YU ISSN 0015—3206
FZKAAA 18 (1) 23 (1986)

ANALYSIS OF AVALANCHE BREAKDOWN CHARACTERISTICS IN COMPLEMENTARY *p-n* JUNCTIONS

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Received 30 October

UDC 538.93

Original scientific paper

A general analysis of the difference between avalanche breakdown voltages ΔU_B of complementary semiconductor *p-n* junctions, when the ratio α_n/α_p of coefficients of electron (α_n) and hole (α_p) ionization is the function of electric field (E) and temperature (T) is presented. The numerical results for $\Delta U_B(T, N, P)$ are given for Si complementary abrupt junctions. A particular attention is paid to temperature dependences: the dependence of the difference ΔU_B of breakdown voltage on temperature was given and it was shown that changes can be considerable (up to 30%) especially for lower impurity concentrations.

1. *Introduction*

The problem of the avalanche breakdown of asymmetric abrupt complementary *p-n* semiconductor junctions has been dealt with by Urgell[1,2], Urgell and Leguerre[3] and Leguerre and Urgell[4], where the difference between breakdown voltages of the above structures was pointed out. The proof derived by Urgell[2], starting from the breakdown conditions

$$\int_0^W a_p \cdot e^{\int_0^x (\alpha_p - \alpha_n)\, dx'} dx = Y = 1, \qquad (1)$$

$$\int_0^W a_n \cdot e^{\int_0^x (\alpha_p - \alpha_n)\, dx'} dx = X = 1, \qquad (2)$$

hold for one-sided abrupt *p-n* complementary junctions only, where $\frac{a_n}{a_p} > 1$ for silicon, and $U_{B\,P^+_-N} < U_{B\,N^+_-P}$.

Urgell[1] has numerically obtained for Si $U_{B\,N^+_-P} = (1.07 \pm 0.01)\, U_{B\,P^+_-N}$. In material having $\frac{a_n}{a_p} < 1$ (for example Ge) we have $U_{B\,N^+_-P} < U_{B\,P^+_-N}$. In papers by Urgell and Leguerre[3] and by Leguerre and Urgell[4] it was found that there is a difference in the multiplication factor M of complementary structures and the values of Miller's exponent are determined.

In calculation of U_B and M by Urgell[1,2], Urgell and Leguerre (1974), and Leguerre and Urgell[4] the following expression for the ionization coefficients was used

$$a_{n,p} = A \cdot \mathrm{e}^{-\left(\frac{b}{|E|}\right)^m}, \tag{3}$$

where A, b and m are constants. This expression does not include explicitly the temperature dependence, so that it is not suitable for numerical analysis of temperature influence on U_B.

This paper will present a more general proof of the difference of avalanche breakdown voltages existing in complementary semiconductor *p-n* junctions, based on an arbitrary profile of impurities in the depletion region. Starting from the expression for the ionization coefficients of electrons and holes, which in a suitable way includes the field and temperature dependence (see Ref. 4) the results of a numerical analysis of the difference of breakdown voltages in function of temperature and concentration will be presented for the case of complementary Si abrupt $p^+ - n$ and $n^+ - p$ junctions with a uniform distribution of impurities.

2. *Theoretical model*

Let us observe the $p - n$ junction with an arbitrary profile of the charge density ϱ and electric field E in the depletion region (see Fig. 1b and 1a).

The best way is to substitute variable x in the breakdown condition relation by electric field E given by Tjapkin and Petković[10]. Starting from Poisson's equation

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{\varrho(x)}{\varepsilon \cdot \varepsilon_0}, \tag{4}$$

and using the relative variable

$$\eta = \frac{E}{E_0}, \quad \left(\eta_m = \frac{E_m}{E_0}\right),$$

with arbitrary E_0, or

$$E_0 = \left(\frac{3\hbar\,\omega_0 \cdot W_i}{4q^2\,\lambda_{00}^2} \cdot \coth\frac{\hbar\,\omega_0}{2kT}\right)^{1/2}$$

