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## LETTER TO THE EDITOR

# INTERPLAY OF CHIRAL VACUUM EFFECTS AND PROJECTIONS IN THE TOPOLOGICAL BAGS

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The projection procedure for chiral solitons and nonrelativistic quark models as given by Manohar has a nontrivial generalization to quark bags with chiral boundary condition if vacuum effects are taken into account. We show that the interplay between the chiral vacuum effects and the projections leads to potentially important contributions to matrix elements of observables.

Skyrmions and topological two-phase models attracted much attention recently[1]. One of their peculiar properties is that they do not have good quantum numbers of spin and isospin and therefore cannot correspond to any physical particles. Only their sum $\vec{K} = \vec{J} + \vec{I}$ is a good quantum number. Up to now, only the $K = 0$ solution for the Skyrme model is known. Such $K = 0$ topological solitons (and the related topological bags) can be viewed as a mixture of physical states with same spin and isospin, $I = J$. When we want to make statements about the physical states in our chiral topological models, we must extract them in some way uot of the $K = 0$ objects from which we start.

In the chiral soliton model this problem was for the $SU(2)$ case solved by Adkins et al.[2] and for $SU(3)$ later on by Guadagnini[3] by introducing the time-dependent collective coordinates summarized in the matrix $A(t) = a_0(t) + i\tau \cdot \vec{a}(t)$. The collective coordinates $a$ were used, by adiabatically rotating the soliton configuration,

$$U_0(\vec{x}) \rightarrow U(\vec{x}, t) = A^{-1}(t)\, U_0(\vec{x})\, A(t) \tag{1}$$

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to define the states of good spin and isospin as rotational excitations on top of the classical Skyrmion solution.

A somewhat more standard (from nuclear physics point of view) projection procedure in $SU(2)$, involving directly rotation $D$-matrices, was used by Jackson et al.[4] for extracting NN-interaction from skyrmion-skyrmion interaction. Manohar[5] generalized the projection technique to $SU(3)$ proving that the group theoretic structure of the chiral soliton model was identical to that of the naive non-relativistic quark model in the limit of large number of colours, $N_c$. The states of good $SU(3)_{flavour} \times SU(2)_{spin}$ quantum numbers are defined with the help of the standard projection operator which is just the integral over $SU(3)$ group (the integration over $SU(2)_{spin}$ can be lumped into $SU(3)_{flavour}$ since $\vec{I} = -\vec{J}$ for the hedgehog):

$$|R, IiY;\ Jm\rangle = \mathcal{N} P_{IiY,\ J-mY_H} W_H \tag{2a}$$

where

$$P_{IiY,\ I'i'Y'} = \int \mathrm{d}\mu(f)\, D^{R*}_{IiY,\ I'i'Y'}(f) \hat{f}_{cl}, \quad f \in SU(3). \tag{2b}$$

$R$'s label $SU(3)$ representations (octet and decuplet for physical baryons), $I$ and $i$ are isospin and third component of isospin, $Y$ is hypercharge and $J$ and $m$ spin and its projection. $W_H$ is either *hedgehog* (i. e. $K = 0$) quark wave function (with the flavour-spin content given simply as a product of $N_c$ single quark *hedgehogs*),

$$W_H = h \times h \times \ldots \times h = \frac{1}{\sqrt{2}}(u_\downarrow - d_\uparrow) \times \frac{1}{\sqrt{2}}(u_\downarrow - d_\uparrow) \times \ldots \times \frac{1}{\sqrt{2}}(u_\downarrow - d_\uparrow) \tag{3}$$

or the *hegdehog* soliton shape. ($h$ is the single quark *hedgehog*, $h = \frac{1}{\sqrt{2}}(u_\downarrow - d_\uparrow)$). $Y_H = 1$ is the hypercharge of such *hedgehog* configurations. $f_{cl}$ is the rotation operator. (The subscript $cl$ indicates that it is classical operator, as opposed to second-quantized one, which we will need later.) $D^{R*}_{IiY,I'i'Y'}(f)$ is a $SU(3)$ rotation matrix. $\mathcal{N}$ is the normalization. (For $N_c \to \infty$, $\mathcal{N} = (dimR)^{1/2}$.)

