YU ISSN 0015-3206
FZKAAA 18 (2) 197 (1986)

LETTER TO THE EDITOR

DERIVATION OF MARSHALEK BOSON FOR $SU(6)$ QUANTUM CASE

GEORGI KYRCHEV

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria

and

VLADIMIR PAAR

Prirodoslovno-matematički fakultet, University of Zagreb, Marulićev trg 19, 41000 Zagreb, Yugoslavia.*

Received 19 May 1986

UDC 539.12

Original scientific paper

Explicit expressions for the creation and annihilation operators of the Marshalek boson are derived for the $SU(6)$ quantum case employing the intertwining operator between the Holstein-Primakoff and Schwinger spaces.

In last decade the approach to nuclear structure based on $SU(6)$ dynamical symmetry has been investigated for even-even nuclei (boson system)[1-3], odd-even nuclei (boson-fermion system)[3,4] and, recently, for odd-odd nuclei (boson-fermion-fermion system)[5-9].

In discussions about the Schwinger and Holstein-Primakoff spaces for the collective degree of freedom the following correspondence was employed[10,11]

$$|s^{N-n}\, d^n\rangle_{SR} \rightarrow |a^N b^n\rangle_{HPR}$$

introducing in the Holstein-Primakoff vector a new boson a, which is fixed for all irreducible representations for a particular nucleus. Thus, it is a dummy object

* Assisted by U. S.-Yugoslav Joint Fund for Scientific and Technological Cooperation in cooperation with the U. S. National Science Foundation under Grant No. YOR 83/074.

in a description of nuclear properties of a particular nucleus; the corresponding degree of freedom in this case is irrelevant and hence frozen. However, when considering physical processes which connect different nuclei, as for example two-nucleon transfer reactions, it is convenient to introduce an operator which leads from the irreducible representations of one into the irreducible representations of the other nucleus. Of course, this operator disappears in the finite expressions for the relevant matrix elements[12].

In the classical limit this Marshalek boson can be rigorously defined for the $SU(n)$ group[10,13,14].

The $SU(6)$ quantum case was considered by heuristically introducing the Marshalek boson[12], carrying out the analogy with the $SU(6)$ classical case in the sense that the classical forms have been used as guidelines to construct the corresponding quantum operators. In this way the expressions for the Marshalek boson in terms of s and d bosons have been obtained. This enabled the mapping of non-$SU(6)$ operators from the Schwinger realization $\equiv$ IBM[1] into the Holstein-Primakoff realization $\equiv$ TQM[2,3].

In the present note the Marshalek boson is rigorously derived for the $SU(6)$ quantum-mechanical case, without resorting to the classical limit.

In the full scope of $SU(6)$-type boson realizations of the quadrupole collective algebra important role is played by the interwining operator[15] which connects Schwinger and Holstein-Primakoff realizations of the $SU(6)$ Quadrupole Collective Algebra (QCA) built from the generalized coordinates, impulses and their generators[2,16]. The intertwining operator is:

$$P_{N0} = \sum_{\{n_\nu\}} |[N_0]\,\{n_\nu\}\rangle_{HPR} \quad {}_{SR}\langle\{n_\nu\}\, n_s\, [N_0]| \tag{1}$$

with

$$\begin{gathered} n_s + n_d = N_0 \\ n_d = \sum_{\nu=-2}^{2} n. \end{gathered} \tag{2}$$

Here the basis vectors in Schwinger realization (SR) and Holstein-Primakoff realization (HPR) are:

$$|[N_0]\, n_s\, \{n_\nu\}\rangle_{SR} = \prod_{\nu=-2}^{2} (n_\nu!)^{-\frac{1}{2}} (d_\nu^+)^{n_\nu} (n_s!)^{-\frac{1}{2}} (s^+)^{n_s}\,|0\rangle \tag{3}$$

$$|[N_0]\,\{n_\nu\}\rangle_{HPR} = (N_0!)^{-\frac{1}{2}} (a^+)^{N_0} \prod_{\nu=-2}^{2} (n_\nu!)^{-\frac{1}{2}} (b_\nu^+)^{n_\nu}\,|\,0\rangle. \tag{4}$$

The operators s^+, d_ν^+, b_ν^+ and a^+ are the creation operators of s-boson, d-boson, quadrupole phonon and Marshalek boson, respectively.

