YU ISSN 0015—3206
FZKAAA 18 (4) 379 (1986)

## LETTER TO THE EDITOR

# ON THE TWO-SOLITON SOLUTION OF A COUPLED SYSTEM IN THREE DIMENSIONS

ASESH ROY CHOUWDHURY and SRIKANTA SASMAL

*High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta — 700 032, India*

Received 9 May 1986

UDC 532.59

Original scientific paper

We have analysed the solitary wave solution of two coupled nonlinear partial differential equations in three dimensions. The system essentially consists of a Kadomstev-Petvisshville equation coupled through derivative terms to a nonlinear Schrödinger equation in three dimensions. Such a set of equations are of interest in plasma turbulence. The one and two soliton solutions are obtained through the method of Hirota.

In recent years there have been various attempts to extend the various class of integrable equations to two space dimensions. Examples of equations that are amenable to the procedure of inverse scattering transform in two space and one time dimension are the KP equation, modified KP equation, Davey-Stewartson system and many others. Here we show that KP equation when coupled to non-linear Schrödinger equation in three dimensions can sustain of coupled equation is another example of integrable equations in three dimension.

The equations under consideration reads

$$\frac{\partial^2 u}{\partial y^2} - \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} + 2bu\frac{\partial u}{\partial x} + c\frac{\partial^3 u}{\partial x^3}\right) = p\frac{\partial^2}{\partial x^2}|\Phi|^2$$

$$\mathrm{i}\frac{\partial \Phi}{\partial t} = u\Phi + \nabla^2\Phi + m|\Phi|^2\Phi, \tag{1}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

In the following analysis we follow the method of Hirota for obtaining the soliton like solutions to (1).

Let us set

$$u = 2(\log f)_{xx}, \qquad b = 3c$$

and $\Phi = \frac{g}{f}$. Then equation (1) is transformed to the following bilinear form

$$(D_y^2 - D_x D_t - aD_x^2 - cD_x^4 + s) f \cdot f = pgg^*$$

$$(\mathrm{i}D_t - D_x^2 - D_y^2)\, g \cdot f = \lambda g \cdot f \qquad (2)$$

$$(D_y^2 + \lambda) f \cdot f = mg \cdot g^*$$

where $\lambda$ and $s$ are constants of integration.

For obtaining solitary wave solutions we set

$$f = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \ldots$$

$$g = a_0\, \mathrm{e}^{\mathrm{i}\Theta} (1 + \varepsilon g_1 + \varepsilon^2 g_2 + \ldots) \qquad (3)$$

with

$$\Theta = \mu x + \nu y + \varkappa t$$

and

$$f_1 = \mathrm{e}^{\eta_1}; \quad \eta = \sigma_1 (x + \gamma_1 y + \omega_1 t); \quad g_1 = \mathrm{e}^{\eta_1 + 2\mathrm{i}\delta}.$$

Then we obtain, by keeping terms of order $\varepsilon$ only

$$\lambda = ma_0^4; \quad s = pa_0^2$$

$$-\varkappa + \mu^2 + \nu^2 = ma_0^2$$

$$\sigma_1^2 (\gamma_1^2 - \omega_1 - a - c\sigma_1^2) = 2pa_0^2 (\cos^2 \delta - 1). \qquad (4)$$

So if we assume the series gets terminated after the $\varepsilon$ term, we get the one soliton solution as

$$\Phi = \frac{g}{f} = \frac{a_0\, \mathrm{e}^{\mathrm{i}\Theta} (1 + g_1)}{1 + f_1} \qquad (5)$$

where we have set

$$\varepsilon = 1.$$

So finally

$$\Phi = \frac{a_0\, e^{i\Theta}(1 + e^{\eta_1 + 2i\delta})}{(1 + e^{\eta_1})} \tag{6}$$

the parameters involved in (6) are connected by equation (4).

To obtain the two-soliton solution, we continue our series up to $\varepsilon^2$ term in equation (3).

Then equating terms occuring as coefficients of $\varepsilon^2$ we get

$$\begin{aligned}(D_y^2 - D_x D_t - aD_x^2 - cD_x^4 + s)(f_2 \cdot 1 + f_1 \cdot f_1 + 1 \cdot f_2) &= \\ = pa_0^2 (g_2 \cdot 1 + g_1 \cdot g_1^* + 1 \cdot g_2^*)\end{aligned} \tag{7}$$

$$\begin{aligned}(iD_t - D_x^2 - D_y^2)\, e^{i\Theta}(g_2 \cdot 1 + g_1 \cdot f_1 + 1 \cdot f_2) &= \\ = \lambda e^{i\Theta}(g_2 \cdot 1 + g_1 \cdot f_1 + 1 \cdot f_2)\end{aligned} \tag{8}$$

$$(D_y^2 + \lambda)(f_2 \cdot 1 + f_1 \cdot f_1 + 1 \cdot f_2) = ma_0^2 (g_2 \cdot 1 + g_1 \cdot g_1^* + 1 \cdot g^*). \tag{9}$$

