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LETTER TO THE EDITOR

EFFECTIVE ELECTRON MASS FROM WARD IDENTITY WITH NON-LOCAL INTERACTION

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The Ward identity relation

$$\lim_{\mathbf{k}\to 0}\left[\lim_{\omega\to 0}\tilde{\Lambda}_{\mathbf{k}\omega}(\mathbf{p})\right]=\frac{m}{m^*}\frac{1}{z_{pF}}\frac{\varkappa}{\varkappa_F}$$

is used to obtain the effective electron mass m^*, at metallic density by solving the integral equation for the proper vertex function, $\tilde{\Lambda}_{\mathbf{k}\omega}(\mathbf{p})$ with a non-local form for the interaction. We find that for sodium metal $m^*/m = 1.14$, which is in close agreement with the experimental value of 1.11 ± 0.08.

In the last few decades many attempts have been made to obtain a suitable form for the dielectric function, prevalent at metallic densities. Recently we derived an expression for the dielectric function, $\varepsilon(\mathbf{k}, \omega)$, in the static approximation, in terms of the proper vertex function $\tilde{\Lambda}_{\mathbf{k}\omega}(\mathbf{p})$, given as[1]:

$$\varepsilon(\mathbf{k}, \omega) = 1 + 2\, V_k \sum_{\mathbf{p}'} g_{\mathbf{k}\omega}(\mathbf{p}')\, \tilde{\Lambda}_{\mathbf{k}\omega}(\mathbf{p}'). \tag{1}$$

The proper vertex function, satisfies an integral equation[1]:

$$\tilde{\Lambda}_{k\omega}(\mathbf{p}) = 1 - \sum_{\mathbf{p}'} \tilde{I}(\mathbf{p}, \mathbf{p}')\, g_{k\omega}(\mathbf{p}')\, \tilde{\Lambda}_{k\omega}(\mathbf{p}') \tag{2}$$

the kernel of which contains the interaction term $\tilde{I}(\mathbf{p}, \mathbf{p}')$, which characterizes an effective electron-hole interaction. Here

$$g_{k\omega}(\mathbf{p}) = (f_{\mathbf{k}+\mathbf{p}} - f_{\mathbf{p}})/(\omega + i\eta + E_{\mathbf{p}} - E_{\mathbf{p}+\mathbf{k}}) \tag{3}$$

where $E_{\mathbf{p}} = p^2/2m + \sum_{\mathbf{p}'} \tilde{I}(\mathbf{p}, \mathbf{p}') f_{\mathbf{p}'}$, η is a positive infinitesimal quantity, $f_{\mathbf{p}}$ is the Fermi function and $\sum_{\mathbf{p}}$ is a shorthand for $(2\pi)^{-3} \int \mathrm{d}^3 p$. We proposed a solvable model based on the separable form of the kernel leading to a non-local form of the static interaction term namely

$$\tilde{I}(\mathbf{p}, \mathbf{p}') = -4\pi\, \mathrm{e}^2\, \lambda^2/(p^2 + \beta^2)(p'^2 + \beta^2) \tag{4}$$

where λ^2, β^2 are screening parameters[2].

Using Eq. (4) for $\tilde{I}(\mathbf{p}, \mathbf{p}')$ in the integral equation (2) for the proper vertex function we obtain[2]:

$$\tilde{\Lambda}_{k\omega}(\mathbf{p}) = 1 + \frac{4\pi e^2 \lambda^2}{(2\pi)^3 (p^2 + \beta^2)}\, c_{k\omega} \tag{5}$$

where

$$c_{k\omega} = \int d^3 p'\, g_{k\omega}(\mathbf{p}')\, \tilde{\Lambda}_{k\omega}(\mathbf{p}')\, \frac{1}{(p'^2 + \beta^2)}. \tag{6}$$

The solution of the integral equation for $\tilde{\Lambda}_{k\omega}(\mathbf{p})$, becomes:

$$\tilde{\Lambda}_{k\omega}(\mathbf{p}) = 1 + \frac{4\pi e^2 \lambda^2}{(p^2 + \beta^2)}\, I(\beta)/(1 - 4\pi e^2 \lambda^2 I'(\beta)) \tag{7}$$

and the expression for the dielectric function, $\varepsilon(\mathbf{k}, \omega)$, is obtained as:

$$\varepsilon(\mathbf{k}, \omega) = 1 + 2V_k I_1 + \frac{2V_k \cdot 4\pi e^2 \lambda^2 [I(\beta)]^2}{1 - 4\pi e^2 \lambda^2 I'(\beta)} \tag{8}$$

where

$$I_1 = \int \frac{\mathrm{d}^3 p'}{(2\pi)^3}\, g_{k\omega}(\mathbf{p}')$$

$$I(\beta) = \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} g_{k\omega}(\mathbf{p}') \frac{1}{(p'^2 + \beta^2)} \tag{9}$$

and

$$I'(\beta) = -\frac{\partial}{\partial \beta^2} I(\beta) \tag{10}$$

with

$$g_{k\omega}(\mathbf{p}) = (f_{\mathbf{p}+\mathbf{k}} - f_{\mathbf{p}}) \Big/ \left\{ \frac{p^2}{2m} - \frac{(\mathbf{p}+\mathbf{k})^2}{2m} + \omega + i\eta \right\}. \tag{11}$$

The values of real and imaginary parts of I_1, $I(\beta)$ and $I'(\beta)$ are given in our earlier paper[2].

