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THE POLARIZATION OF MONOCHROMATIC LIGHT AFTER REFLEXION ON A PLANPARALLEL PLATE

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The states of the polarization of light are calculated for the light reflected from a planparallel plate when the incident light is linearly polarised. The reflected light is generally elliptically polarized. The azimuth and ellipticity of this light are calculated for plates of a uniaxial crystal und applied to a special case of isotropic plates.

1. Introduction

It has been shown[1,2] that after the transmission of linearly polarized monochromatic light through a planparallel plate of a uniaxial crystal the transmited light is generally elliptically polarized and that the azimuth of greater oscillation ellipse axis as well as the ellipticity varies with incident light angle. Even at small incident angles the changes are significant. On reaching the plate one part of the light is reflected and it is of interest to analyze the changes of the polarization after reflection of monochromatic light on the planparallel plate. This problem is discussed in the present paper. The calculation is carried out for a planparallel plate of uniaxial crystal and applied to a special case of isotropic plates.

2. Formulation of the problem

It is assumed that the incident light is monochromatic with the wavelength λ and amplitude 1 and is linearly polarized with the azimuth of oscillation ψ_i. The light has two components, the parallel one

$$E_{ip} = \cos \psi_i \tag{1a}$$

and the normal one

$$E_{in} = \sin \psi_i. \tag{1b}$$

The component of reflected light is proportional to the incident light with the reflection factor r

$$E = r E_i. \tag{2}$$

This factor was calculated by Wolter[3] and for vacuum as the incident and transmited medium it reads

$$r = \frac{(g_0^2 - g_1^2)\exp i\varphi + (g_1^2 - g_0^2)\exp(-i\varphi)}{(g_0 + g_1)^2 \exp i\varphi - (g_1 - g_0)^2 \exp(-i\varphi)}. \tag{3}$$

By introducting

$$U = \frac{1}{2}\left(\frac{g_0}{g_1} - \frac{g_1}{g_0}\right) \tag{4a}$$

$$R = \frac{1}{2}\left(\frac{g_0}{g_1} + \frac{g_1}{g_0}\right) \tag{4b}$$

Eq. (3) can be cast in the form

$$r = \frac{iU}{\operatorname{ctg}\varphi + iR}. \tag{5}$$

Two waves propagate in a planparallel uniaxial plate: the ordinary one (index o) and the extraordinary one (index e), and their propagation depends on the optical axis. The simplest case is when the optical axis is parallel to the sides of the plate. The optical axis is normal to the incident plane for all angles of the incident light, and the extraordinary wave has always the refractive index n_e. The ordinary wave has always the same refractive index n_0. Due to this position of the optical axis, the ordinary wave oscillates parallel to the incident plane and the extraordinary wave oscillates normal to incident plane.

The incident angle is denoted by α, and the angles of the light propagation in plate by β_0 and β_e. In accordance with Wolter the equation for g_0 reads

$$g_0 = \cos \alpha \tag{6}$$

and for g_1 there are two values

$$g_{1p} = \frac{\cos \beta_p}{n_p} = \frac{\cos \beta_0}{n_0} \tag{7a}$$

$$g_{1n} = n_n \cos \beta_n = n_e \cos \beta_e \tag{7b}$$

and Eq. (4) gives

$$\left.\begin{matrix} U_0 \\ R_0 \end{matrix}\right\} = \frac{1}{2}\left(\frac{n_0 \cos \alpha}{\cos \beta_0} \mp \frac{\cos \beta_0}{n_0 \cos \alpha}\right) \tag{8a}$$

$$\left.\begin{matrix} U_e \\ R_e \end{matrix}\right\} = \frac{1}{2}\left(\frac{\cos \alpha}{n_e \cos \beta_e} \mp \frac{n_e \cos \beta_e}{\cos \alpha}\right). \tag{8b}$$

In Eq. (3) the value φ is defined by

$$\varphi_{0,e} = \frac{2\pi}{\lambda} d\, n_{0,e} \cos \beta_{0,e} \tag{9}$$

where d is the thickness of the planparallel plate.

3. *Theoretical analysis*

If Eq. (5) is substituted into Eq. (2), the components of the reflected light read:

$$E_p = (R_0 + \mathrm{i} \operatorname{ctg} \varphi_0) \frac{U_0 \cos \psi_t}{R_0^2 + \operatorname{ctg}^2 \varphi_0} \tag{10a}$$

$$E_n = (R_e + \mathrm{i} \operatorname{ctg} \varphi_e) \frac{U_e \sin \psi_t}{R_e^2 + \operatorname{ctg}^2 \varphi_e}. \tag{10b}$$

These components are complex due to fact that reflected light is elliptically polarized, and the basic parameters of polarization have to be determined.