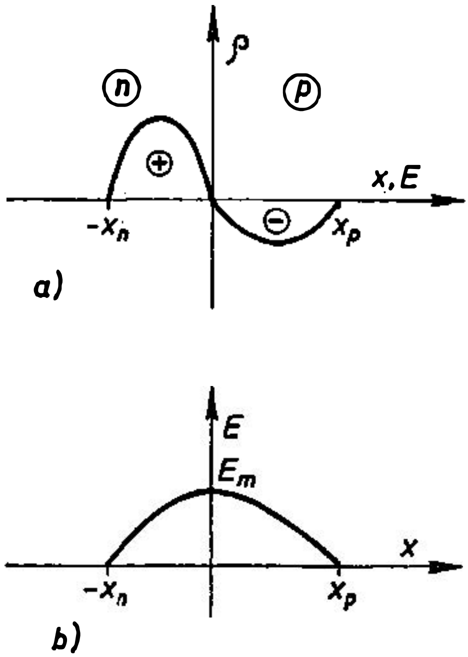

Fig. 1. a) Charge density distribution $\varrho(x)$ in space charge region. b) Field distribution $E(x)$; $x_p - x_n = W$.

and with

$$D_0 = \varepsilon_0\, \varepsilon E_0; \qquad \varrho_p(x) < 0; \qquad \varrho_n(x) > 0,$$

where $\hbar\,\omega_0$ is the optical phonon energy, W_i — ionization energy, E_B — electric field when the breakdown occurs, λ — the mean free-path of carriers, the breakdown condition (1) and (2) could be written in the form[10)]

$$D_0 \int\limits_0^{\eta_m} a_p(\eta) \left[\frac{1}{|\varrho_p|} \underset{(p)}{\mathrm{e}^{-D_0 \int\limits_0^{\eta} \frac{\alpha(\eta)}{|\varrho_p|} \mathrm{d}\eta' + f_W}} + \frac{1}{\varrho_n} \underset{(n)}{\mathrm{e}^{D_0 \int\limits_0^{\eta} \frac{\alpha(\eta)}{\varrho_n} \mathrm{d}\eta'}} \right] \mathrm{d}\eta \equiv Z \to 1 \qquad (5)$$

where the first and second term in the brackets represent contributions of p- and n-regions, respectively, and

$$f_W = D_0 \int\limits_0^{\eta_m} \left(\frac{1}{\varrho_n} + \frac{1}{|\varrho_p|} \right) \alpha \cdot \mathrm{d}\eta', \qquad \alpha = a_n - a_p. \qquad (6)$$

If η_m tends to $\eta_B = \frac{E_B}{E_0}$ i. e. when Z tends to unity the values of avalanche breakdown field E_B of $p-n$ junctions are obtained. Using (5) and (4) we can calculate the breakdown voltage dependence on the junction profile and on the ratio of coefficients of electron ionization α_n and hole ionization α_p, i. e. indirectly on carrier concentration and temperature. In that sense condition (5) is general. From (5) we see that the temperature dependence of the breakdown results not only from $\alpha_{n,p}(T)$ but from the dependence of dielectric constant* $\varepsilon(T)$ appearing in D_0 (which cannot be explicitly seen from (1) and (2)).

Complementary $p-n$ junctions are defined as junctions for which the transformations

$$\varrho_{nc}(x) = |\varrho_p(x)|,$$

$$|\varrho_{pc}(x)| = \varrho_n(x),$$

are valid, where ϱ_{pc} and ϱ_{nc} are charge densities in the complementary junction. So, condition (5) remains valid for the complementary junction upon the substitution: $Z \to Z_c$; $|\varrho_p| \to \varrho_n$; $\varrho_n \to |\varrho_p|$, while η_m and f_W remain unchanged (two particular equations were dealt with by Urgell[2]). In the case of one-sided abrupt junctions condition (5) can be substituted by

$$Z = D_0 \int_0^{\eta_m} \frac{\alpha_p}{\varrho_p} e^{f_W - D_0 \int_0^{\eta} \frac{\alpha}{|\varrho_p|} d\eta'} d\eta, \tag{7}$$

$$f_W = D_0 \int_0^{\eta_m} \frac{\alpha}{|\varrho_p|} d\eta', \tag{8}$$

for $n^+ - p$ structure, and

$$Z_c = D_0 \int_0^{\eta_m} \frac{\alpha_p}{\varrho_n} e^{D_0 \int_0^{\eta} \frac{\alpha}{\varrho_n} d\eta'} d\eta, \tag{9}$$

for $p^+ - n$ structure. So, we have for one-sided complementary junctions ($\varrho_n = |\varrho_p|$)

$$\Delta Z = Z_c - Z = D_0 \int_0^{\eta_m} \frac{\alpha_p}{\varrho_n} \left(e^{D_0 \int_0^{\eta} \frac{\alpha}{\varrho_n} d\eta} - e^{D_0 \int_0^{\eta_m} \frac{\alpha}{\varrho_n} d\eta'} \right) d\eta. \tag{10}$$

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* This dependence was ignored in our calculations.

