

${}^4\text{He}$ ENERGY LEVELS IN THE ENERGY RANGE $13 \text{ MeV} < E_x < 60 \text{ MeV}$

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The ${}^4\text{He}$ energy level spectrum is investigated within the 33 MeV to 60 MeV excitation energy range by utilizing the generalized R -matrix methodology of Lane and Robson. R -matrix calculations predict numerous $J^\pi = 0^+, 1^+, 2^+, 3^+, 4^+, 0^-, 1^-, 2^-, 3^-$ and 4^- levels within the investigated energy range. The levels tend to be broad and overlapping which will complicate their experimental investigation.

1. Introduction

Previously, we have performed a series of calculations¹⁻⁷⁾, using the generalized R -matrix methodology of Lane and Robson⁸⁾, to study the level scheme of the ${}^4\text{He}$ system. These calculations illustrated the importance of including binary breakup clusters in the model equations as well as the possibility of the existence of resonance states in addition to those provided in the compilation of Fiarman and Meyerhof⁹⁾. We have also provided R -matrix wave functions and shell-model configurations for newly suggested resonances^{5,7)}. Most of our work¹⁻⁷⁾ has addressed the energy region below 33 MeV which spans the region of the levels of Fiarman and Meyerhof. However, less work has focused on the energy region above 33 MeV excitation energy.

In this paper, we report the results of generalized R -matrix calculations within the energy range between 33 MeV and 60 MeV excitation energy. This energy region has been the subject of experimental efforts^{10,11)} and it is hoped that the present publication will be an aid in future experimental work to quantify energy levels in the ${}^4\text{He}$ system.

2. Theory and formulation

The essential features of the model used in this paper have been outlined previously¹⁻⁷). The model is constructed within a $4\hbar\omega$ model space which is considerably larger than the $2\hbar\omega$ model spaces usually utilized in ${}^4\text{He}$ studies. Model basis states are completely antisymmetrized, translationally invariant harmonic oscillator eigenfunctions. The eigenenergies are obtained from the solution of the equation⁸⁾

$$\sum_{\lambda'} [\langle \lambda | H - E | \lambda' \rangle_0^{a_c} + \sum_c \gamma_{\lambda c} (b_{\lambda' c} - b_c) \gamma_{\lambda' c}] A_{\lambda'} = 0 \quad (1)$$

where $|\lambda\rangle$ are the model basis states, a_c are the channel radii, A_{λ} are expansion amplitudes obtained by solving Eq. (1), and the other quantities appearing in Eq. (1) are defined in Refs. 1-7. The sum over channels (c) includes all contributions of $4\hbar\omega$ or less oscillator excitation from all three binary breakup channels — i. e. $p + {}^3\text{H}$, $n + {}^3\text{He}$ and $d + {}^2\text{H}$.

Specific formulae for the resonance positions (E_R^μ) and level widths (Γ_R^μ) may be defined as

$$E_R^\mu = \text{Re} [E_\mu - \xi_\mu(E_R^\mu)] \quad (2)$$

and

$$\Gamma_R^\mu = -2\text{Im} [E_\mu - \xi_\mu(E_R^\mu)] \quad (3)$$

where E_μ is defined as the solution of Eq. (2) and ξ_μ is defined in terms of known R -matrix energies and widths E_μ , $\gamma_{\mu c}$ and standard Coulomb radial functions^{12,13}). The reader should refer to Refs. 1-7 for details of the application of Eqs. (1)-(3) to the ${}^4\text{He}$ system.

Our model results can also be directly compared with shell model solutions by setting all channel radii equal to infinity. In the limit $a_c \rightarrow \infty$, Eq. (1) becomes

$$\sum_{\lambda'} \langle \lambda | H - E_{sm} | \lambda' \rangle_0^\infty A_{\lambda'} = 0 \quad (4)$$

where E_{sm} is the shell-model level energy. It should be noted that the shell-model limit only provides energy position information. The level width can not be determined in the shell-model limit.