If $A_{p,n}$ are the amplitudes and $\delta_{p,n}$ the phases of the reflected light, the complex amplitude of the light can be written in the form $E_{p,n} = A_{p,n} \exp(\mathrm{i}\, \delta_{p,n})$ and in comparison with (10) this gives

$$\left.\begin{aligned} A_p \cos \delta_p &= \frac{U_0 R_0 \cos \psi_t}{R_0^2 + \operatorname{ctg}^2 \varphi_0}, \\ A_p \sin \delta_p &= \frac{U_0 \operatorname{ctg} \varphi_0 \cos \psi_t}{R_0^2 + \operatorname{ctg}^2 \varphi_0}, \end{aligned}\right\} \tag{11a}$$

$$\left.\begin{aligned} A_n \cos \delta_n &= \frac{U_e R_e \sin \psi_t}{R_e^2 + \operatorname{ctg}^2 \varphi_e}, \\ A_n \sin \delta_n &= \frac{U_e \operatorname{ctg} \varphi_e \sin \psi_t}{R_e^2 + \operatorname{ctg}^2 \varphi_e}. \end{aligned}\right\} \tag{11b}$$

The basic parameters of the elliptical polarization are the azimuth ψ of the greater axis of the ellipse and the ellipticity ϑ defined by

$$\operatorname{tg} \vartheta = b/a \tag{12}$$

where b and a are half-axes of the ellipse. These parameters are given by[4]:

$$\operatorname{tg} 2\psi = \operatorname{tg} 2\xi \cos \delta \tag{13}$$

$$\sin 2\vartheta = \sin 2\xi \sin \delta \tag{14}$$

with

$$\operatorname{tg} \xi = A_n/A_p \tag{15}$$

$$\delta = \delta_p - \delta_n. \tag{16}$$

Eqs. (13) and (14) can be cast into the form

$$\operatorname{tg} 2\psi = \frac{2}{A_p^2 - A_n^2} (A_p \cos \delta_p \, A_n \cos \delta_n + A_p \sin \delta_p \, A_n \sin \delta_n), \tag{17}$$

$$\sin 2\vartheta = \frac{2}{A_p^2 + A_n^2} (A_p \sin \delta_p \, A_n \cos \delta_n - A_p \cos \delta_p \, A_n \sin \delta_n). \tag{18}$$

Application of Eq. (11) and the use of

$$V = U_e/U_0 \tag{19}$$

gives the following expressions:

$$\operatorname{tg} 2\psi = \frac{2V \operatorname{tg} \psi_i (R_0 R_e + \operatorname{ctg} \varphi_0 \operatorname{ctg} \varphi_e)}{R_e^2 + \operatorname{ctg}^2 \varphi_e - V^2 \operatorname{tg}^2 \psi_i (R_0^2 + \operatorname{ctg}^2 \varphi_0)}, \tag{20}$$

$$\sin 2\vartheta = \frac{2V \operatorname{tg} \psi_i (R_e \operatorname{ctg} \varphi_0 - R_0 \operatorname{ctg} \varphi_e)}{R_e^2 + \operatorname{ctg}^2 \varphi_e + V^2 \operatorname{tg}^2 \psi_i (R_0^2 + \operatorname{ctg}^2 \varphi_0)}. \tag{21}$$

These equations define the basic parameters for elliptical polarization of the reflected light.

4. *Discussion*

The position on the greater axis of the ellipse of oscillation is calculated by using Eq. (20). We have calculated numerically the dependence of the position of the greater axis on the incident angle. As an example we take a planparallel calcite plate of thickness $d = 2$ mm that has a refractive index $n_0 = 1.6561$ and $n_e = 1.4856$

for the wavelenght of He-Ne LASER $\lambda = 633$ nm. The optical axis of calcite is parallel to the sides of the plate. The incident light is linearly polarized and the azimuth of the oscillation is $\psi_i = 30°$. The results are given for incident angles in the interval $\alpha = 0°$ to $\alpha = 5° = 300'$ with steps of 1'. Fig. 1 is a graphical re-