In order to show which breakdown voltage U_B or U_{BC} (the breakdown voltage of complementary structure) is greater, in the general case, it is suitable to start from (5) and carry out a partial integration, introducing

$$\alpha \equiv \alpha_n - \alpha_p = \alpha_p (k - 1), \tag{11}$$

$$k = \frac{\alpha_n}{\alpha_p} = f(E).$$

So we get*

$$Z = \frac{e^{f_W} - 1}{k(\eta = 0) - 1} + \int_0^{\eta_m} \frac{-\frac{dk}{d\eta} d\eta}{(k-1)^2} (e^{f_W - I_p} - e^{I_n}), \tag{12}$$

$$I_{n,p} = D_0 \int_0^{\eta} \frac{\alpha}{\varrho_{n,p}} d\eta. \tag{13}$$

For a complementary junction, for the same U, i. e., η_m, Z_c would be obtained for $I_{pc} = I_n$, and $I_{nc} = I_p$, while $f_{Wc} = f_W$. The difference between Z_c and Z would be

$$\Delta Z = Z_c - Z = 4 \int_0^{\eta_m} \frac{-\frac{dk}{d\eta}}{(k-1)^2} \cdot e^{\frac{f_W}{2}} \cdot \sinh \frac{f'_W - (I'_p + I'_n)}{2} \cdot \sinh \frac{I'_p - I'_n}{2} d\eta. \tag{14}$$

Since $(\alpha \to |\alpha|)$

$$f'_W - (I'_p + I'_n) \equiv D_0 \int^{\eta_m} |\alpha| \left(\frac{1}{|\varrho_p|} + \frac{1}{\varrho_n} \right) d\eta \tag{15}$$

is always positive we get finally that the sign of $\Delta Z \equiv Z_c - Z$ is determined by the signs of expressions

$$I'_p - I'_n \equiv I_{np} = D_0 \int_0^{\eta} |\alpha| \left(\frac{1}{|\varrho_p|} - \frac{1}{\varrho_n} \right) d\eta, \tag{16}$$

and

$$\frac{dk}{d\eta} \equiv k'. \tag{17}$$

* In the paper by Urgell[2] the term with $k\,(x = W)$ in the denominator appears inadequately.

Relation (14) enables, also, a more sophisticated definition of asymmetrical (complementary) junction of the arbitrary profile:

- for $P^+ - N$ junction $\frac{1}{|\varrho_p(E)|} < \frac{1}{\varrho_n(E)}$ or quite general $I'_p < I'_n$, i. e. $I_{np} < 0$;
- for $N^+ - P$ junction $\frac{1}{\varrho_n} < \frac{1}{|\varrho_p|}$ or quite general $I_{np} > 0$.

A generalized symmetric junction would be the one having $I_{np} = 0$, so that $U_B = U_{BC} (Z = 0)$, irrespective of the fact that it is not necessary that $\varrho_n = |\varrho_p|$ (for the same E), neither $k' = 0$.

In Si $\alpha_n > \alpha_p$ i. e., $a > 0$ and $k > 1$, while in Ge $a < 0$ and $k < 1$.

Using (14) we find that $\Delta Z = 0$ when $\frac{\mathrm{d}k}{\mathrm{d}\eta} = 0$, i. e., the difference of breakdown voltages will exist if $k [E(x)] = \frac{\alpha_n [E(x)]}{\alpha_p [E(x)]} \neq$ const (and if $I_{np} \neq 0$). The sign of the integral on the right hand side of expression (14) determines which structure, $n^+ - p$, or $p^+ - n$, will have greater breakdown voltage at a given temperature. For silicon $\frac{\mathrm{d}k}{\mathrm{d}\eta} > 0$ so that $\Delta Z > 0$, i. e. $Z_c > Z$ for the same U. That means that the structure $p^+ - n$ will fulfill the condition $Z_c \to 1$ for a smaller voltage value $U = U_{BC}$, i. e. it will be

$$U_{BC} \equiv U_{B\,P^+_-N} < U_B \equiv U_{B\,N^+_-P},$$

while for Ge ($\mathrm{d}k/\mathrm{d}\eta < 0$) the opposite is valid.