The matrix element of an operator $\hat{A}$ is then given as the double integral over $SU(3)$ manifold:

$$\langle a|\hat{A}|b\rangle = \mathcal{N}^2 \int \int d\mu(f)\, d\mu(g)\, D^{R_a}_{aH}(g)\, D^{R_b^*}_{bH}(f) \int W_H^+ \hat{g}^+_{cl} \hat{A} \hat{f}_{cl} W_H d^3x. \tag{4}$$

$a$ and $b$ stand for the quantum numbers of states $|a\rangle$ and $|b\rangle$, and $H$ for those of the hedgehog.

Let $\hat{A}$ transform as an irreducible tensor operator under $SU(3)$ transformations. We will also assume the limit of large number of colours $N_c$, which simplifies the integration by essentially killing off one of the integrals. One can then derive the key result (first proved by Manohar[5] for both the chiral topological solitons and $K = 0$ nonrelativistic quark states), namely the expression for the matrix element of any tensor operator between the physical states in the large $N_c$-limit:

$$\langle R_1, I_1\, i_1 Y_1; J_1\, m_1\, | \hat{A}^{R,J}_{I i Y, m} | R_2, I_2\, i_2 Y_2; J_2\, m_2 \rangle = \left( \frac{dim\, R_1}{dim\, R_2} \right)^{1/2} C^{I I_1 I_2}_{i i_1 i_2}\, C^{J J_1 J_2}_{-m -m_1 -m_2}$$

$$\sum_\gamma \begin{pmatrix} R & R_1 & \Big| & R_2^\gamma \\ IY & I_1 Y_1 & \Big| & I_2 Y_2 \end{pmatrix} \begin{pmatrix} R & R_1 & \Big| & R_2^\gamma \\ J0 & J_1 Y_H & \Big| & J_2\, Y_H \end{pmatrix} \int W_H^+ \hat{A}^{R,J}_{J-m0,m} W_H d^3x. \tag{5}$$

$C$'s are Clebsch-Gordan ceofficients. The symbols after the sum over the $SU(3)$ representations $R^\gamma$ are $SU(3)$ isoscalar factors as defined in de Swart[6]. The formula (5) resembles strongly to Wigner-Eckart theorem, with the *hedgehog* matrix element playing the role of a reduced matrix element.

Let us for example take $\hat{A}$ to be the third component of magnetic moment operator:

$$(\vec{\mu})_3 = \frac{1}{2} \int (\vec{r} \times \bar{\psi}\, \vec{\gamma}\, Q\, \psi)_3\, \mathrm{d}^3 r. \tag{6}$$

The change operator is

$$Q = I_3 + \frac{Y}{2} = \frac{\lambda_3}{2} + \frac{\lambda_8}{2\sqrt{3}}. \tag{7}$$

so that we must decompose $(\vec{\mu})_3$ into two contributions to get the irreducible tensor components, one isovector and the other isoscalar. Using the spherical components,

$$(\vec{\mu})_3 = (\vec{\mu})_3^{I=1} + (\vec{\mu})_3^{I=0} = \mu^{8,1}_{100,0} + \frac{1}{\sqrt{3}} \mu^{8,1}_{000,0}. \tag{8}$$

Now we can apply formula (5) to magnetic moments. For the proton ($I = J = 1/2$),

$$\mu_p = -\frac{4}{15} \int W_H^+\, \mu^{8,1}_{100,0}\, W_H\, \mathrm{d}^3 x. \tag{9}$$

Also, (5) yields exactly the same ratios for the magnetic moments as obtained for unbroken $SU(3)$ in the pure chiral soliton model by Adkins and Nappi[7] by the projecting through adiabatic time-dependent rotations in isospin space[2]:

$$\begin{gathered} \mu_n = -\frac{3}{4}\mu_p \quad \mu_{\Xi^0} = -\frac{3}{4}\mu_p \quad \mu_{\Xi^-} = -\frac{1}{4}\mu_p \\ \mu_{\Sigma^-} = -\frac{1}{4}\mu_p \quad \mu_{\Sigma^0} = \frac{3}{8}\mu_r \quad \mu_{\Sigma^+} = \mu_p \\ \mu_{\Lambda^0} = -\frac{3}{8}\mu_p \quad \langle \Lambda^0 \uparrow | \mu_3 | \Sigma^0 \uparrow \rangle = 3\frac{\sqrt{3}}{8}\mu_p \quad \mu_{\Delta^{++}} = \frac{15}{8} u_p. \end{gathered} \tag{10}$$

These ratios are not very well satisfied experimentally, but (as shown in Ref. 7) can be improved by introduction of $SU(3)$ breaking.