To solve these system of bilocal equations we set

$$\begin{aligned} f_1 &= a_1 e^{\eta_1} + a_2 e^{\eta_2} \\ g_1 &= a_1 e^{\eta_1 + 2i\delta_1} + a_2 e^{\eta_2 + 2i\delta_2} \\ f_2 &= b_3\, e^{\eta_1 + \eta_2} \\ g_2 &= a_3\, e^{\eta_1 + \eta_2 + 2i(\delta_1 + \delta_2)} \end{aligned} \tag{10}$$

where

$$\eta_i = \sigma_i (x + \omega_i t + \gamma_i y).$$

Substituting in (7), (8) and (9) we generate following relations between the various parameters occuring in (10)

$$b_3 s_1 + 2a_1 a_2 s_2 = pa_0^2 [a_3 \cdot 2\cos(\delta_1 + \delta_2) + 2a_1 a_2 \cos(\delta_1 - \delta_2)] \tag{11}$$

$$\begin{aligned} s_1 &= (\sigma_1\gamma_1 + \sigma_2\gamma_2)^2 - (\sigma_1 + \sigma_2)(\sigma_1\omega_1 + \sigma_2\omega_2) - (\sigma_1 + \sigma_2)^2 a - \\ &\quad - c(\sigma_1 + \sigma_2)^4 + 2pa_0^2 \\ s_2 &= (\sigma_1\gamma_1 - \sigma_2\gamma_2)^2 - (\sigma_1 - \sigma_2)(\sigma_1\omega_1 - \sigma_2\omega_2) - (\sigma_1 - \sigma_2)^2 a - \\ &\quad - c(\sigma_1 - \sigma_2)^4 + 2pa_0^2 \end{aligned} \tag{12}$$

$$b_3 T_1 + 2a_1 a_2 T_2 = ma_0^2 \{a_3\, 2\cos(\delta_1 + \delta_2) + 2a_1 a_2 \cos(\delta_1 - \delta_2)\} \tag{13}$$

where

$$T_1 = (\sigma_1\gamma_1 + \sigma_2\gamma_2)^2 + 2ma_0^2$$

$$T_2 = (\sigma_1\gamma_1 - \sigma_2\gamma_2)^2.$$

So when equations (11), (12) and (13) determine the new parameters, the two-soliton solutions are

$$\Phi = \frac{a_0\, e^{i\Theta} [1 + a_1 e^{\eta_1+2i\delta_1} + a_2 e^{\eta_2+2i\delta_2} + a_3 e^{\eta_1+\eta_2+2i(\delta_1+\delta_2)}}{1 + a_1 e^{\eta_1} + a_2 e^{\eta_2} + b_3 e^{\eta_1+\eta_2}}$$

and

$$u = 2\frac{\partial^2}{\partial_x{}^2}\{\log (1 + a_1 e^{\eta_1} + a_2 e^{\eta_2} + b_3 e^{\eta_1+\eta_2})\}. \tag{14}$$

The forms of the two soliton solution given in (14) clearly indicates that it has got all the properties usually ascribed to a multisoliton states.

Further properties and forms of the multisoliton solutions and conservation laws of such system will be discussed in a future communication.

## References


1) V. E. Zakharov, Lecture in the volume *Soliton*, Edited by R. K. Bullough and P. J. Caudrey, Current Topics in Physics. (Springer Verlag New York—Berlin);
2) M. J. Ablowitz and A. Fokas, In *Nonlinear Phenomena* (Mexico) (Springer Verlag, New York——Berlin);
3) R. Hirota, Talk in *Backlund Transformation*, Lecture Notes in Physics Vol. 515 (Springer Verlag, New York—Berlin).


## DVO-SOLITONSKO RJEŠENJE VEZANOG SISTEMA U TRI DIMENZIJE


ASESH ROY CHOWDHURY i SRIKANTA SASMAL

*High Energy Physics Division, Department of Physics, Jadavpur University,*

*Calcutta — 700 032, India*



UDK 532.59

Originalni znanstveni rad



Analizirali smo solitonsko rješenje dviju vezanih nelineranih parcijalnih diferencijalnih jednadžbi u tri dimenzije. Sistem se u osnovi sastoji od Kadomstev-Petvisshvilleove jednadžbe, koja je preko derivatnog člana vezana na nelinearnu Schrödingerovu jednadžbu u tri dimenzije. Ovakav skup jednadžbi je od interesa za fiziku turbulentne plazme. Jedno od dva solitonska rješenja dobiveno je Hirotovom metodom.