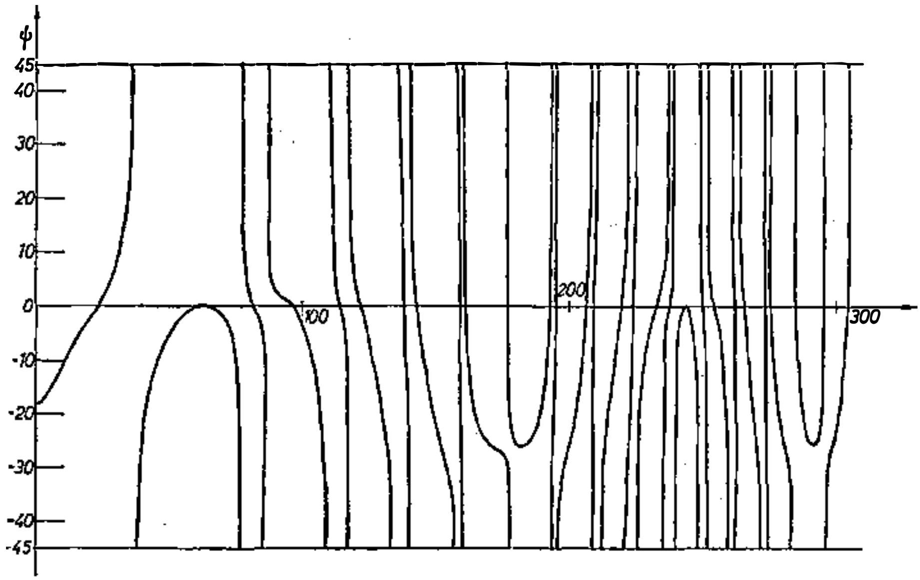

Fig. 1. Dependence of the azimuth of the greater axis of ellipse with incident angle in reflected light.

presentation of the numerical analysis, and it consists of a large number of curves that lie between $-45°$ and $45°$. At these boundaries, the curves jump from one to the other boundary. This behaviour is a result of numerical calculation for which there is no physical interpretation. For example, when the azimuth exceeds 45° than 2ψ becomes larger then 90° and the tangent becomes negative. When $2\psi > 90°$ the result of Eq. (20) is positive again and above 135° the result is negative. With that fact in mind, there is no jump and the rotation of the greater axis is continuous as is shown in Fig. 2. We have determined the points on the circle for every azimuth step of 5', and every point denotes the incident angle in minutes that gives the azimuth. The direction of the greater axis of the oscillation ellipse is determined by connection of point for an incident angle with the centre, as that is shown for the incident angle $\alpha = 0°$.

The positions of the points show that ellipse is rotating in the positive direction, but for incident angles between 60' and 65' the rotation stops and turns into negative direction. That rotation exceeds a little one and half revolutions and at 175' again starts in the positive direction and continues analogously. Fig. 1. shows that in the interval up to 300' the rotation of the greater axis of the ellipse changes the direction four times altogether. Therefore, in reflected light the greater axis of the oscillation ellipse rotates in one or an other direction, which differs essentially from he case of the transmitted light. For comparison we have calculated in the trans-