Alternatively, we can refrain from the large $N_c$ approximation used in deriving (5) and keep the expression for the matrix elements at the level of integrals, i. e. formula (4). Namely, if we eventually evaluate the *hedgehog* matrix element for $N_c = 3$ in order to find not only ratios but magnitudes of magnetic moments, we will see that it is a quite crude approximation. In the case of $SU(2)$ the integration is for some simpler operators doable exactly, although it becomes very cumbersome because of proliferation of Euler angles. In the case of $SU(3)$ we can still hope to do the integrals numerically without large-$N_c$ approximation.

We want to show that the projection procedure as outlined by Manohar[5] and its results (4) and (5) apply not only to the non-relativistic quark model, but in a certain generalized sense to all the models where the second-quantized quark fields necessarily come into play, for instance the topological chiral bag model[8,9,10,11]. (Of course, validity of these results for the classical valence quarks in the bag is pretty trivial, but we will see that taking into account the second quantization gives us also a contribution in the form of vacuum asymmetry.)

The topological chiral bag model consists of two phases: a quark bag and the incomplete topological soliton outside the bag, and is described by the Lagrangian density

$$\mathscr{L}_{HCBM} = \overline{\psi}(x)\left(\frac{i}{2}\gamma_\mu\overrightarrow{\partial}^\mu - \frac{i}{2}\gamma_\mu\overleftarrow{\partial}^\mu - B\right)\psi(x)\,\Theta_{V_{in}}(x) - \frac{1}{2}\overline{\psi}U_5\psi\Delta_S +$$

$$+ \left[\frac{F_\pi^2}{16}\mathrm{Tr}\,(\partial_\mu U\,\partial^\mu U^+) + \dots\right]\Theta_{V_{out}}(x), \tag{11}$$

where

$$U = e^{i\vec{\Theta}(x)\cdot\vec{\tau}}, \quad U_5 = e^{i\vec{\Theta}(x)\cdot\vec{\tau}\gamma_5}. \tag{12}$$

Here $F_\pi = 186$ MeV is the pion decay constant, and the dots stand for a term which must be present to stabilize the soliton energetically. We do not write it explicitly, since although there are several possible choices[12,2,13,14], our discussion is general and does not depend on which particular variety of the model we choose. Actually, if the bag is big enough, (if radius $R > 0.45$ fm) the presence of the bag itself is enough to stabilize the soliton and no explicit stabilizing term is needed[11].

The variation of the Lagrangian (9) give us not only equations of motion, but also boundary conditions on the bag surface $S$, of which the so called linear boundary condition

$$-i\hat{r}\cdot\vec{\gamma}\,\psi(R) = e^{i\vec{\tau}\cdot\vec{\Theta}(R)\gamma^5}\psi(R) \text{ on } S \tag{13}$$

is of particular importance, because it makes the quark energy spectrum dependent on $\Theta(R)$, the chiral angle at the bag surface.

The most remarkable fact about this model is that because of this dependence of the energy spectrum on the chiral angle, the baryon number is shared between the incomplete topological soliton outside and the quarks inside the bag[8,9]

$$B_{out} = \frac{1}{\pi}\left[\Theta(R) - \sin\Theta(R)\cos\Theta(R)\right] \tag{14}$$

$$B_{in} = B_{val} + B_{vac} = 1 - \frac{1}{\pi}\left[\Theta(R) - \sin\Theta(R)\cos\Theta(R)\right]. \tag{15}$$

Because of the vacuum asymmetry of the quark spectrum, the baryon number of three valence quarks gets depleted. Through the boundary condition (10) the chiral angle $\Theta(R)$ on the bag surface shifts the quark energy spectrum and causes the vacuum asymmetry of the baryon number. The baryon number fractionation is the most striking example of how crucial quantum effects are in the chiral bag models. The question we pose is: will results (4) and (5) be modified by these effects, and if yes, how?