For the rigorous introduction of Marshalek boson the relevant operators in Schwinger realization are

$$\hat{N} = s^+ s + \sum_\mu d_\mu^+ d_\mu \tag{5}$$

$$E_s = (s\, s^+)^{-\frac{1}{2}}\, s. \tag{6}$$

Let us consider the mapping of the number operator (5) defined in the Schwinger space onto the Holstein-Primakoff space:

$$\hat{N}_{map} = P \hat{N} P^{+}, \tag{7}$$

where P is the full projector

$$P = \sum_{N} P_{N} \tag{8}$$

and P_N is given by Eq. (1).

Inserting (1), (5) and (8) into (7) straightforward calculation gives

$$P \hat{N} P^{+} = a^{+} a P P^{+}. \tag{9}$$

Now we take the matrix elements of Eq. (9) in the HPR basis:

$$_{HPR}\langle| P \hat{N} P^{+} |\rangle_{HPR} = {}_{HPR}\langle| a^{+} a P P^{+} |\rangle_{HPR}. \tag{10}$$

Using (1), (8) it follows:

$$_{HPR}\langle| P \hat{N} P^{+} |\rangle_{HPR} = {}_{SR}\langle| \hat{N} |\rangle_{SR}, \tag{11}$$

$$_{HPR}\langle| a^{+} a P P^{+} |\rangle_{HPR} = {}_{HPR}\langle| a^{+} a |\rangle_{HPR}. \tag{12}$$

Thus we have

$$(\hat{N})_{SR} = (a^{+} a)_{HPR}, \tag{13}$$

where the l. h s. acts in SR and r. h. s. in HPR.

Similarly, we have derived the operator equations

$$P E_s \hat{N}^{\frac{1}{2}} P^{+} = a P P^{+}, \tag{14}$$

$$P \hat{N}^{\frac{1}{2}} E_s^{+} P^{+} = a^{+} P P^{+}. \tag{15}$$

Taking the matrix elements of (14), (15) in HPR basis we obtain

$$(E_s \hat{N}^{1/2})_{SR} = (a)_{HPR}, \tag{16}$$

$$(\hat{N}^{1/2} E_s^{+})_{SR} = (a^{+})_{HPR}, \tag{17}$$

where the l. h. s. and r. h. s. operators in parentheses act in SR and HPR spaces, respectively. The equations (16), (17) present the explicit forms for the creation and annihilation operators of Marshalek boson, expressed in terms of s and d bosons. In this way we have explicitly derived Marshalek boson.

It should be pointed out that Marshalek boson plays an important role as an essential ingredient of a link between Holstein-Primakoff realization (Truncated Quadrupole Phonon Model — *TQM*) and Schwinger realization (Interacting Boson Model — *IBM*). This link is an element of the full scope of boson realizations of *SU* (6) Quadrupole Collective Algebra (*QCA*)[2] which is built from the generalized quadrupole collective coordinates, momenta and their commutators. As will be presented in our forthcoming publication[17], the full scope of boson realizations of *QCA* incorporates construction of Dyson realization of *QCA*, derivation of Holstein-Primakoff realization via Dyson realization, relation between the Holstein-Primakoff and Schwinger spaces using Marshalek boson and the bijective bicontinuous intertwining operator between Holstein-Primakoff and Schwinger realization.