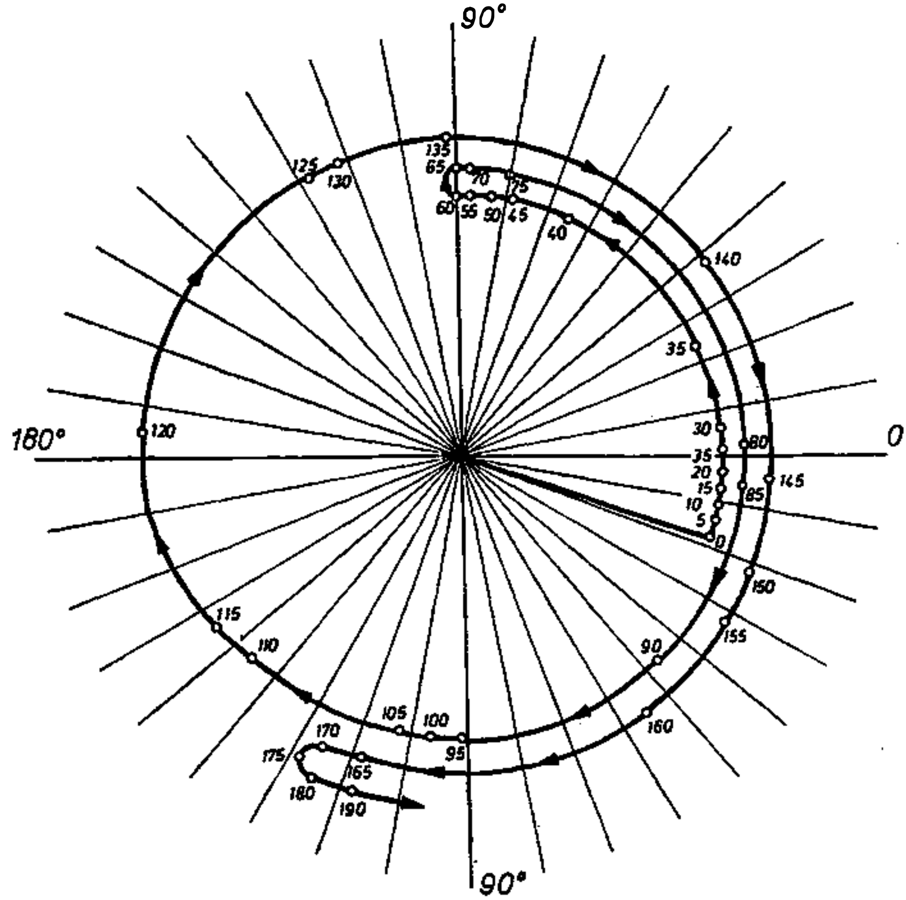

Fig. 2. Position of the greater axis of the oscillation ellipse in reflected light.

mitted light for the same plate by using the approach presented in Ref. 2, and the results are given in Fig. 3. It is shown that the azimuth ψ is allways between $-45°$ and $45°$ and never exceeds these values. The azimuth exhibits an oscillatory behaviour composed of two oscillations; one with a large period and amplitude and other with a small period and amplitude. This shows that the changes of the greater axis of the oscillation ellipse azimuth of transmitted light is quite different from that of the reflected light.

The second basic parameter of the polarization is its ellipticity. This is calculated from Eqs. (12) and (21). The calculation is performed for the example given and the results are shown graphicaly in Fig. 4. The curve shows the changes of ellipticity as a function of the incident angle and the curve resembles a modular oscillation. Due to this the curve intersects several times the abscissa axis. The ellipticity for these incident angles is equal to zero and the reflected light is linearly polarized. Each change of the ellipticity sign means the change of direction of rotation of the oscillation ellipse.

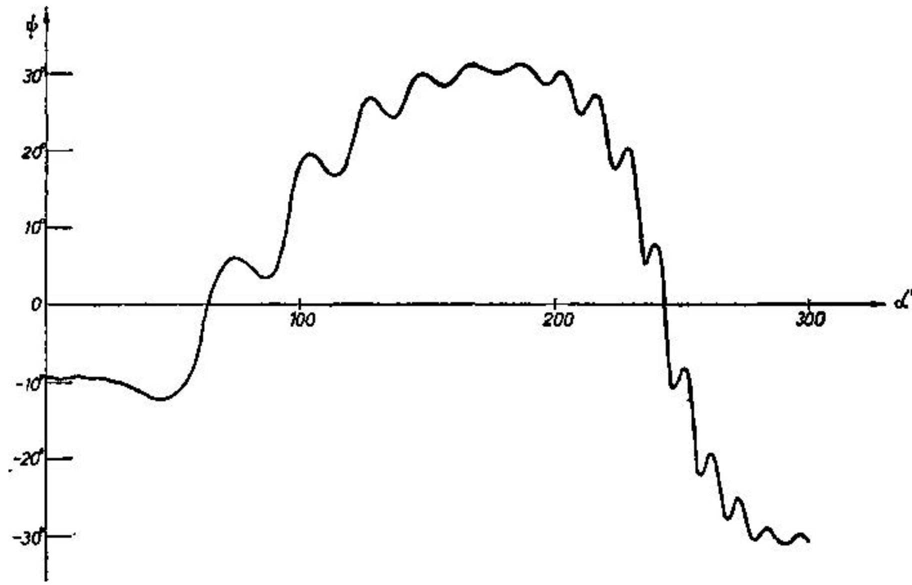

Fig. 3. Dependence of the azimuth of the ellipse greater axis with incident angle in transmitted light.