Baryon numbers fractionation is probably the single most important effect of the vacuum asymmetry, but baryon number is a $SU(3)$ single and is therefore not affected by projections. To provide additional motivation for analyzing the interplay of the vacuum effiects and the projection procedure in the quark sector of the chiral bag, we should analyze an operator whose matrix elements do depend on the projecting out the states of good spin, isospin and hypercharge. So, let us consider again the magnetic moment of the proton. If we apply result (9) to $N_c$ valence quarks which occupy $K = 0$ (*hedgehog*) state $W_H$, we obtain for the quark contribution to the magnetic moment

$$\mu_p^q = \frac{4}{15}\frac{N_c}{3}\frac{R\left[3\Omega - 3\sin\Omega\cos\Omega - 2\Omega\sin^2\Omega\right]}{4\Omega^4\left[j_0^2(\Omega) + j_1^2(\Omega) - 2j_0(\Omega)j_1(\Omega)/\Omega\right]}. \tag{16}$$

$\Omega \equiv E_i[\Theta(R)]R$ is the solution of the eigenvalue equation

$$j_0^2(\Omega) - j_1^2(\Omega) - 2j_0(\Omega)j_1(\Omega)\tan\Theta(R) = 0 \tag{17}$$

which comes from the linear boundary condition (13) and gives the valence quark energy as a function of the chiral angle.

Let us first compare the result (16) with the one from the MIT model. If we take the MIT limit $\Theta(R) = 0$, then $\Omega = \Omega_0 = 2.04$, the lowest MIT bag eigenmode. However, (16) does not reduce to the MIT result, because it was obtained by using large $N_c$ approximation which simplified $SU(3)$ integration by killing a lot of group space that would contribute to exact integration. Indeed, for $N_c = 3$, in the limit $\Theta(R) = 0$, $\Omega = \Omega_0$, (16) gives us a result 5/2 times smaller than the MIT result,

$$(\mu_p^q)_{IMT} = \frac{2}{3}\frac{R\left[3\Omega_0 - 3\sin\Omega_0\cos\Omega_0 - 2\Omega_0\sin^2\Omega_0\right]}{4\Omega^4\left[j_0^2(\Omega_0) + j_1^2(\Omega_0) - 2j_0(\Omega_0)j_1(\Omega_0)/\Omega_0\right]} =$$

$$= \frac{R}{12}\frac{4\Omega_0 - 3}{\Omega_0(\Omega_0 - 1)} \tag{18}$$

which for reasonable bag radii ($R \leq 1$ fm) is somewhat too small anyway ($\sim 1.9$ $M_p$, where $M_p$ = proton magneton). However, since we can keep the expression for the projected matrix element in the form of integrals like (4) which we in principle can do numerically, this is less of a trouble than what happens with (16) when we dial the chiral angle $\Theta(R)$ to finite values. One might think that for $\Theta(R) \neq 0$ the soliton contribution to the magnetic moment will make the situation better. It does, but increasing $\Theta(R)$ decreases the energy $\Omega$ dramatically. $\mu_p^q$ from (16) falls to zero linearly with $\Omega$ as $\Theta(R)$ approaches $\pi/2$, the chiral angle where valence quarks sink into Dirac sea.

Although the solution contribution is fairly larger than the valence quark contribution (16) (especially if we take the experimental value of the pion decay constant, $F_\pi = 186$ MeV, and not a smaller value as favoured by some authors[2,13]), its increase with decreasing $R$ cannot make up for the loss in $\mu_p^q$ as $\Omega$ falls.

Actually, consistently projecting inside and outside sector in the two-phase model is somewhat more complicated than just projecting inside and outside separately[15]. Therefore, our discussion strictly speaking applies only to the quark sector, i. e. to the quark bag with the chiral boundary condition (13). However, the contributions from the soliton sector which we are going to show are meant to serve only as a qualitative illustration. So, we again apply large $N_c$ projection formula (5):

$$\mu_p^{sol} = -\frac{4}{15}\mu_H^{sol} = -\frac{4}{15}\left(-\frac{2\pi}{3}\right)F_\pi^2 \int_R^\infty \mathrm{d}r\, r^2\, [\sin^2\Theta(r) + \ldots] \qquad (19)$$

where dots indicate the contribution from some stabilizing term, which we will not include here because it is not needed for large bag radii. Then, for instance,