References

1) A. Arima and F. Iachello, Phys. Rev. Lett. **35** (1975) 1069;
2) D. Janssen, R. V. Jolos and F. Dönau, Nucl. Phys. **A224** (1974) 93;
3) V. Paar, S. Brant, L. F. Canto, G. Leander and M. Vouk, Nucl. Phys. **A378** (1982) 41;
4) F. Iachello and O. Scholten, Phys. Rev. Lett. **43** (1979) 679;
O. Scholten, Thesis, University of Groningen, Groningen (1980);
5) S. Brant, V. Paar and D. Vretenar, Z. Phys. **A319** (1984) 355;
6) V. Paar, in *Capture Gamma-Ray Spectroscopy and Rel. Top.*, Am. Inst. Phys., New York (1985);
7) T. Hübsch, V. Paar and D. Vretenar, Phys. Lett. **151B** (1985) 320;
8) P. Van Isacker, J. Jolie and K. Heyde, Phys. Rev. Lett. **54** (1985) 653;
9) A. B. Balantekin and V. Paar, Phys. Lett. **169B** (1986) 9;
10) J. P. Blaizot and E. R. Marshalek, Nucl. Phys. **A309** (1978) 422;
11) E. R. Marshalek, Nucl. Phys. **A347** (1980) 253;
12) G. Kyrchev and V. Paar, Nucl. Phys. **A395** (1983) 61;
13) R. L. Hatch and S. Levit, Phys. Rev. **C25** (1982) 614;
14) R. Gilmore, C. M. Bowden and L. M. Narducci, Phys. Rev. **A12** (1975) 1019;
15) M. T. Naimark, *Normed Algebras*, Amsterdam, Noordhoof (1972);
16) G. Kyrchev, Nucl. Phys. **A349** (1980) 416;
17) G. Kyrchev and V. Paar, Ann. Phys. (1986) (in print).

IZVOD MARSHALEKOVOG BOZONA ZA *SU* (6) KVANTNI SLUČAJ

GEORGI KYRCHEV

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria

i

VLADIMIR PAAR

Prirodoslovno-matematički fakultet, Sveučilište u Zagrebu, Marulićev trg 19, 41000 Zagreb, Yugoslavia

UDK 539.12

Originalni znanstveni rad

Izvedeni su eksplicitni izrazi za operatore stvaranja i poništenja Marshalekovih bozona u *SU* (6) kvantnom slučaju koristeći operator povezivanja između Holstein--Primakoffovog i Schwingerovog prostora.

YU ISSN 0015-3206
FZKAAA 18 (2) 201 (1986)

LETTER TO THE EDITOR

A COMMENT ON DEEP INELASTIC NEUTRINO SCATTERING IN THE QUARK-CONFINING POTENTIAL MODELS*

SVJETLANA FAJFER and KENAN SURULIZ

Institut za fiziku, University of Sarajevo, 71000 Sarajevo, Yugoslavia

Received 17 March 1986

UDK 539.12

Original scientific paper

We calculate structure functions for ν-N deep inelastic scattering using quark-confining potential models.

In addition to the electromagnetic lepton-nucleon scattering, an important source of information about nucleon structure is the study of neutrino-nucleon scattering.

In the previous note[1] we presented a calculation of nucleon structure functions for e-N deep inelastic scattering using quark-confining potential models. Here we apply the same method to the investigation of ν-N deep inelastic scattering.

The main features of the model are exposed in Ref. 1. For neutrino scattering the virtual photon is replaced by the W boson, and corresponding changes are made in the interaction Lagrangian. We denote the polarization vectors of the W boson going in the z-direction by

$$\varepsilon_\mu^{(R)} = \frac{-1}{2}(1, \mathrm{i}, 0, 0)$$

$$\varepsilon_\mu^{(L)} = \frac{1}{2}(1, -\mathrm{i}, 0, 0) \qquad (1)$$

$$\varepsilon_\mu^{(S)} = \frac{1}{\sqrt{q^2}}(0, 0, \nu, \mathrm{i}q_z),$$

* Supported in part by grants from the NSF and SIZ-nauke BiH.

otherwise the conventions are the same as in Ref. 1. The cross-section for the basic process $Wq \to q'$ is

$$\mathrm{d}\sigma = C\, \bar{\varepsilon}_\mu\, \varepsilon_\nu\, (L_{\mu\nu} + L^5_{\mu\nu}) \tag{2}$$

with

$$L_{\mu\nu} + L^5_{\mu\nu} = \int \frac{\mathrm{d}^2 k_T}{k_z + q_z}\, (l_{\mu\nu} + l^5_{\mu\nu}). \tag{3}$$

The $l_{\mu\nu}$ are of the same form as for the electromagnetic scattering, and $l^5_{\mu\nu}$ have additional γ_5 terms (we take only the dominant transition and set the K-M matrix equal to 1).