In expression (14) the temperature appears as a parameter in η_m, k and all other quantities depending on ionization coefficients.

3. *Numerical results and analysis*

The calculation of the avalanche breakdown voltage is performed for silicon one-sided abrupt $p^+ - n$ and $n^+ - p$ complementary junctions with constant profile of impurities for concentrations of the lower doped side of the junction from 10^{15} to 10^{17} cm^{-3} and the temperature range of 100—460 K. For the calculation of the ionization coefficient, expression of Crowell and Sze[5] is used

$$\alpha_{n,p} = \frac{1}{\lambda_{n,p}} \cdot \mathrm{e}^{a_{n,p} \cdot x_{n,p}^2 + b_{n,p} \cdot x_{n,p} + b_{n,p}} \tag{18}$$

Since the carriers mean free path $\lambda_{n,p}$ and the ionization energy W_i are temperature dependent, the change $k = \frac{\alpha_n}{\alpha}$ is much pronounced, especially for weak fields (see Fig. 2). This change alters the value of the difference ΔU_B of breakdown voltages.

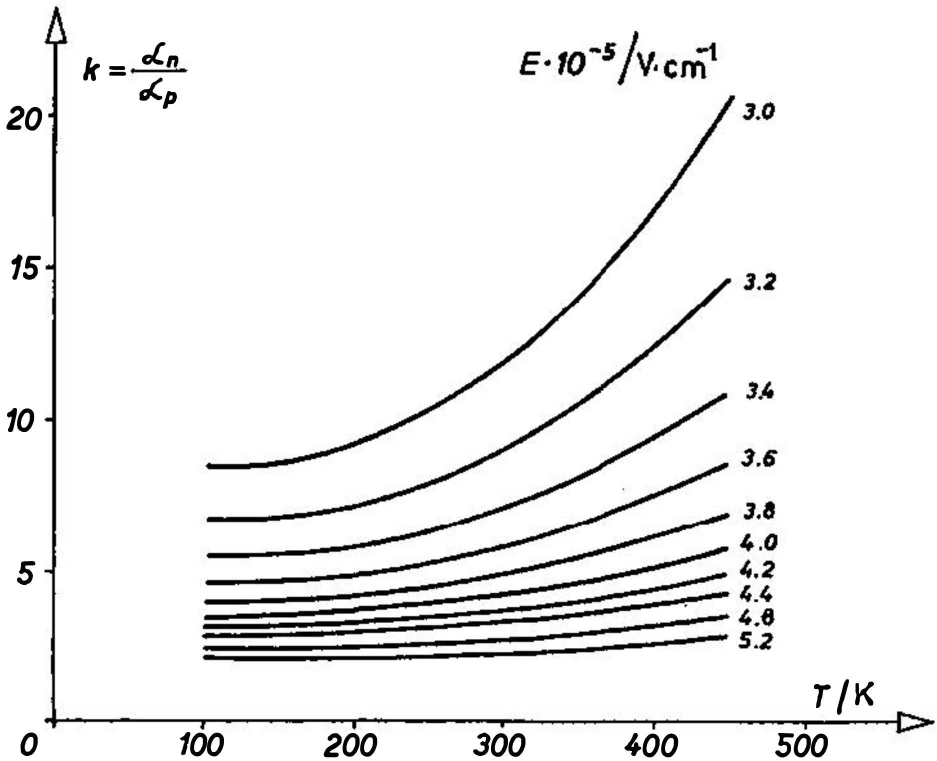

Fig. 2. Temperature dependence of ionization coefficients ratio $k = \frac{\alpha_n}{\alpha_p}$ for different values of electric field for Si.

The breakdown and complementary breakdown voltages (U_B and U_{BC}) are determined numerically from the conditions $X, Y = 1$. (In numerical calculations it was assumed that these conditions were fulfilled when X and Y were between 0.99999 and 1).

The results obtained for $\Delta U_B = U_{B\,n^+-p} - U_{B\,p^+-n}$ in function of temperature and carrier concentration of the lower doped side of abrupt one-sided $p - n$ junction ($p^+ \gg n, n^+ \gg p$) are graphically presented in Figs. 3 and 4 (the concentration dependence is contained in $\varrho_{n,p}$ i. e. $\varrho^c_{p,n}$; $\varrho_n = e \cdot N$, $|\varrho_p| = e \cdot P$).

