YU ISSN 0015-3206
FZKAAA 18 (2) 161 (1986)

ALPHA DECAY WIDTHS OF HEAVY NUCLEI

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Received 1 September 1985

Revised manuscript received 5 May 1986

UDC 539.164

Original scientific paper

Previous treatments of α-decay widths of rare-earth nuclei are extended to those α-spectra which contain transitions with α-angular momentum $L = 0$. As before, the barrier is taken to be the non-local α-nucleus potential with exchange term, superposed on the usual Coulomb potential and reduced widths hF_L calculated. It is found that for almost all odd parity transitions $F_L/F_0 \geqslant 1$ as expected and for even parity transitions F_L may be less than F_0. The results are discussed.

1. Introduction

Study of α-decay widths has engaged much attention[1-5] as it is an important informative factor about nuclear structure. Theoretical formulas for α-decay widths are derived by using overlap of shell model wave functions[1-3]. The α-decay reduced widths can also be calculated directly from the relationship:

$$F_L = \lambda_L / P_L \tag{1}$$

where λ_L is partial decay constant for the mode of decay involving α-angular momentum L and F_L and P_L being corresponding internal transition probability and penetrability factor, respectively. However, since neither of the factors F_L

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and P_L is experimentally measurable, assumption has to be made on either of them to obtain information about the other. In shell model calculations of α-decay widths, the barrier is taken to be either purely electrostatic or that superposed by a static α-nucleus potential. On the other hand, the momentum dependence of nulcear force has long been known from scattering experiments and other sources. From the above considerations it was suggested[6] that in the treatment of α-decay also, the α-nucleus potential should be represented by an appropriate non-local potential. This view point is also confirmed from the works of Mang and Fliessbach[1,7] who have shown that the relative motion of the α-particle and product nucleus obeys non-local equation. Further, it is known[8] that the exchange of nucleons between two nuclei involves non-local interaction. Also, the effect of non-locality has been demonstrated[9] in other nuclear phenomena. However, to avoid the mathematical complications involved in the Schrödinger equation with non-local potential prescriptions have been made for a local approximation of non-local interaction. But in the one body model of α-decay theory, WKB solution of integro-differential Schrödinger equation is a reasonably good approximation. Further justification of this approach lies in the fact that the calculated values of penetrabilities with non-local barrier largely reproduce observed decay data consistently over wide range of nuclei in trans-lead[12], trans-uranium[13] and rare-earth[14] regions together with F_L/F_0 values within reasonable range[14] as expected from other sources such as studies of α-spectroscopic amplitudes[2], while on the other hand Coulomb barrier or that superposed by static α-nucleus potential leads to results in much disagreement[15].

Poggenburg et al.[16] pointed out the limitations of shell model calculations for higher angular momentum. Watt et al.[17] questioned the method of evaluating α-spectroscopic amplitudes by finding the overlap of the wave functions of the product and α-particle each of which is taken to be of harmonic oscillator type and the ambiguities of optical model potential parameters are well known[18-21]. Soinski et al.[22] found that the experimental data of ^{253}Es and ^{255}Fm for higher α-wave cannot be accounted for by coupled channel treatment and assumed non-zero projections ($m_L \neq 0$) of orbital angular momenta along the nuclear symmetry axis. Since sufficient data are not available to determine $m_L \neq 0$ amplitudes, they assumed that the escape of an α-particle makes the principle axis of the core to tilt. The $m_L \neq 0$ components are than generated by rotating the coordinate system. However, here too, the experimental data for hindrance factors for neighbouring even-even nuclei have been used for calculating tilt angle.

Winslow[23] used level spacing approach to determine α-formation factor with a square well as barrier encountered in α-decay but the level spacing approaches have not been very satisfactory, the observed relative intensities are not explained in these approaches.