The integration over quark transverse momentum k_T is easily performed in the Bjorken limit. The $L_{\mu\nu}$ are essentially the same as before, and $L^5_{\mu\nu}$ are

$$L^5_{ij} = \mathrm{i} e_{ij3}\, N^2 \exp\left\{-\frac{k_z^2}{\xi^2}\right\}\left\{\left(\frac{m_N x + m}{E + m}\right)^2 \pi\xi^2 + \frac{\pi\xi^4}{(E+m)^2}\right\}, \text{ other } L^5_{\mu\nu} = 0. \tag{4}$$

Now it is easy to find the cross-sections for different polarizations of W bosons:

$$\sigma^{(L)} = 2\pi\, C\, \xi^2\, N^2 \exp\left\{-\frac{(E - m_N x)^2}{\xi^2}\right\}\left\{\left(\frac{(m_N\, x + m)}{E + m}\right)^2 + \frac{\xi^2}{(E - m)^2}\right\}$$

$$\sigma^{(R)} = 0 \tag{5}$$

$$\sigma^{(S)} = \pi\, C\, \xi^2\, \frac{m_N^2\, x^2}{q^2}\, N^2 \exp\left\{-\frac{k^2}{\xi^2}\right\}\left\{\left(1 + \frac{E - m_N x)}{E + m}\right)^2 + \frac{\xi^2}{(E+m)^2}\right\}.$$

If the q^2 dependence of the W propagator is neglected (which is justified in this case), the structure functions F_i are simply expressed in terms of $\sigma^{(L)}$, $\sigma^{(R)}$, $\sigma^{(S)}$ and then calculated explicitly:

$$F_1(x) = \frac{2m_N\, K}{\sqrt{\pi}\, \xi}\, \frac{(m_N x + m)^2 + \xi^2}{(E + m)^2 + \xi^2} \exp\left\{-\frac{\xi^2}{(E - m_N x)^2}\right\}$$

$$F_2(x) = x F_1(x) \tag{6}$$

$$F_3(x) = F_1(x),$$

in the Bjorken limit.

Besides the Callan-Gross relation here we also have the connection of the third structure function F_3 with the first two. This is a manifestation of the chiral symmetry (which emerges in the Bjorken limit).

The structure function $F_3(x)$ (or rather $xF_3(x)$) is plotted in Fig. 1, for three potential model wave functions described in Ref. 1. The general shape of the xF_3 vs. x curve is in agreement with experimental results[2] in the case of the HOS model[3].

