T_s}} Z(\zeta_s) \right] \tag{25}$$

and

$$\lambda_{D_s} = \frac{V_{T_s}}{\omega_{p_s}}, \quad V_{T_s} = \sqrt{\frac{2 KT_s}{m_s}}, \quad \omega_{p_s} = \sqrt{\frac{4 \pi n_0 e_s^2}{m_s}}$$

$$\hat{\omega}_s = \frac{\hat{\varepsilon}_s k_{\perp} KT_s}{m_s \Omega_s}, \quad \zeta_s = \frac{\omega - n\Omega_s}{k_{\parallel} V_{T_s}}$$

$$Z(\zeta_s) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \frac{e^{-x^2}}{x - \zeta_s}$$

$I_n(x)$ being the modified Bessel function of the first kind and order n.

Using (10), equation (17) gives

$$\varphi_1(\vec{k}, \omega) = \frac{4\pi}{k^2} \sum_s \left[\sum_{l=-\infty}^{\infty} (-i)^l J_l(-\mu_s)\, \varrho_{s1}(\vec{k}, \omega - l\,\omega) \right]. \tag{26}$$

Neglecting the ion-response to the high-frequency field, (26) gives

$$\varphi_1(\vec{k}, \omega - l\omega_0) = \frac{4\pi}{k^2} \sum_{m=-\infty}^{\infty} (-i)^{m-l} J_{m-l}(-\mu_0)\, \varrho_{e1}(\vec{k}, \omega - m\,\omega_0) + $$

$$+ \frac{4\pi}{k^2} \varrho_{i1}(\vec{k}, \omega - l\omega_0) \tag{27}$$

where $\mu = \mu_e$.

3. The dispersion relation

Using (27), (24) gives

$$\varepsilon_e(\omega)\,\varrho_{e1}(\omega) = -\chi_e(\omega)\left[\sum_{l=-\infty}^{\infty} (-\mathrm{i})^l J_l(\mu)\,\varrho_{i1}(\omega - l\omega_0)\right] \tag{28}$$

$$\varepsilon_i(\omega)\,\varrho_{i1}(\omega) = -\chi_i(\omega)\left[\sum_{l=-\infty}^{\infty} (\mathrm{i})^l J_l(\mu)\,\varrho_{e1}(\omega - l\omega_0)\right] \tag{29}$$

where

$$\varepsilon_s(\omega) \equiv 1 + \chi_s(\omega) \tag{30}$$

we have used the relation

$$\sum_{l=-\infty}^{\infty} J_l(a)\,J_{m-l}(b) = J_m(a+b)$$

and have suppressed the argument k.

(2)8and (29) can be expressed differently as

$$\left.\begin{aligned}\varepsilon_e(\omega - l\omega_0)\,\varrho_{e1}(\omega - l\omega_0) &= -\chi_e(\omega - l\omega_0)\left[\sum_{m=-\infty}^{\infty} J_{m-l}(\mu)\,\varrho_{i1}(\omega - m\omega_0)\right]\\ \varepsilon_i(\omega - l\,\omega_0)\,\varrho_{i1}(\omega - l\,\omega_0) &= -\chi_i(\omega - l\omega_0)\left[\sum_{m=-\infty}^{\infty} J_{m-l}(\mu)\,\varrho_{e1}(\omega - m\omega_0)\right]\end{aligned}\right\} \tag{31}$$

where the charge densities have been redefined to absorb the factor containing i.

Following Okamoto et al.[5], let us take the oscillating field to be weak so that $|\mu| \ll 1$. Then one may retain in (31) only the terms corresponding to $m - l = 0$, ± 1. Thus, (31) gives

$$\left.\begin{aligned}&\varrho_{e1}(\omega - (\omega_0) + \Gamma_e(\omega - \omega_0)\,[J_0(\mu)\,\varrho_{i1}(\omega - \omega_0) + J_1(\mu)\,\varrho_{i1}(\omega)] = 0\\ &\varrho_e(\omega) + \Gamma_e(\omega)\,[-J_1(\mu)\,\varrho_{i1}(\omega - \omega_0) + J_0(\mu)\,\varrho_{i1}(\omega) + J_1(\mu)\,\varrho_{i1}(\omega + \omega_0)] = 0\\ &\varrho_{e1}(\omega + \omega_0) + \Gamma_e(\omega + \omega_0)\,[-J_1(\mu)\,\varrho_{i1}(\omega) + J_0(\mu)\,\varrho_{i1}(\omega + \omega_0)] = 0\end{aligned}\right\} \tag{32}$$

$$\left.\begin{aligned}&\varrho_{i1}(\omega - \omega_0) + \Gamma_i(\omega - \omega_0)\,[J_0(\mu)\,\varrho_{e1}(\omega - \omega_0) - \varrho_{e1}(\omega)] = 0\\ &\varrho_{i1}(\omega) + \Gamma_i(\omega)\,[J_1(\mu)\,\varrho_{e1}(\omega - \omega_0) + J_0(\mu)\,\varrho_{e1}(\omega) - J_1(\mu)\,\varrho_{e1}(\omega + \omega_0)] = 0\\ &\varrho_{i1}(\omega + \omega_0) + \Gamma_i(\omega + \omega_0)\,[\varrho_{e1}(\omega) + J_0(\mu)\,\varrho_{e1}(\omega + \omega_0)] = 0\end{aligned}\right\} \tag{33}$$