From what has been stated above, it is evident that one of the essential factor that emerges in α-decay is the choice of a realistic α-nucleus potential. In recent years some progress have been made in generating nucleus-nucleus potential from nucleon-nucleon potential by single folding model[24-27] and double folding model[28]. However, a common difficulty with both these methods are that the effects of antisymmetrization and saturation are not taken care of. Fleckner and Mosel[29] have demonstrated the effect of antisymmetrization by using Skyrme interaction[30]. Sinha[31] has estimated antisymmetrization effect by folding in a two body effective interaction with Slater density matrix of one of the nuclei and the density of the other. Eisen and Day[32] used Slater density matrix of both the nuclei. However, both the methods require correction in finite nuclear matter.

Recently some progress have been made for α-decay absolute intensity calculations[33,20,18]. Furman et al.[18] considered the effect of finite size of α-particle on absolute α-decay widths and found that these effects are very sensitive on the choice of shell model configurations and it was also pointed out that the results given therein must be taken qualitatively. To summarise the results in this line it can be stated that the problems of α-formation probability and α-nucleus potential have not yet been fully resolved.

In view of these, it would be of interest to check to what extent the non-local formalism reproduces observed decay data. It has already been shown[34] that the type of non-locality used by Frahn and Lemmer[35] leads to too large change of penetrability factor indicating momentum dependence in α-nucleus potential is somewhat different than that envisaged in effective mass approximation. Also it is known[36] that the potential used by Brueckner and Gammel[37] yields binding energies of ^{3}H and ^{3}He which are not consistent with experimental data. Hence, an alternative form of α-nucleus potential was suggested[12] which yielded closer agreement with experimental results[12,13,14] as mentioned earlier compared to purely Coulomb or static barriers.

In view of these successes, recently non-local barrier theory was applied to some neutron deficient lead and sub-lead nuclei[38]. However, there it was not possible to study the trend of variation of F_L within the same spectra as reliable data about α-fine structures are not available. Hence, the purpose of the present paper is to calculate penetrability factor P_L using non-local potential together with exchange term. Using these values of P_L, F_L are calculated directly from the relationship $F_L = \lambda_L / P_L$. The calculated values of P_L and hF_L are listed in Table 1. For comparisional purpose, hF_L with static barrier are also given in the same table. Secondly, the trend of variation of the ratio F_L/F_0 is given in Table 2 and discussed in Section 4.

2. *Penetrability factor*

The barrier encountered by an α-wave is,

$$V_{Int} = 2(Z-2)\,e^2/r + U_{N,L}(\vec{r}, \vec{r}') \qquad (2)$$

where $\vec{r}\,(= r, \Theta, \varphi)$ is the position of the α-particle with respect to the centre of mass of the system, $\vec{r}'\,(= r', \Theta', \varphi')$ represents some position of the α-particle other than $\vec{r}$ and Z is the charge number of the parent. In the region of deformed trans-uranium nuclei, the electrostatic and α-nucleus potential were taken to be anisotropic[13,39] but for the nuclei in the region $202 < 230$ studied here have been considered to be spherical and hence the potentials isotropic. The reason for considering these nuclei spherical is that the nuclei in the neighbourhood of doubly closed Pb^{208} are less likely to be deformed and it has been shown by Lemmer[40] that short ranged non-local interaction counteracts to some extent the effects of deformation. Further, reduced widths of some nuclei in this region have been calculated[41] from the overlap of shell model wave functions.