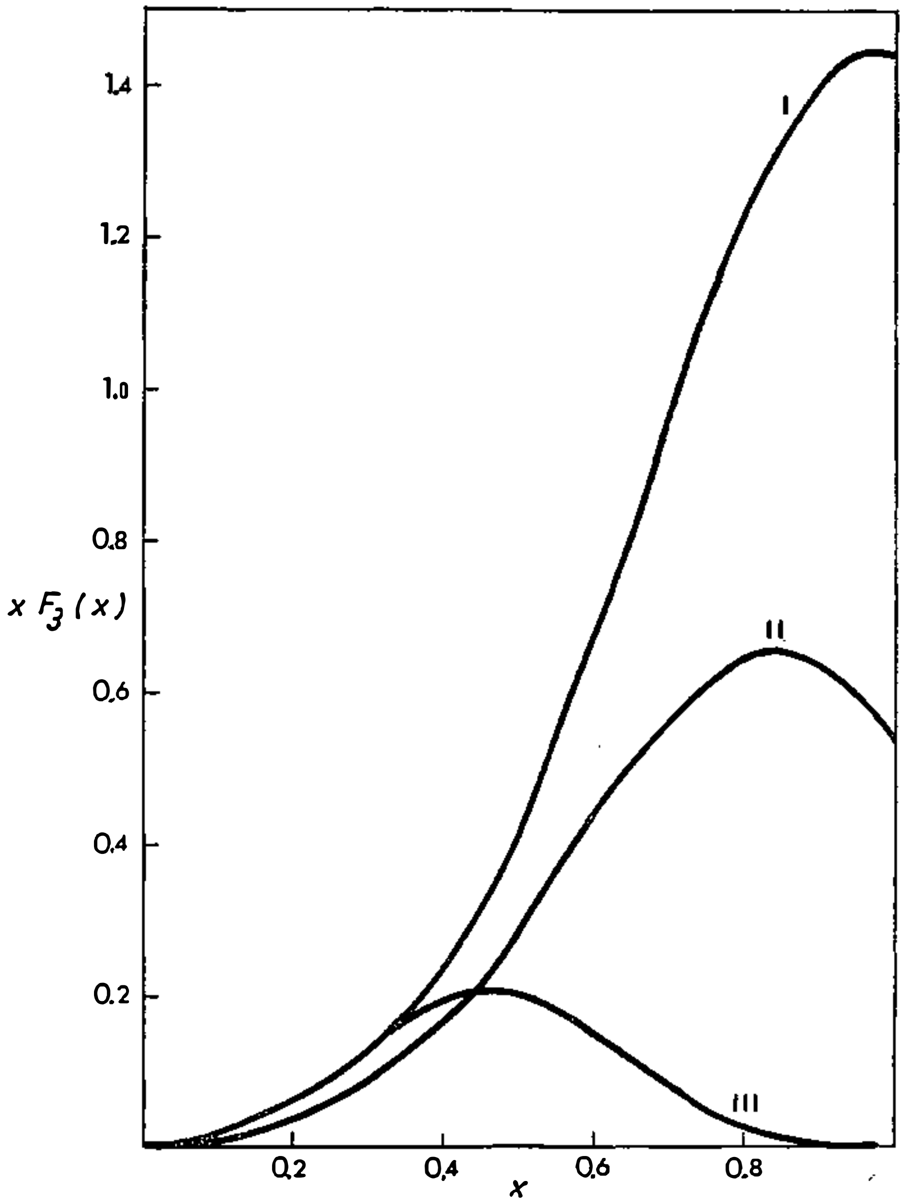

Fig. 1. The structure function $x\ F_3\ (x)$ is shown as a function of x. The normalization is arbitrary. The curve I corresponds to the model described in Ref. 1, the curve II is drawn for the same set of confining parameters with zero quark mass and the curve III describes the structure function behaviour in the HOS model[3].

In conclusion we can say that there are no additional terms coming from the use of quark-confining potential model which means that in the leading term approximation F_i give typical parton-like behaviour.

This approach suffers from several drawbacks, e. g. the oversimplification of the basic picture of the nucleon: we neglect a part of sea quark contributions which are dominant in the $x \to 1$ region. Still, the study of deep inelastic scattering in this sense completes the investigation of the use of the quark-confinement models in hadronic processes.

References

1) S. Fajfer and K. Suruliz, Lett. Nuovo Cimento **42** (1985) 39;
2) K. Kato and Y. Shimizu, Prog. Theor. Phys. **68** (1982) 862;
3) P. Colić, J. Trampetić and D. Tadić, Phys. Rev. D **26** (1982) 2286.

KOMENTAR O DUBOKO-NEELASTIČNOM RASPRŠENJU NEUTRINA U POTENCIJALNIM MODELIMA KVARKOVSKOG KONFAJNMENTA

SVJETLANA FAJFER i KENAN SURULIZ

Institut za fiziku, 71000 Sarajevo

UDK 539.12

Originalni znanstveni rad

Strukturne funkcije za ν-N duboko-neelastično raspršenje proračunate su u potencijalnim modelima kvarkovskog konfajnmenta.