$$-\text{at } R = 1\ \text{fm},\ \mu_p^q = 0.59M_p,\ \mu_p^{sol} = 0.96M_p,\ \rightarrow \mu_p = 1.56M_p, \qquad (20a)$$

$$-\text{at } R = 0.8\ \text{fm},\ \mu_p^q = 0.32M_p,\ \mu_p^{sol} = 1.17M_p,\ \rightarrow \mu_p = 1.49M_p, \qquad (20b)$$

and for smaller radii the falloff of $\mu_p^q$ towards zero gets still faster until it apparently even turns negative with the valence quark energy around $R = 0.5$ fm. This unphysical behaviour comes from including only the valence quarks in the domain where the contribution of the vacuum asymmetry becomes dominant and should also be included. However, operators like magnetic moments are not $SU(3)$-blind like the baryon number. In the rest of this article we will show how one can proceed to isolate in principle the vacuum asymmetry in the case of a second-quantized operator $\hat{A}$ which does not commute with $SU(3)$ rotations which are involved in the projection procedure.

In order to incorporate the vacuum effects, we must acknowledge the fact that our bag is populated by second-quantized quark fields $\psi$. The *hedgehog* on which rotations in (4) act is no longer the wave function of the valence quarks; it must become a second quantized state:

$$W_H \rightarrow |H\rangle = b_h^{1\dagger}\, b_h^{2\dagger}\, b_h^{3\dagger}\, |D\rangle. \qquad (21)$$

$|D\rangle$ is the vacuum (or »Dirac sea«) state. The operator $A$ becomes

$$\hat{A} \to A = \int \psi^{\dagger}(x)\,\hat{A}\,\psi(x)\,\mathrm{d}^3x. \tag{22}$$

Rotations acting on quarks must from classical rotation operators $\hat{f}_{cl}$ change to quantum operators $\hat{f}_Q$,

$$\hat{f}_{cl} = \mathrm{e}^{-\mathrm{i}\alpha^a T^a} \to \hat{f}_Q = \mathrm{e}^{\mathrm{i}\,\alpha^a Q^a}, \tag{23}$$

where the conserved normal ordered charge

$$Q^a = \int :j_0^a(x):\mathrm{d}^3x = \int :\psi^{\dagger}(x)\,T^a\,\psi(x):\mathrm{d}^3x \tag{24}$$

becomes the generator of $SU(N)$ transformations.

With the substitutions (21) and (23) in the basic formula (4), we see that the second-quantized projection procedure becomes highly problematical. An immediate practical difficulty is that it is not clear how to perform the integration over the group space without expanding fields, operators and rotations explicitly in the creation and annihilation operators, which would result in very clumsy expressions. More importantly, the whole scheme can be undermined even in principle because of the interplay of the vacuum $|D\rangle$ with the quantized quark fields sitting in the rotation operators. It is not immediately clear that in the matrix element $\langle H|\hat{g}_Q^{\dagger} : A : \hat{f}_Q|H\rangle$ the creation and annihilation operators will not give rise to unwanted particle-hole excitations which would prevent us from defining the physical states with well-defined quantum numbers.

To show that the procedure is still well defined, it is useful to separate immediately the vacuum asymmetry of the observables through the Bogoliubov transformation. The normal ordered version of $A$ at $\Theta(R) = 0$, denoted by $: A :_0$, gives 0 when it acts on the trivial vacuum:

$$: A :_0 |D\rangle_0 = 0. \tag{25}$$

However, we want to know how $: A :_0$ acts on a confined state with $\Theta(R) \neq 0$. If we perform the Bogoliubov transformation which takes us from the $\Theta(R) = 0$ basis (let us call it $|m\rangle$) to some other basis $|a\rangle$ at $\Theta(R) \neq 0$, the normal-ordered operator $: A :_0$ splits into two parts:

$$: A :_0 = : A :_\Theta + \langle A\rangle_\Theta. \tag{26}$$

$: A :_\Theta$ is the operator normal ordered with respect to the redefined Dirac vacuum at $\Theta(R)$,