TABLE 1

Nucleus	Spin and parent	Parity product	L of alpha	$T_{Partial}/s$	E/MeV	R_t/fm	P_L	Reduced width/MeV Static	Reduced width/MeV Non-Local
1	2		3	4*	5	6	7*	8*	9*
$^{210}_{83}$Bi[a)]	1^-	1^-	0	1.37 (+14)	4.739	8.78	8.75 (−30)	1.23 (−6)	2.39 (−6)
		2^+	1	2.05 (+14)	4.776	8.78	1.57 (−30)	0.57 (−6)	8.88 (−6)
$^{202}_{85}$At[a)]	3^+	3^+	0	153.00	6.257	8.70	4.98 (−23)	0.198	0.376
		5^-	3	3.06 (+3)	6.357	8.70	5.98 (−24)	0.104	1.568
$^{210}_{85}$At[a,b)]	4^-	4^-	0	4.66 (+7)	5.458	8.78	1.11 (−26)	2.84 (−3)	5.50 (−3)
		5^+	1	5.54 (+7)	5.542	8.78	4.22 (−27)	1.04 (−3)	0.016
		6^+	3	5.44 (+7)	5.626	8.78	6.22 (−27)	9.10 (−4)	0.014
$^{222}_{88}$Ra[a)]	0^+	0^+	0	39.75	6.671	8.93	2.61 (−22)	0.145	0.276
		2^+	2	858.00	6.344	8.92	6.26 (−24)	0.257	0.533
$^{224}_{88}$Ra[c)]	0^+	0^+	0	3.30 (+5)	5.784	8.92	2.97 (−26)	0.150	0.291
		2^+	2	6.42 (+6)	5.538	8.92	8.86 (−28)	0.244	0.516
$^{226}_{88}$Ra[a)]	0^+	0^+	0	5.24 (+10)	4.863	8.92	1.69 (−31)	0.158	0.313
		2^+	2	8.97 (+11)	4.671	8.92	4.62 (−33)	0.321	0.691
$^{226}_{90}$Th[a,b)]	0^+	0^+	0	1.89 (+3)	6.444	8.95	4.29 (−24)	0.188	0.352
		1	1	8.81 (+4)	6.204	8.95	3.83 (−26)	0.055	0.850
$^{227}_{90}$Th[d)]	$3/2^+$	$3/2^+$	0	5.57 (+7)	6.115	8.95	1.45 (−25)	1.82 (−4)	3.50 (−4)
		$3/2^-$	1	8.08 (+10)	6.065	8.95	8.70 (−27)	2.57 (−7)	40.76 (−7)$^+$
		$1/2^+$	2	6.60 (+6)	6.146	8.95	1.13 (−25)	1.82 (−3)	3.83 (−3)
$^{228}_{90}$Th[a,b)]	0^+	0^+	0	6.03 (+7)	5.517	8.95	1.34 (−28)	0.186	0.354
		1^-	1	1.08 (+10)	5.300	8.95	7.84 (−30)	0.020	0.394
		3^-	3	1.51 (+11)	5.349	8.95	1.67 (−31)	7.00 (−3)	0.113
$^{230}_{90}$Th[c)]	0^+	0^+	0	3.17 (+12)	4.769	8.95	3.44 (−33)	0.131	0.262
		2^+	2	3.04 (+13)	4.702	8.96	6.44 (−34)	0.198	0.429

TABLE 1. (contd.)

1	2		3	4	5	6	7	8	9
$^{227}_{91}Pa^{d)}$	$5/2^-$	$5/2^-$	0	3.75 (+3)	6.580	8.95	5.94 (−24)	0.066	0.128
		$7/2^+$	1	2.38 (+4)	6.470	8.95	2.10 (−25)	0.636	0.575
$^{228}_{91}Pa^{a),b)}$	4^+	4^+	0	1.00 (+7)	6.152	8.95	1.03 (−25)	1.40 (−4)	2.75 (−3)
		2^-	3	9.02 (+7)	6.195	8.95	5.30 (−27)	3.96 (−4)	5.92 (−3)
		3^-	1	1.73 (+7)	6.209	8.95	1.42 (−26)	7.20 (−4)	0.011
		3^-	1	1.79 (+7)	6.223	8.95	1.65 (−20)	5.47 (−4)	8.80 (−3)
		0^-	5	8.29 (+7)	6.247	8.95	2.22 (−27)	9.50 (−4)	0.015
$^{230}_{92}U^{e)}$	0^+	0^+	0	2.66 (+6)	5.992	8.96	4.50 (−27)	0.122	0.239
		2^+	2	5.59 (+6)	5.917	8.96	1.08 (−27)	0.222	0.474
		4^+	4	4.76 (+8)	5.758	8.96	4.29 (−29)	0.056	0.140

* Figures in bracket in column 4, 7, 8 and 9 indicate power of 10.

+ Exceptional case.

a) Ref. 44; b) Ref. 45; c) Ref. 46; d) Ref. 47; e) Ref. 48.

P_L and reduced widths.