where,

$$\Gamma_s(\omega) \equiv \frac{\chi_s(\omega)}{1 + \chi_s(\omega)}.$$

Let us assume $|\mu(\omega_{pi}/\omega_0)| \ll 1$, (this assumption was also made but not stated by Okamoto et al.[5]), and keep only terms to $O(J_1^2)$. Then (32) becomes (on using (33)),

$$\varrho_{e1}(\omega-\omega_0) + \Gamma_e(\omega-\omega_0)\,[-J_0^2(\mu)\,\Gamma_i(\omega-\omega_0)\,\varrho_{e1}(\omega-\omega_0) -$$
$$- J_1(\mu)\,\Gamma_i(\omega)\,\{J_1(\mu)\,\varrho_{e1}(\omega-\omega_0) + J_0(\mu)\,\varrho_{e1}(\omega) - J_1(\mu)\,\varrho_{e1}(\omega+\omega_0)\}] = 0$$
$$\varrho_{e1}(\omega) + \Gamma_e(\omega)\,[-J_0(\mu)\,\Gamma_i(\omega)\,\{J_1(\mu)\,\varrho_{e1}(\omega-\omega_0) + J_0(\mu)\,\varrho_{e1}(\omega) -$$
$$- J_1(\mu)\,\varrho_{e1}(\omega+\omega_0)\}] = 0$$

$$\varrho_{e1}(\omega+\omega_0) + \Gamma_e(\omega+\omega_0)\,[J_1(\mu)\,\Gamma_i(\omega)\,\{J_1(\mu)\,\varrho_{e1}(\omega-\omega_0) + J_0(\mu)\,\varrho_{e1}(\omega) -$$
$$- J_1(\mu)\,\varrho_{e1}(\omega+\omega_0)\} - J_0^2(\mu)\,\Gamma_i(\omega+\omega_0)\,\varrho_{e1}(\omega+\omega_0)] = 0. \quad (34)$$

One obtains, from (34), the dispersion relation

$$[D(\omega-\omega_0) - J_1^2(\mu)\,\Gamma_e(\omega-\omega_0)\,\Gamma_i(\omega)] \times [D(\omega)\,D(\omega+\omega_0) +$$
$$+ J_1^2(\mu)\,\Gamma_e(\omega+\omega_0)\,\Gamma_i(\omega)] + J_1^2(\mu)\,\Gamma_e(\omega)\,\Gamma_e(\omega-\omega_0)\,\Gamma_i^2(\omega)\,D(\omega+\omega_0) = 0 \quad (35)$$

where,

$$D(\omega) = 1 - J_0^2(\mu)\,\Gamma_e(\omega)\,\Gamma_i(\omega).$$

Approximating $J_0^2(\mu) \approx 1$, (35) can be simplified to

$$D(\omega) = -J_1^2(\mu)\,\Gamma_i(\omega)\left[\frac{\Gamma_e(\omega-\omega_0)}{D(\omega-\omega_0)} + \frac{\Gamma_e(\omega+\omega_0)}{D(\omega+\omega_0)}\right]. \quad (36)$$

Approximating further

$$D(\omega) \approx \frac{\varepsilon(\omega)}{[1+\chi_e(\omega)]\,[1+\chi_i(\omega)]}, \quad J_1(\mu) \approx \frac{\mu}{2}$$

(36) gives

$$\varepsilon(\omega) = -\frac{\mu^2}{4}\,\chi_i(\omega)\,[1+\chi_e(\omega)]\left[\frac{\chi_e(\omega-\omega_0)}{\varepsilon(\omega-\omega_0)} + \frac{\chi_e(\omega+\omega_0)}{\varepsilon(\omega+\omega_0)}\right]. \quad (37)$$

Note that the dispersion relation given by Okamoto et al.[5] is apparently in error, who missed the terms $\chi_e(\omega \pm \omega_0)$ on the right hand side in (37).

4. *The dynamic stabilisation of drift waves*

Observe that the left hand side in (37) describes the ordinary drift modes in the absence of the oscillating field. Let us now consider cases wherein the latter becomes parametrically effective so that $\varepsilon(\omega-\omega_0) \approx 0$. Then, assuming that

$$\left|\frac{\omega-\omega_0}{k_{\parallel}\,V_{T_e}}\right| \gg 1, \quad \left|\frac{\omega}{\omega_0}\right| \ll 1, \quad \left|\frac{\hat{\omega}_e}{\omega_{p_e}}\right| \ll 1$$

one obtains from (25) and (30),

$$\varepsilon(\omega - \omega_0) \approx - \frac{2\omega_{pe}}{(\omega - \omega_0)^2}(\omega - \Delta + \mathrm{i}\,\Gamma_1) \tag{38}$$

where,

$$\Delta \equiv \omega_0 - \omega_{pe}\sqrt{1 + 3k^2\lambda_{De}^2}$$

$$\Gamma_1 \equiv \sqrt{\pi}\,\frac{(\omega_{pe}\sqrt{1 + 3k^2\lambda_{De}^2} - \hat{\omega}_e)}{3k^3\lambda_{De}^3}\,\mathrm{e}^{-\frac{1}{n^2\lambda_{De}^2}}.$$

