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Superposition Methods for Topology Optimization for Non-Concurrent Loads

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Abstract: Problems with non-concurrent loads with variable load directions are often encountered in engineering. While dynamic topology optimization more realistically solves such problems, it is also much more computationally expensive and requires complex mathematical formulations and solvers. Hence, in this paper, we compared the performances of three superposition methods for fusing topologies obtained for various load directions. The top88 algorithm was used to generate 91 initial topologies using MATLAB, which were then joined via the superimposition method, weighted-density parameter, and the revolutionary superposition layout method. The performances of superposed topologies were then analysed by calculating their compliances for various load conditions via the finite element method also using MATLAB. The results have shown that the superimposition method and the weighted-density parameter method yield rather similar results. Both methods provided structures with lower total compliances compared to the revolutionary superposition layout method, making them more suitable for problems where stiffness is paramount.

Keywords: non-concurrent loads; superposition methods; topology optimization; variable load direction

1 INTRODUCTION

Topology optimization is a method enabling optimal distribution of material with respect to a given criterion, most often compliance of the structure [1]. In a general case, the domain within which the material is distributed is defined, along with support positions and load properties (position, direction, magnitude), and material volume fraction [2]. The problem is formulated in a way that it is necessary to find the material distribution that minimizes the compliance of the structure subjected to volume constraint and additional constraints (filled or void elements, supports) [3].

Many algorithms for topology optimization are available as open source and are most often based on the SIMP approach (solid isotropic material with penalization). SIMP is a density-based approach that transforms the discrete to continuous variables, reducing the problem complexity [4]. The majority of methods for topology optimization are still focused on static optimization in which the domain and loads are static. However, dynamic topology optimization, which considers time-dependent loads and dynamic responses, is gaining significance. It is particularly attractive in fields such as aerospace, automotive, and seismic engineering, where structures are subjected to dynamic forces [5].

Dynamic loading conditions are also encountered when optimizing mechanisms that are subjected to non-concurrent loads with variable directions, such as slider-crank systems or manipulator links [6]. While there were attempts to solve such problems by considering only the static load in the worst-case position [7], such an approach results in limited design quality. Hence, methods accounting for multiple mechanism positions were developed and are mostly based on superposing the results of multiple static topology optimization processes [8–10].

Srinivas and Javed [8, 9] proposed the superimposition method to improve the structural performance of manipulators subjected to varying operational conditions. The method joins topologies optimal for different load cases into a unified topology [9]. The same authors proposed the weighted-density parameter approach which fuses multiple optimized topologies based on the element-wise sensitivity and volume fraction [11]. Element sensitivities are measured by summing the density parameters of each optimal topology for all the elements. Finally, Alkalla et al. [10] proposed the revolutionary superposition layout (RSL) method which combines multiple independent optimal designs to generate a topology that will meet the performance criteria of all loads.

In this article, the authors compared the available approaches to superposition of topology-optimized structures subjected to variable load directions. Firstly, the optimal topologies for each step of the observed case problem, in which the load direction was varied, were obtained via top88 [12]. Next, topologies were joined using three available approaches – the superimposition method [9]. weighted-density parameter [11], and the revolutionary superposition layout (RSL) [10]. The compliances and volume fractions were calculated for each of the outputs and were compared.

While previous studies have explored individual methods, a direct comparative analysis of their performance and structural effectiveness was lacking. By implementing these methods on a benchmark problem and evaluating their compliance results, this work offers valuable insights into their relative advantages and limitations. The findings contribute to the broader understanding of superposition-based topology optimization and provide practical guidance for selecting appropriate methods based on structural and computational considerations.

2 CASE STUDY

The example structure used in a case study is a structure supported on the left side (encastre, all degrees of freedom fixed) and subjected to a concentrated load F = 100 on the right side (no unit; see Section 3.4). The load is applied in the middle of the right domain edge and has variable direction. The direction was varied from 0° (horizontal load action) to 90° (vertical). Topology optimizations were carried out at load increments of 1° which resulted in a total of 91 optimized geometries. The domain size and shape, applied supports, and load are shown in Fig. 1.