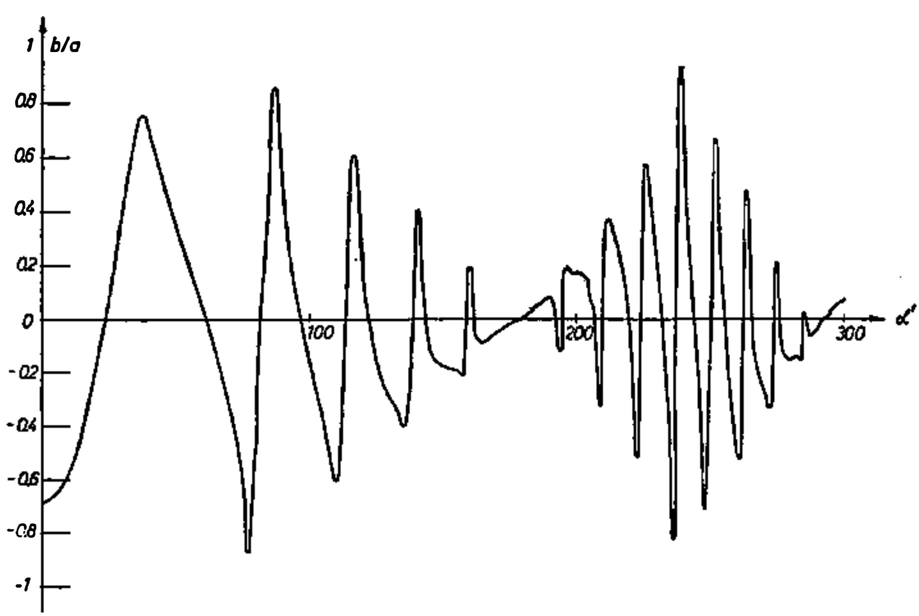

Fig. 4. Dependence of the ellipticity with incident angle in reflected light.

5. The optical axis of the plate is in the incident plane

The crystal plate is cut to have the optical axis normal to the parallel sides, and the optical axis always lies in the incident plane. In that case the ordinary wave oscillates normal to the incident plane and the extraordinary wave oscillates parallel to the incident plane. This case is opposite to the previous one, and the results can be obtained by an interchange of indices o and e in Eqs. (20), (21) and (8). It should be noted that the refractive index of the extraordinary wave n_e changes with the incident angle in accordance with the rules of crystal optics.

6. Isotropic plate

The phenomenon of isotropic plate is obtained as a special case of the crystal plate for

$$n_0 = n_e = n \tag{22}$$

from which follows

$$\beta_0 = \beta_e = \beta, \qquad \varphi_0 = \varphi_e = \varphi. \tag{23}$$

In this case Eq. (8) assume the form

$$\left.\begin{matrix} U_p \\ R_p \end{matrix}\right\} = \frac{1}{2}\left(\frac{n\cos\alpha}{\cos\beta} \mp \frac{\cos\beta}{n\cos\alpha}\right), \tag{24a}$$

$$\left.\begin{matrix} U_n \\ R_n \end{matrix}\right\} = \frac{1}{2}\left(\frac{\cos\alpha}{n\cos\beta} \mp \frac{n\cos\beta}{\cos\alpha}\right), \tag{24b}$$

and Eqs. (19), (20) and (21) become:

$$V = U_n/U_p, \tag{25}$$

$$\operatorname{tg} 2\psi = \frac{2V \operatorname{tg}\psi_i (R_n R_p + \operatorname{ctg}^2\varphi)}{R_n^2 + \operatorname{ctg}^2\varphi - V^2 \operatorname{tg}^2\psi_i (R_p^2 + \operatorname{ctg}^2\varphi)}, \tag{26}$$

$$\sin 2\vartheta = \frac{2V \operatorname{tg}\psi_i (R_n - R_p)\operatorname{ctg}\varphi}{R_n^2 + \operatorname{ctg}^2\varphi + V^2 \operatorname{tg}^2\psi_i (R_p^2 + \operatorname{ctg}^2\varphi)}. \tag{27}$$

It can be seen that the reflected light from an isotropic plate is generally elliptically polarized.

In the case of light at normal incidence, one has:

$$\alpha = \beta = 0$$

and Eq. (24) takes on the form:

$$\left.\begin{matrix} U_p \\ R_p \end{matrix}\right\} = \frac{1}{2}\left(n \mp \frac{1}{n}\right),$$

$$\left.\begin{matrix} U_n \\ R_n \end{matrix}\right\} = \frac{1}{2}\left(\frac{1}{n} \mp n\right).$$

Due to this:

$$R_p = R_n = R, \qquad U_p = -U_n, \qquad V = -1,$$

and Eqs. (26) and (27) give

$$\psi = -\psi_i, \qquad \vartheta = 0. \tag{28}$$

In this case the reflected light is linearly polarized and the azimuth of the incident light is negative.

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POLARIZACIJA MONOHROMATSKE SVJETLOSTI NAKON REFLEKSIJE NA PLANPARALELNOJ PLOČICI

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Određeno je stanje polarizacije svjetlosti nakon odbijanja na planparalelnoj pločici od jednoosnog kristala. Pokazano je da je odbijena svjetlost općenito eliptički polarizirana i određeni su azimut i eliptičnost takve svjetlosti. Opći rezultati su specijalizirani na pločice od izotropnih materijala.