Thus the Schrödinger equation takes the form,

$$\frac{\hbar^2}{2m} \nabla_r^2 \psi(\vec{r}) + [E_L \psi(r) - 2(Z-2) e^2/r] = \int \psi(\vec{r}')\, U_{N,L}(r, \vec{r}')\, d\vec{r}' \tag{3}$$

where m is the mass of the α-particle and E_L is the decay energy for the mode involving α-angular momentum L and corrected for electron screening and recoil. The spherically symmetric non-local α-nucleus potential is taken to be as before[38])

$$U_{N,L}(\vec{r}, \vec{r}') = V(r)\, \delta_b(\vec{r}, \vec{r}') \tag{4}$$

where $V(r)$ is the static part of the potential and the non-local part is,

$$\delta_b(\vec{r}, \vec{r}') = \pi^{-3/2} b^{-3} \exp[-(\vec{r} - \vec{r}')^2/b^2] \tag{5}$$

b, being the range of non-locality. Since in general nuclear force is exchange in nature, a mixed potential has been used of which the ordinary part is denoted by S. Expanding $U_{N,L}(\vec{r}, \vec{r}')$ in terms of Legendre polynomials and using Eqs. (4) and (5), the radial part R_L of the α-wave is obtained as,

$$\frac{d^2R_L}{dr^2} + (2m/\hbar^2)[E_L - 2(Z-2)e^2/r + n(x) f(r) + \frac{\hbar^2}{2m}(L(L+1)/r^2)(1 + $$

$$+ cn(x) f(r))]\, R_L(r) = 0 \tag{6}$$

where,

$$V_{\pm} = V_0 [S + (1 - S)(-1)^L],$$

$V(r) = V_0 f(r)$[42]), the static part of the potential with form factor

$$f(r) = \exp - [(r - 1.17A^{1/3})/0.574],$$

A being the mass number of the product nucleus:

$$n(x) = V_{\pm} [1/2 \cdot (1 + erf(x))]$$

$$x = (r - R_t)/b, \qquad c = mb^2/2\hbar^2.$$

From Eq. (6) and using the WKB method of approximation, the penetrability factor is obtained as,

$$P_L = \exp\{ - (8m/\hbar^2)^{1/2} \int_{R_t}^{R_{0,L}} [2(Z-2)e^2/r - n(x) f(r) - E_L + $$

$$+ \frac{\hbar^2}{2m}(L(L+1)/r^2)(1 + cn(x) f(r))]^{1/2}]\, dr\}, \tag{7}$$

where R_t is the inner turning point and $R_{0,L} = 2(Z-2)e^2/E_L$ is the outer turning point.

3. *Results*

The integral (7) has been calculated by using the modified Simpson's rule (cf. Ref. 38) with 161 strips and the *erf* function with 61 strips. Using greater number of strips was found to be unproductive, P_L was found to be insensitive to the variation of b in the range 0.5 fm $< b <$ 0.9 fm and so an average value of 0.7 fm has been used. R_t has been calculated by an iterative method and the mixture proportion S was set at 60 percent (cf. Ref. 38). Calculated values of P_L are listed in column (7) of Table 1 and reduced widths (hF_L) calculated from Eq. (1) using experimental data for λ_L are given for both static and non-local barriers in column (8) and (9), respectively, of Table 1. Sources for decay data are mentioned in appropriate places.

Secondly, reduced width ratios, F_L/F_0 are listed in Table 2 and for comparisonal purpose, those with static barrier are also given in column (3) of the same table. Results are discussed in the next section.

TABLE 2

Nucleus	L	F_L/F_0	
		Static	Non-local
^{210}Bi	1	0.46	3.70
^{202}At	3	0.52	4.17
^{210}At	1	0.36	2.90
	3	0.32	2.64
^{222}Ra	2	1.72	1.92
^{226}Ra	2	2.03	2.20
^{226}Th	1	0.29	2.41
^{227}Th	1	0.001	0.01$^+$
	2	10.00	10.81
^{228}Th	1	0.11	1.11
	3	0.03	0.32
^{230}Th	2	1.51	1.63
^{227}Pa	1	0.54	4.49
^{228}Pa	1 (E = 6.209 MeV)	0.51	3.99
	1 (E = 6.223 MeV)	0.39	3.19
	3	0.26	2.19
	5	0.67	5.55
^{230}U	2	1.83	1.98
	4	0.46	0.58

$^+$ Exceptional case.