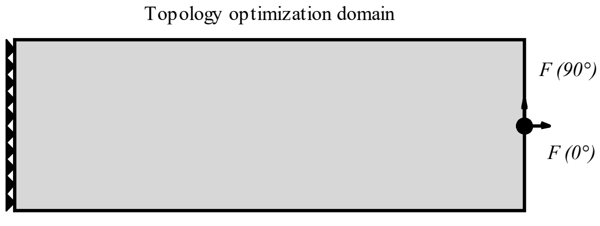


Figure 1 Example problem – domain size, support locations, and load positions

The domain size was 300 × 100 elements, and the topology optimization was carried out using the top88 algorithm [12]. The algorithm settings were as follows – volume fraction was 0.3, penalty factor was 3, as well as the minimum filter radius $r_{\min}$. The density-based filtering method was used with a modified SIMP approach. The move factor was 0.2 and the maximum number of iterations for each load position was set to 100. Additionally, the Poisson ratio of the selected material was taken as 0.3.

The optimal topologies were obtained for all 91 cases and were saved in one 3D matrix with a size of 91 × 100 × 300. Said 3D matrix was denoted as *distributions* and was used as input in the following steps.

3 METHOD

Four methods for superposition of topology-optimized structures for non-concurrent loads were compared (methods are briefly outlined in sections 3.1 to 3.3). To do so, it was first necessary to obtain optimal topologies for the observed case found according to the process outlined in Section 2. The matrix *distributions* was used as input for all of the compared methods.

Since methods for superpositioning topologies generally result in greater volume densities compared to the base topologies, their volume fractions were limited to volume fraction v_f = 0.5. For this reason, aiming to facilitate the comparison between the results of superposed topologies and topologies for specific loads, optimal topologies were also obtained using v_f = 0.5 for load angles 0°, 30°, 60°, and 90°.

3.1 Superimposition Method

The method outlined in [8] was used to implement the superimposition method. First, optimal topologies contained in the matrix *distribution* were loaded and *superimposition* MATLAB function was carried out. The function summed the densities of all the elements along the first dimension (number of topologies).

After the densities of elements across topologies were summed up, normalization to a scale [0, 1] was carried out to ensure that the realistic material density was used. Next, the target threshold needed to achieve the previously selected volume fraction was calculated. The final design was obtained after eliminating all the elements below the target threshold. The mathematical model for the superimposition method used in this paper is given as [8]:

$$\boldsymbol{T}_{\text{sup}} = \sum_{k=1}^{91} \boldsymbol{T}_k, \quad \boldsymbol{T}_{\text{nor}} = \frac{1}{91 \cdot v_{\text{f}}} \boldsymbol{T}_{\text{sup}}, \quad \boldsymbol{T}_{\text{RP}} = \left[\boldsymbol{T}_{\text{nor}} \right]^{\tau}, \tag{1}$$

where $\boldsymbol{T}_k$ is the matrix containing element densities of k^{th} optimal topology, k is the number of topologies (k = 91 in this paper), $\boldsymbol{T}_{\text{sup}}$ is the matrix containing $\boldsymbol{T}_{\text{nor}}$ is the matrix containing the normalized topologies,

It should be noted that the additional image processing was omitted in this paper due to space constraints.

3.2 Weighted-Density Parameter

The weighted density method was outlined in [11] and the associated mathematical model was given in Eq. (2). To carry out the weighted-density parameter approach, it is first necessary to determine the weight number of each element W_{eij}. Its value is calculated by summing the density values of specific elements for all optimal topologies x_{eij}, where e is the topology and i and j represent the number of rows and columns, respectively. The numerical factor T is calculated based on the number of topologies that are being superposed and the volume fraction. Finally, binary density values are obtained after comparing the element weights; these larger than 0 were taken as material.

$$W_{eij} = \frac{\sum_{k=1}^{91} T_{kij} - T}{T}, \quad T = 91 \cdot \left(1 - v_{\text{f}}\right), \quad x_{eij} = \begin{cases} 1 & W_{eij} \geq 0 \\ 0 & W_{eij} < 0 \end{cases} \quad i = 1, 2, ..., 100; j = 1, 2, ..., 300. \tag{2}$$

3.3 Revolutionary Superposition Layout

The RSL method [10] is a systematic approach for optimizing structures subjected to non-concurrent multi-load conditions. As was the case in previous two methods, the optimal topology is first determined for individual load cases separately, using any topology optimization algorithm, and the final design is obtained by combining them.