3. Results and discussion

In this section, model results are summarized in Tables 1-6 for the energy region $33 \text{ MeV} < E_{sm} < 60 \text{ MeV}$. These results are also compared with the shell-model results of Szydlik et al.¹⁴). Szydlik et al. performed a $2\hbar\omega$ ${}^4\text{He}$ shell-model calculation and attempted to compensate for model space truncation effects by calculating in second-order perturbation theory modifications to the two-body interaction matrix elements representing excitations into the $4\hbar\omega$ model space. In

Ref. 1, it was shown that the perturbative treatment was not able to compensate fully for the differences in $2\hbar\omega$ and $4\hbar\omega$ basis sizes. However, the results of Szydlik et al. do provide approximate level positions and will be compared with our complete $4\hbar\omega$ basis treatment. This comparison will further illustrate the differences between the perturbative and complete treatment of the $4\hbar\omega$ solution in ${}^4\text{He}$.

Before discussing detailed model results, a discussion of the impact of omitting three- and four-body breakup channels on the level width and level position is warranted. In Ref. 5, an analysis of $A = 4$ model results indicated that multi-particle breakup channels ($n + p + {}^2\text{H}$ and $2n + 2p$) could have a sizable impact on the calculated model widths. In particular, a factor of 2 effect was noted for the ${}^4\text{H}$ and ${}^4\text{Li}$ systems^{1,5,15,16}). The addition of three- and four-body breakup channels will likely have a relatively modest impact on the level position, since the addition of three binary breakup channels has a relatively small effect on the shell-model (no channel) level energy^{15,16}). Binary breakup channels lead to about a 15% impact on the shell model levels for the states summarized in Tables 1–6.

The levels summarized in Tables 1–6 are generally broad but about 10% of these levels are narrow (< 0.01 MeV). Even with the factor of 2 uncertainty in level width noted above, the levels still present an opportunity for experimental observation. Since the width of a level depends on its lifetime, these narrow resonances are indicative of states with lifetimes considerably longer than the current set of broad levels in ${}^4\text{He}^9$). Physically, the states with narrow widths will be characterized by inhibited transitions which lead to the long lifetimes noted above.

Positive parity levels

Positive parity levels in the energy region between 33 MeV and 60 MeV excitation energy are summarized in Tables 1–6. Levels with $J^\pi = 0^+ - 4^+$ are found to be predicted to lie within this energy range by our model calculations.

Table 1 summarizes the Szydlik et al. $4\hbar\omega$ shell-model (R -matrix calculation with no channel contributions), and $4\hbar\omega$ R -matrix energy level predictions (with

TABLE 1.

(J^π, T)	Excitation energy (MeV)			R -matrix ^{c)} width (MeV)
	Szydlik et. al.	R -matrix ^{a)} (shell-model)	R -matrix ^{b)} (with channels)	
$(0^+, 1)$	41.74	37.61	40.32	10.24
$(0^+, 0)$	44.24	41.75	43.41	8.77
$(0^+, 0)$	50.25	47.39	49.20	2.78
$(0^+, 2)$	50.87	48.66	48.90	0.01
$(0^+, 0)$	59.31	52.55	53.60	6.74
$(0^+, 1)$	62.02	55.78	56.92	< 0.01
$(0^+, 2)$	62.56	56.68	57.38	< 0.01
$(0^+, 0)$	—	57.14	65.78	9.69

0^+ levels in ${}^4\text{He}$.

a) E_{sm}^μ determined from Eq. (4).

b) E_R^μ determined from Eq. (2).

c) Γ_R^μ determined from Eq. (3).

channel contributions) for 0^+ levels. This table also provides the R -matrix energy level widths for the 0^+ states in ${}^4\text{He}$ between 33 MeV and 60 MeV excitation energy. The shell-model calculations (Szydlik et al. and R -matrix with no channel contributions) predict the same 0^+ level order, but the actual energy level positions differ by as much as 7 MeV. In addition, the R -matrix levels tend to lie lower in excitation energy than their perturbative counterparts.