Reduced width ratios for static and non-local barriers.

4. *Discussion*

As mentioned, evidence in support of the values of hF_L given in column (9) of Table 1 is based on the consistency with which the non-local barrier method accommodates the observed relative intensities in terms of P_L/P_0 together with values of reduced width ratios F_L/F_0 within expected range. It is intended here for further indication in support of the present method. For this purpose F_L values and reduced width ratios, both with static and non-local barriers are compared and discussed below.

i) It is seen from Table 2, that for all odd-parity transitions (with two exceptions of $L = 1$ in ^{227}Th and $L = 3$ in ^{228}Th), the ratio $F_L/F_0 > 1$ as expected[2], while on the other hand static barrier yields values of F_L/F_0 incompatibly low.

ii) It may be seen from Table 2, that for even L, the ratios with static and non-local barriers are not much different. It may however be noticed that the non-local values are consistently higher than the values from static potential by about ten percent. Although detailed studies are required for its explanation, not much different results from both the potentials is not unexpected since for even L exchange effect is absent and the concerned L being small, non-local effects should also be small.

But when the compared groups are of opposite parity, the results with static non-exchange barrier are much different than those with non-local barrier, the latter being within expected range indicating that exchange effect is quite significant. In other areas of nuclear physics such as in scattering, it has also been observed[43] that the effective potential requires an odd-even L component for fitting scattering data.

iii) It is seen from Table 1 that for even L the values of reduced widths with static barrier are smaller by a factor of about half than those with non-local barrier while for odd L, the reduced widths with the first barrier are incompatibly lower, namely about one fifteenth of the non-local values.

iv) It is found that within an α-spectra, values of P_L for higher L are in general greater than the for $L = 0$, irrespective of whether the latter is a transition to ground state or not. This result has also been found for rare-earth nuclei[14] indicating that L dependence (or non-locality) of the α-nucleus interaction is an important factor for higher L.

5. *Conclusion*

The results may be summarised as follows:

i) When the compared groups are of opposite parity, the values of reduced width ratios are in conformity with the expected trend[2], $F_L > F_0$. But when the compared groups are of same parity, the ratios with the two barriers are not much different. Yet the non-local values are about ten percent higher than those from static barrier as expected.

ii) The value of F_L/F_0 vary from 2.2 to 4.2 with an average value of 3.2. For $L = 5$, $F_L/F_0 \approx 5.55$ and for some other cases also reduced width ratios do not exactly correspond to the simple relation, $F_L/F_0 = (2L + 1)$ obtained from the studies of α-spectroscopic amplitudes[2]. The reason for this may lie in the fact that the above mentioned relationship was obtained for the decay of light nucleus $^{16}O \rightarrow {}^{12}Ne$ for which both the decaying and product nuclei can be considered as assemblage of four and three α-particles, respectively, while none of the heavy nuclei considered here is so giving rise to configurational difference. Still, it is worthwhile to note that the trend $F_L/F_0 > 1$ is reproduced.

Acknowledgments

One of the authors (S. M. C.) gratefully acknowledges the grant of a scholarship by Indian School of Mines, Dhanbad during the tenure of which this work was done. The computing facilities extenled by F. P. D. I. L., Sindri is also thankfully acknowledged.

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ŠIRINE α-RASPADA U TEŠKIM JEZGRAMA

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UDK 539.164

Originalni znanstveni rad

Raniji tretman širina α-raspada jezgara rijetkih zemalja proširen je na α-sprektre koji sadrže prijelaze s angularnim momentom α-čestice $L = 0$. Kao i ranije, barijeru predstavlja nelokalni α-jezgra potencijal s članom izmjene superponiran na uobičajeni Coulomb-ov potencijal, te su izračunate reducirane širine F_L. Nađeno je da je $F_L/F_0 > 1$ za gotovo sve prijelaze neparne parnosti prema očekivanju, dok za prijelaze parne parnosti F_L može biti manji od F_0. Rezultati su diskutirani.