It is well known that the proper vertex function $\tilde{\Lambda}_{k\omega}(\mathbf{p})$ satisfies different[3] Ward identity relations depending on the order of taking the limits $\mathbf{k} \to 0$ and $\omega \to 0$. For instance one of the Ward identity relations requires:

$$\lim_{\omega \to 0} \left[\lim_{\mathbf{k} \to 0} \tilde{\Lambda}_{k\omega}(\mathbf{p}) \right] = z_{\mathbf{p}}^{-1} \tag{12}$$

where $z_{\mathbf{p}}$ is the wave function renormalization constant. In the static limit $z_{\mathbf{p}}$ tends to unity for all $\mathbf{p}$. Thus in this limit the Ward identity relation Eq. (12) becomes:

$$\lim_{\omega \to 0} \left[\lim_{\mathbf{k} \to 0} \tilde{\Lambda}_{k\omega}(\mathbf{p}) \right] = 1 \tag{13}$$

for all $\mathbf{p}$.

This condition is satisfied in our model. We see from Eq. (7) that $I(\beta)$ given by Eq. (9) vanishes in the limit $\mathbf{k} \to 0$ for non-vanishing ω. This follows from the use of Eq. (11) for $g_{k\omega}(\mathbf{p})$ which occurs in equation for $I(\beta)$.

The other Ward identity relation is[4]:

$$\lim_{\mathbf{k} \to 0} \left[\lim_{\omega \to 0} \tilde{\Lambda}_{k\omega}(\mathbf{p}) \right] = \frac{m}{m^*} \frac{1}{z_{pF}} \frac{\varkappa}{\varkappa_F} \tag{14}$$

where m, m^* respectively are the mass and effective mass of the electron, $\varkappa/\varkappa_F$ is the compressibility ratio of the interacting and free electron gas and z_{pF} is the wave function renormalization constant at Fermi surfaces. If we perform the integration for $I(\beta)$ and take the limit $\omega = 0$, $\mathbf{k} \to 0$, we get

$$\lim_{\mathbf{k} \to 0} \left[\lim_{\omega \to 0} \tilde{\Lambda}_{k\omega}(\mathbf{p}) \right] = 1 + \frac{z_1}{z_2} \tag{15}$$

where z_1 and z_2 are the following:

$$z_1 = \frac{k_{FT}^2 \lambda^2}{8\pi(p^2+\beta^2)} \left\{ \frac{2\pi(k_F^2 - \beta^2)}{k_F^2(\beta^2+k_F^2)} \ln \frac{\beta^2}{\beta^2+k_F^2} + \frac{4\pi}{\beta k_F} \tan^{-1} \frac{2\beta k_F}{\beta^2 - k_F^2} - \right.$$
$$\left. - \frac{2\pi(k_F^2+3\beta^2)}{\beta^2(\beta^2+k_F^2)} \right\}$$

$$z_2 = 1 - \frac{k_{FT}^2 \lambda^2}{\beta^2} \left\{ \frac{(k_F^2 + 2\beta^2)}{4\beta^2 (\beta^2 + k_F^2)} - \frac{(\beta^2 + k_F^2)}{2 [(\beta^2 - k_F^2)^2 + 4k_F^2 \beta^2]} - \right.$$

$$\left. - \frac{\beta^2}{2 (\beta^2 + k_F^2)^2} \ln \frac{\beta^2}{\beta^2 + k_F^2} - \frac{1}{4\beta k_F} \tan^{-1} \frac{2\beta k_F}{(\beta^2 - k_F^2)} \right\}.$$

Here the following abbreviations are used:

$$k_{FT}^2 = \frac{6\pi n e^2}{E_F}, \quad E_F = \frac{k_F^2}{2m} \quad \text{and} \quad k_F^3 = 3n\pi^2.$$

The compressibility ratio $\frac{\varkappa_F}{\varkappa}$ of the free and interacting electron gas is calculated from the following expression[5]:

$$\frac{\varkappa_F}{\varkappa} = \frac{k_{FT}^2}{\lim\limits_{k \to 0} \{k^2 [\varepsilon (\mathbf{k}, 0) - 1]\}}.$$

Making use of Eq. (14), we have calculated m^*/m for different values of r_s [$r_s = \left(\frac{4}{3} \pi n a_0^3\right)^{-1/3}$, n is the density and a_0 the Bohr radius] and these are tabulated in Table 1. In the calculations the screening parameters $\lambda^2 = \beta^2 = 2k_F^2 + k_{FT}^2$ have been used. It may be noted that for sodium metal i. e; $r_s = 3.93$, the calculated value of m^*/m is equal to 1.14, which agrees with the experimental result[6] of $m^*/m = 1.11 \pm 0.08$.

TABLE 1.

	$\lambda^2 = \beta^2 = 2k_F^2 + k_{FT}^2$					
	$r_s = 1$	$r_s = 2$	$r_s = 3$	$r_s = 3.93$	$r_s = 5$	$r_s = 6$
m^*/m	1.010	1.043	1.085	1.140	1.167	1.188

Variation of m^*/m with metallic electron densities represented by r_s.

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PRORAČUN EFEKTIVNE ELEKTRONSKE MASE POMOĆU WARDOVA IDENTITETA S NELOKALNOM INTERAKCIJOM

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Originalni znanstveni rad

Polazeći od Wardova identiteta

$$\lim_{\mathbf{k}\to 0}\ [\lim_{\omega\to 0} \tilde{\Lambda}_{\mathbf{k}\omega}(\mathbf{p})] = \frac{m}{m^*}\frac{1}{z_{pF}}\frac{\varkappa}{\varkappa_F}$$

određena je efektivna masa elektrona u metalima rješavanjem integralne jednadžbe za vršnu funkciju $\tilde{\Lambda}_{\mathbf{k}\omega}(\mathbf{p})$ s nelokalnom interakcijom. Nađeno je da je za natrij $m^*/m = 1{,}14$, što je blizu eksperimentalnoj vrijednosti $1{,}11 \pm 0{,}08$.