$$: A :_\Theta |D\rangle_\Theta = 0. \tag{27}$$

The remainder is a $C$-number

$$\langle A\rangle_{\Theta}=\frac{1}{2}\left(\sum_{a}\langle a-|A|\,a-\rangle-\sum_{a}\langle a+|A|\,a+\rangle\right)-$$
$$-\frac{1}{2}\left(\sum_{a}\langle m-|A|\,m-\rangle-\sum_{a}\langle m+|A|\,m+\rangle\right) \tag{28}$$

which turns out to be equal to the vacuum asymmetry[9,10] since the second line in (28) vanishes by symmetry of the spectrum at $\Theta(R)=0$. Indices $+$ and $-$ indicate positive and negative energy eigenvalues, and $a$ or $m$ denote all quantum numbers which label each particular mode. The manipulations which we used to derive (26)—(28) are unfortunately only formal because we had to deal with unregularized infinite sums (which of course are not well-defined) in order to be able to use the closure relation*. Up to now, we were not able to make the derivation of (28) mathematically air-tight by using the regularized sums from the start. Therefore, when we want to calculate numerically the mode-sum (28), we have to put in a regulator by hand in order to proceed like Vepstas et al.[10] or Mulders[16] for the baryon number asymmetry or the chiral Casimir energy.

In order to keep the analysis less messy, let's stick to the case $N_c=1$. (Generalization to arbitrary $N_c$ is obvious, it is just messy.) Then

$$|H\rangle=b_h^{\dagger}|D\rangle=\int d^3x\psi^{\dagger}(x)\,h(x)\,|D\rangle. \tag{29}$$

(Remember that for $N_c=1$, $W_H(x)=h(x)$.) Using

$$[Q^a,\psi(x)]=-T^a\psi(x)\ \text{and}\ \langle Q^a\rangle=0, \tag{30}$$

we get

$$\hat{f}_Q|D\rangle=e^{i\alpha^aQ^a}|D\rangle=\int\psi^{\dagger}(x)\,|D\rangle\,e^{-i\alpha^aT^a}\,h(x)\,d^3x. \tag{31}$$

In (31) we expressed the quantum rotation of a state through the classical rotation of $h(x)$, the wave function of the valence quark. Then, anticommuting the quark field $\psi^{\dagger}(x)$ through the operator $:A:$ and using the Bogoliubov transformation (26) to get rid of the vacuum state $|D\rangle$ and isolate the vacuum asymmetry $\langle A\rangle_{\Theta}$, we finally get the desired result

$$\langle a|\hat{A}|b\rangle=\mathcal{N}^2\int\int d\mu(f)\,d\mu(g)\,D^{R_a}_{aH}(g)\,D^{R_b^*}_{bH}(f)\int W_H^{\dagger}(x)\,\hat{g}_{cl}^{+}\,\hat{A}\,\hat{f}_{cl}\,W_H(x)\,d^3x+$$
$$+\langle A\rangle_{\Theta}. \tag{32}$$

Therefore, the full second-quantized projection procedure is expressed in terms of the classical rotations acting on the valence hedgehog states and a $\Theta(R)$-dependent vacuum asymmetry which is just a $C$-number. The valence part is just

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*We thank Andreas Wirzba for pointing this out.

(4) (or (5), in $N_c \to \infty$ case) and the explicit separation of the vacuum asymmetry is the new feature.

At this point we have unfortunately not yet been able to evaluate the mode sums involved in expressions for vacuum asymmetries $\langle A \rangle_\Theta$ of such relatively complicated operators as magnetic moments. (Even much simpler quantities as energy are still somewhat controversial and quite hard to obtain — see for instance Refs. 10, 16 and 17, and for most up to date overview, Ref. 18. The baryon number asymmetry is at the moment still the only »easy« vacuum asymmetry.) Therefore, we will have to keep temporarily the somehwat too general formula (32) as the final result. Only after we learn more about handling the vacuum mode sums of the type (28) for general operators, will we be able to apply the general result (32) to the concrete cases.

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# MEĐUIGRA EFEKATA KIRALNOG VAKUUMA I PROJEKCIJA U TOPOLOŠKIM VREĆAMA

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Manoharov projekcioni postupak za kiralne solitone i nerelativistički kvarkovski model ima netrivijalno poopćenje na kiralne kvarkovske vreće ukoliko se uzmu u obzir vakuumski efekti. Pokazano je da sprega između kiralnih vakuumskih efekata i projekcija vodi do potencijalno važnih doprinosa matričnim elementima observabli.