Assuming $V_T \gg \frac{\omega}{k_{\parallel}} \gg V_{T_i}$ (as done by Okamoto et al.[5]) and using (25), (30) and (38), (37) gives

$$k^2\lambda_{De}^2\left[1 + \mathrm{i}\sqrt{\pi}\frac{(\omega - \hat{\omega}_e)}{k_{\parallel}V_{T_e}}\right] - k^2\lambda_{De}^2\frac{\hat{\omega}_e}{\omega} = - \frac{\mu^2/4\,[\omega_{pe}^2(1 + 3k^2\lambda_{De}^2)]}{2\omega_{pe}(\omega - \Delta + \mathrm{i}\Gamma_1)}. \tag{39}$$

Writing

$$\omega \equiv \Omega + \mathrm{i}\gamma \tag{40}$$

(39) gives

$$\gamma = -\sqrt{\pi}\frac{\hat{\omega}_e(\Omega - \hat{\omega}_e)}{k_{\parallel}V_{T_e}} - \frac{\mu^2}{4}\left[\frac{\hat{\omega}_e\,\omega_{pe}^2(1 + 3k^2\lambda_{De}^2)}{4\omega_{pe}(\Omega - \Delta)^2 k^2\lambda_{De}^2}\right]\Gamma_1 \tag{41}$$

which clearly illustrates the stabilising effect of the oscillating electric field on the drift waves. Physically this is apparent that under the condition $\varepsilon(\omega - \omega_0) \approx 0$, the drift wave will be in resonance interaction with the Langmuir wave and the pump electric field and will lose its energy to the latter in the process. Note that this parametric stabilisation, unlike the mechanisms in Fainberg and Shapiro[4], and Okamoto et al.[5], does not involve any requirement of an increase in the frequency of the drift waves in the presence of the oscillating electric field.

5. *Parametric destabilisation processes*

It may be noted that the parametric processes do not always necessarily produce stabilisation. For instance, in a homogeneous plasma, consider processes wherein the high-frequency pump electric field couples low-frequency ion-acoustic modes ($\omega \ll \omega_0$) and high-frequency side-bands ($\omega \pm \omega_0$) — which are electron plasma waves. This leads to a resonant parametric decay of the pump mode. In order to see this, assume as before.

$$\varepsilon(\omega - \omega_0) \approx 0$$

$$\left|\frac{\omega - \omega_0}{k_{\parallel}V_{T_e}}\right| \gg 1, \quad \left|\frac{\omega}{\omega_0}\right| \ll 1, \quad \left|\frac{k_{\parallel}}{k}\right| \ll 1$$

so that one obtains from (25) and (30)

$$\varepsilon(\omega - \omega_0) \approx - \frac{2\,\omega_{pe}^2}{(\omega - \omega_0)^2} (\omega - \Delta + \mathrm{i}\,\Gamma_1). \tag{38}$$

Assuming further that $V_{T_e} \gg \frac{\omega}{k_\parallel} \gg V_{T_i}$ and using (25), (30) and (38), (37) gives

$$k^2 \lambda_{De}^2 \left(1 + \mathrm{i}\sqrt{\pi}\frac{\omega}{k_\parallel V_{T_e}}\right) + \frac{\omega^2 - \Omega_i^2}{\omega_{pi}^2} = - \frac{\mu\,\omega_{pe}^2 (1 + 3\,k^2 \lambda_{De}^2)}{8\,\omega_{pe}(\omega - \Delta + \mathrm{i}\,\Gamma_1)}. \tag{42}$$

Writing,

$$\omega = \omega_r + \mathrm{i}\,\gamma \tag{43}$$

(42) gives

$$\omega_r^2 = \Omega_i^2 + \frac{\omega_{pi}^2}{1 + 1/k^2\lambda_{De}^2} \tag{44}$$

$$\gamma = \frac{\mu^2}{4} \frac{\omega_{pe}^2 (1 + 3\,k^2 \lambda_{De}^2)\,\Gamma_1}{2\,\omega_{pe}(\omega_r - \Delta)^2 \left(\frac{2\,\omega_r}{\omega_{pi}^2} + \frac{\sqrt{\pi}}{k\,V_{T_e}}\right)} \tag{45}$$

which illustrates the parametric amplification produced by the pump.

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DINAMIČKA STABILIZACIJA DRIFTNIH NESTABILNOSTI U NEHOMOGENOJ PLAZMI

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Originalni znanstveni rad

Obrađuje se problem stabilizacije driftnih valova pomoću dodatnog vanjskog visokofrekventnog električnog polja. Uslijed parametarskog djelovanja ovog polja dolazi do efekta stabilizacije odnosno energija driftnih valova se smanjuje a njihova frekvencija ostaje nepromijenjena.