Since the method was intended to be used when there are up to three non-concurrent loads, three cases were used to create superposed topology. These were structures obtained for 0° ($\boldsymbol{D}_0$), 45° ($\boldsymbol{D}_{45}$), and 90° ($\boldsymbol{D}_{90}$) load angles. The mathematical model is given in [10] as:

$$\boldsymbol{D}_{\text{opt}} = \boldsymbol{D}_0 \cup \boldsymbol{D}_{45} \cup \boldsymbol{D}_{90} = \sum_{i=1}^{n} \max_{j=1}^{m} \left\{ x_{ij} \right\}, \tag{3}$$

where $\boldsymbol{D}_{\text{opt}}$ is the final resultant optimum design, $m = 3$ is the number of concurrent loads being observed, and x_{ij} is the highest relative density of element i at any of load cases j.

The method takes the maximum density across optimal topologies for each element. Next, densities were normalized to the interval [0, 1] and thresholding was carried out to ensure that the volume fraction was 0.5.

3.4 Finite Element Method

The finite element method (FEM) was used to analyse the obtained superposed topologies. The FEM analysis used in this paper was implemented in MATLAB. Each finite element was taken as a linear quadrilateral element. Young moduli of elements with material were taken as $E = 1$, while empty elements were assigned value $E_{\min} = 10^{-9}$ to avoid the stiffness matrix from becoming singular. Since no units were assigned to Young moduli, no unit was applied to the concentrated load F. The Poisson's ratio was taken as 0.3 and the material was assumed to be homogenous and isotropic.

The finite element method carried out in this paper was based on the implementation provided in [12]. The domain was comprised of 30000 elements with a total of 60802 degrees of freedom. The applied boundary conditions and loads were identical to those presented in Section 2.

4 RESULTS AND DISCUSSION

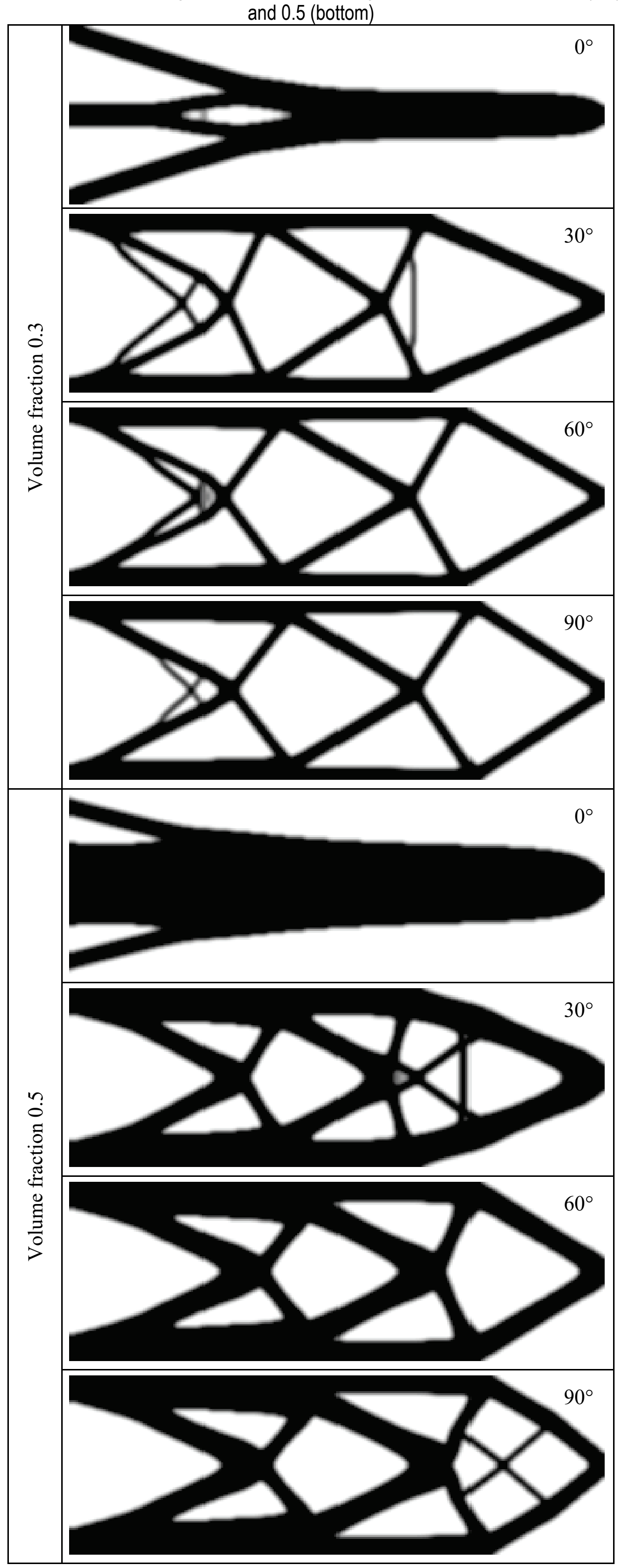
A total of 91 optimal topologies were obtained for the considered case. The optimization was carried out using MATLAB 2010 running on a PC with Intel i5-8500 CPU at 3.00 GHz and 32 GB of RAM. For illustration purposes, four optimal topologies obtained for load angles of 0°, 30°, 60°, and 90° and volume fraction $v_f = 0.3$ are shown in Tab. 1.