On the basis of the results obtained we can conclude that

$$U_{B\,n^+-p} = (1 + x)\, n\, U_{B\,p^+-n}, \tag{19}$$

where $x = 0.04 - 0.07$ depending on concentration of carriers in the range from $10^{15}\ \mathrm{cm^{-3}}$ to $10^{17}\ \mathrm{cm^{-3}}$, which differs somewhat from that given by Urgell[1,2] ($x = 0.06 - 0.08$). The numerical results of the dependence of $x\,(T, N)$ are pre-

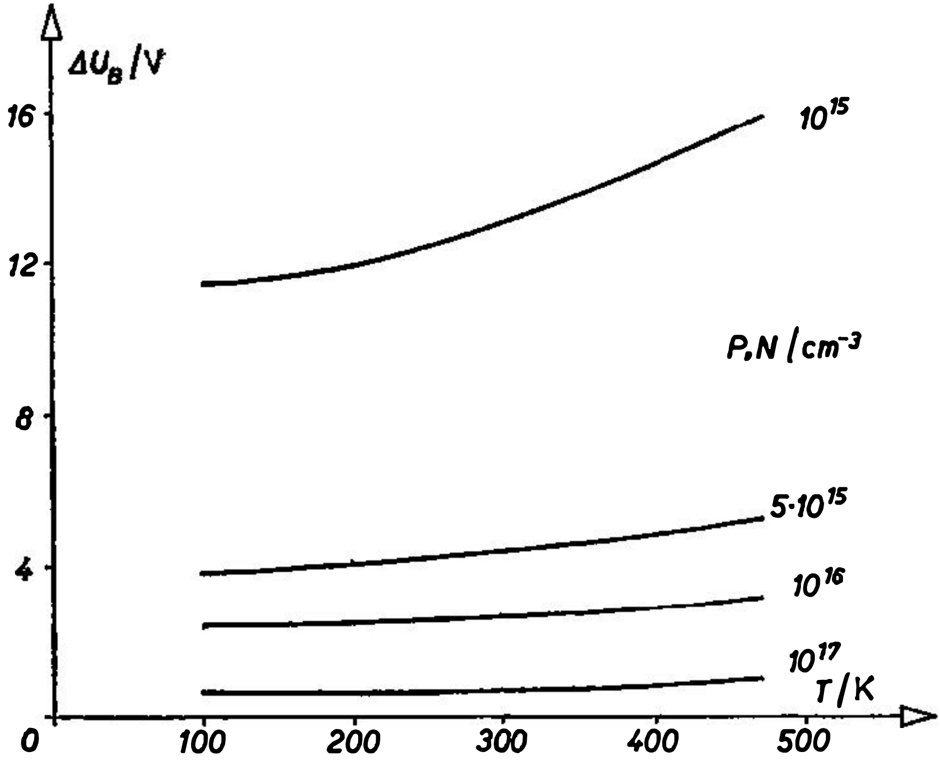

Fig. 3. The dependence ΔU_B on temperature in complementary abrupt one-sided Si $p-n$ junctions for various concentrations of lower doped side.

sented in Table 1. They display the weak dependence of x on T. This is in agreement with results reported by Grehov and Serezhkin[6]. The greatest relative deviations are for the smallest U_B, i. e. for the greatest $N(P)$ and amount to about 7%.

TABLE 1.

N, P cm^{-3}	x			
	100 K	200 K	300 K	460 K
10^{15}	0.0422	0.0422	0,0423	0,0428
$5\cdot10^{15}$	0.0523	0.0533	0.0530	0.0524
10^{16}	0.0546	0.0533	0.0530	0.0524
10^{17}	0.0688	0.0686	0.0685	0.0685

The results of the temperature dependence $\Delta U_B(T)$ (which was not elaborated by other authors) should be particularly stressed because they indicate the following. For the change of T from 100 to 460 K (the full ionization of impurities was assumed even at 100 K) ΔU_B could be changed even by 30% in Si junctions

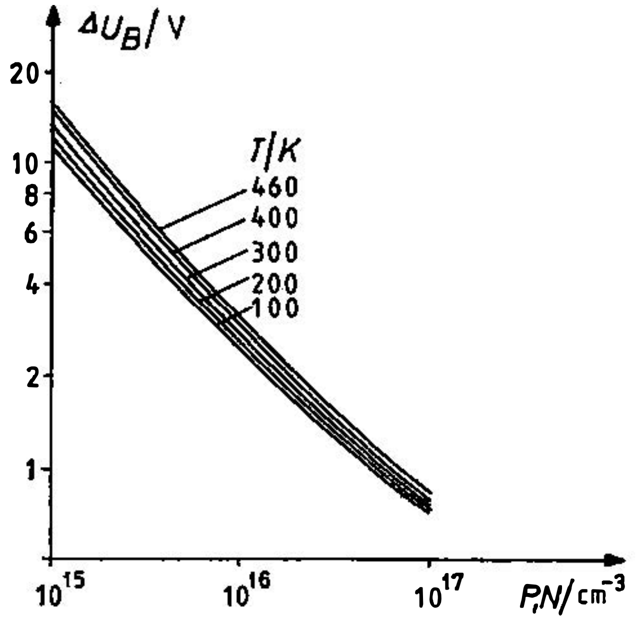

Fig. 4. Dependence ΔU_B on concentration of lower doped side (N, P) in complementary abrupt one-sided Si $p - n$ junctions.

of small impurity concentrations (Fig. 3). We note that in calculations an even more accurate expression was used for ionization coefficients (see (18)) in comparison with expressions used in Refs. 1, 2, 3, 4, 7 and 8.

On the basis of the results obtained it would be interesting to find an approximate analytical dependence $\Delta U_B(N, T)$ similary as by Ramović and Jevtić[9] for $\Delta U_B(N)$. It would enable a simpler analysis of working conditions of semiconductor components in the breakdown voltage region.

4. Conclusion

For a complementary structure of an arbitrary profile of impurities in n and p-region, it was shown generally (expression (14)) that the difference in breakdown voltages of asymmetric $p^+ - n$ and $n^+ - p$ structures will exist, if $k \equiv \frac{a_n}{a_p} \neq$ const. The sign of derivative $\mathrm{d}k/\mathrm{d}\eta$ determines which structure has a greater breakdown voltage.

Numerical results concern Si one-sided abrupt complementary junctions with a uniform distribution of impurities in the lower doped region of semiconductor in the range of impurity concentration of 10^{15} cm^{-3} to 10^{17} cm^{-3}. Making use of expressions for a_n and a_p which include temperature as a parameter, we were able to analyse ΔU_B vs. temperature. It was found that the differences of breakdown voltages increase with temperature increase, more quickly if the impurity concentration is lower. It was also found that between $U_{B\,N^+_-P}$ and $U_{B\,P^+_-N}$ an empirical relation (19) could be established with x varying from 0.04 to 0.07. The numerical results show (Table 1) that this x depends weakly on temperature.

In determining the multiplication factors M_n and M_p (at voltages smaller than the breakdown one), the attention is to be paid to whether one has $p^+ - n$ or complementary junction, to the asymmetry degree and to the temperature.

The results obtained are of particular importance for analyzing the characteristics of semiconductor devices where very asymmetric $p-n$ junction (avalanche photo-diode, high voltage diode and so on) are used.

Acknowledgements

We wish to express our thanks Mrs. Nada Obradović, for the translation of the text into English, to Mrs. Zorica Marković and Mr. Gradimir Petrić, for its technical presentation.

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ANALIZA KARAKTERISTIKA LAVINSKOG PROBOJNOG NAPONA KOMPLEMENTARNIH *p-n* SPOJEVA

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UDK 538.93

Originalni naučni rad

U radu je izvršena generalna analiza zavisnosti razlike između probojnih napona ΔU_B komplementarnih poluprovodničkih *p-n* spojeva u zavisnosti od odnosa koeficijenata jonizacije (a_n/a_p) elektronima (a_n) i šupljinama (a_p), od veličine električnog polja (E) i temperature (T). Izvršen je numerički proračun i prezentirani dobijeni rezultati za $\Delta U_B(T, N, P)$ kod Si komplementarnih spojeva sa strmim prelazom. Posebna pažnja je posvećena temperaturnoj zavisnosti. Nađena je i grafički prikazana promena razlike probojnih napona ΔU_B sa promenom temperature. Ta promena je dosta velika (do 30%), naročito za niže koncentracije slabije dopirane strane *p-n* spoja sa strmim prelazom.