The approximate computational time needed to carry out one optimization was determined using the tic and toc functions native to MATLAB. The time needed to optimize one topology was 44.16 to 49.73 s. On the other hand, carrying out the FEM simulation takes approx. 0.68 s. Lastly, the superposition of optimal topologies was practically instantaneous, so computational times were not calculated.

As shown in Tab. 1, regardless of the volume fraction, solutions obtained for lower angle loads tend to have lower distances between the normal line and the material placement compared to those obtained for larger angles. Such material distribution is more favourable with respect to compression and tension. On the other hand, taller designs fare better when subjected to bending stresses.

When comparing the effects of the volume fraction on the material distributions, the similarities are clear. As the load action angle increases, so does the amount of material placed adjacent to the top and bottom domain edges. The solutions obtained for the same load angle have rather similar outer contours.

Table 1 Optimal topologies obtained for load angles and volume fraction 0.3 (top) and 0.5 (bottom)



It was also observed that the compliance starkly increased as the load application angle was increased, i.e., depending on the type of loading. Under bending (90° load angle), compliance is primarily governed by the flexural stiffness, which depends on the moment of inertia and elastic modulus. When subjected to pure tension (0° angle), the compliance is dictated by axial stiffness, favouring a direct load-carrying path with material distributed along the force direction. Consequently, as observed in the results, topology optimization for bending results in truss-like or beam-reinforced structures, whereas tension-driven optimization produces streamlined, load-aligned designs. For cases where both bending and tension occur, the optimized topology integrates these characteristics, balancing axial stiffness with flexural resistance to achieve an efficient load-bearing structure.

4.1 Superposed Solutions

Optimal topologies obtained for load angles from 0° to 90° and volume fraction 0.3 were superposed using the three above-listed methods. The resulting material distributions are shown in Tab. 2

As can be seen from Tab. 2, superposed topologies obtained using the superimposition method and the weighted-density parameter approach are practically identical. This is due to their approach, which relies on applying a threshold value to the normalized sum of superposed topologies. Both topologies are rather similar to that obtained for load at a 60° angle at volume fraction 0.5 (shown in Table 1), with minor differences, with the key difference being additional holes near the base.

On the other hand, the structure obtained using the revolutionary superposition layout is rather different compared to the original topologies. It also utilizes thin streaks of material, which makes the piece more complex for the manufacturer. Additionally, there are small areas with no material located within the solid domain areas, possibly implying the existence of a checkerboard problem.

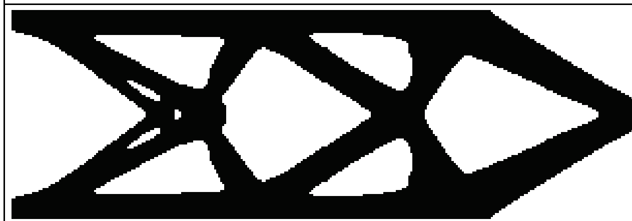
Next, the compliances of the three superposed topologies were compared to compliances of original topology optimized structures at angles 0°, 30°, 60°, and 90° obtained for volume fractions 0.3 and 0.5. The compliances were calculated using the finite element method outlined in Section 3.4. and the results are shown in Tab. 3. The naming convention for the original topology-optimized structures indicates the load angle used during optimization and the corresponding volume fraction. For instance, the structure labeled "Angle 30°, $v_f = 0.3$" was optimized for a load applied at 30° with a volume fraction of 0.3.

As expected, compliances between the superimposition method and the weighted-density parameter approach are identical. When compared to the results of the RSL method, it can be seen that the RSL method yields better results when the load is applied at angles of 0° and 15°. As the load angle increases, the superimposition method and the weighted-density parameter approach have better results.

However, unexpected results were obtained as the structure obtained for 30° angle and volume fraction 0.5 gives better results than all the superposed topologies, except for RSL at 0° angle. Such results imply that there is no need for superposition, as one of the original topologies is shown to have better results. This might be significant since, even though the superposition itself has a low computational cost, generating the necessary topologies is very resource-intensive. Similar results were obtained for the 60° angle structure ($v_f = 0.5$), which had lower compliances than all the superposed topologies at angles of 30° and over.

Table 2 Superposed topologies obtained using three different methods

Superimposition method



Weighted-density parameter

Revolutionary superposition layout

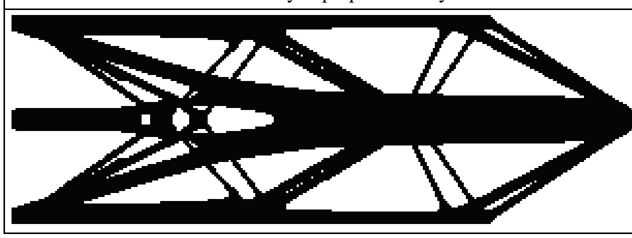


Table 3 Compliances of structures depending on the load angle applied to them (value divided by 10^5 for formatting reasons)

The method used to obtain the structure	Load angle						
	0°	15°	30°	45°	60°	75°	90°
Superimposition	1.37	2.59	5.91	10.5	15	18.3	19.5
Weighted-density	1.37	2.59	5.92	10.5	15	18.3	19.5
RSL	1.04	2.54	6.62	12.2	17.8	21.8	23.3
Angle 0°, $v_f = 0.3$	1.41	15.4	53.5	106	158	196	210
Angle 30°, $v_f = 0.3$	2.6	4.53	9.78	17	24.1	29.4	31.3
Angle 60°, $v_f = 0.3$	3.7	5.47	10.3	16.9	23.6	28.4	30.2
Angle 90°, $v_f = 0.3$	4.38	6.1	10.8	17.2	23.6	28.3	30
Angle 0°, $v_f = 0.5$	0.896	5.01	16.3	31.6	47	58.2	62.3
Angle 30°, $v_f = 0.5$	1.25	2.4	5.53	9.82	14.1	17.2	18.4
Angle 60°, $v_f = 0.5$	1.81	2.89	5.85	9.89	13.9	16.9	18
Angle 90°, $v_f = 0.5$	2.36	3.42	6.32	10.3	14.2	17.1	18.2

4.2 Limitations

The limitations of the study at hand should be addressed. The study at hand was also focused on the 2D space, which is not true for most engineering problems. While all the methods can be easily extended to 3D space, doing so would significantly increase the computational cost. Increasing the dimensionality to 3D space would introduce an additional degree of freedom to each node, greatly expanding the global stiffness matrix. Since topology optimization is an iterative process and since it is necessary to generate a large number of topologies for different load angles, it might be necessary to either reduce the domain size or use parallel computing functions.

Moreover, the obtained results are only illustrative, as the more thorough comparison warrants using multiple examples to confirm the robustness of the findings. Additional examples should be introduced with various types of loads domain sizes and shapes. Finally, the application of various filters on the superposed results was not considered in this paper. Introducing the use of filters will result in increased manufacturability of specimens.

5 CONCLUSION

This study compared three superposition methods—Superimposition, Weighted-Density Parameter, and Revolutionary Superposition Layout (RSL)—for topology optimization under non-concurrent loads with variable load directions. The methodology involved generating optimal topologies for 91 different load angles using the top88 algorithm and subsequently combining them using the three superposition approaches. The performance of the resulting structures was evaluated in terms of compliance under various loading conditions using the finite element method.

The results demonstrated that the Superimposition and Weighted-Density Parameter methods produce nearly identical topologies, both achieving lower compliance values compared to the RSL method in most loading scenarios. However, the RSL approach exhibited superior stiffness when the load direction was aligned with the structure's primary load-carrying path (e.g., at 0°). Interestingly, the optimal topology for a single load angle of 30° and volume fraction of 0.5 outperformed all superposed topologies in several cases, suggesting that worst-case load conditions might be a viable alternative to superposition in some applications.

These findings indicate that while superposition methods provide a computationally efficient alternative to full dynamic topology optimization, their effectiveness depends on the specific loading conditions. Furthermore, the study highlights the need for further research into hybrid approaches that incorporate adaptive volume constraints and filtering techniques to refine superposed topologies.

Future work will focus on extending these methods to three-dimensional structures and applying them in multi-body system optimization, where topology changes due to motion and load variation are more complex. Additionally, investigating the integration of superposition methods with gradient-based optimization strategies could enhance their adaptability and computational efficiency in more complex engineering problems.

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