The inclusion of channels into the R -matrix model generally does not alter the level energy by more than a few MeV¹⁾. However, the $(0^+, 0)$ 57.14 MeV (sm) level is shifted to 65.78 MeV when reaction channels are included. In addition, the inclusion of channels shifted the energy level order for the $(0^+, 0)$ 47.39 MeV (sm) and $(0^+, 2)$ 48.66 MeV (sm) levels.

The 0^+ energy levels are generally found to be broad and overlapping. A notable exception are the $(0^+, 2)$ levels which are narrow and on the order of 10 keV or less. The 0^+ levels also appear to be either broad (3 to 10 MeV) or very narrow (10 keV or less).

TABLE 2.

(J^π, T)	Excitation energy (MeV)			R -matrix width (MeV)
	Szydlik et. al.	R -matrix (shell-model)	R -matrix (with channels)	
$(1^+, 1)$	39.21	36.98	36.71	10.71
$(1^+, 0)$	44.19	41.64	42.06	0.30
$(1^+, 1)$	45.52	41.93	45.52	0.26
$(1^+, 1)$	48.13	45.54	48.93	0.79
$(1^+, 0)$	48.51	46.48	46.95	7.45
$(1^+, 1)$	54.01	48.93	49.04	0.78
$(1^+, 1)$	57.23	52.15	52.37	0.09
$(1^+, 2)$	57.98	53.77	54.02	0.09
$(1^+, 0)$	58.06	54.03	54.25	0.21
$(1^+, 1)$	59.01	54.47	54.71	<0.01
$(1^+, 1)$	60.48	55.45	55.70	0.05

1^+ levels in ${}^4\text{He}$.

The 1^+ (Table 2), 2^+ (Table 3) and 3^+ (Table 4) levels generated from our $4\hbar\omega$ shell-model calculations also lie below the corresponding levels of Szydlik et al. The differences between the R -matrix ($4\hbar\omega$ shell-model) and Szydlik et al. predictions are smaller than noted for 0^+ levels, but these differences are still on the order of a few MeV. The 4^+ level (Table 4) is an exception to the trend noted above with the R -matrix level exceeding the Szydlik et al. excitation energy prediction by about one MeV.

The addition of channels also alters the ordering of levels within the 1^+ , 2^+ and 3^+ spectra. The shifting of levels from their shell model positions are most dramatic in the 2^+ spectrum.

Level widths for the 1^+ levels are generally similar to the 0^+ widths. As with the 0^+ spectrum, the 1^+ levels can be grouped into two groups characterizing their width — i. e. broad, 8 to 11 MeV levels, and narrow, levels with widths

less than 1 MeV. Similar combinations of broad and narrow levels are also noted for the 2^+ and 3^+ spectra.

TABLE 3.

(J^π, T)	Excitation energy (MeV)			R -matrix width (MeV)
	Szydlik et al.	R -matrix (shell-model)	R -matrix (with channels)	
$(2^+, 0)$	40.69	39.69	40.57	6.03
$(2^+, 1)$	43.42	41.76	43.36	0.96
$(2^+, 1)$	44.85	42.32	46.63	5.58
$(2^+, 0)$	47.49	44.50	45.47	7.13
$(2^+, 1)$	48.58	45.06	46.57	0.37
$(2^+, 0)$	49.53	46.48	51.15	11.30
$(2^+, 1)$	52.75	50.29	50.56	0.55
$(2^+, 2)$	52.99	50.37	51.09	0.44
$(2^+, 0)$	53.49	51.33	51.56	0.01
$(2^+, 1)$	56.80	53.18	53.65	0.18
$(2^+, 2)$	56.98	53.43	53.39	2.82
$(2^+, 0)$	59.12	54.47	54.71	<0.01

2^+ levels in ${}^4\text{He}$.

TABLE 4.

(J^π, T)	Excitation energy (MeV)			R -matrix width (MeV)
	Szydlik et al.	R -matrix (shell-model)	R -matrix (with channels)	
$(3^+, 1)$	42.79	41.49	42.23	0.08
$(3^+, 0)$	42.98	42.08	49.47	4.36
$(3^+, 0)$	47.55	45.45	46.78	6.92
$(3^+, 1)$	48.89	47.61	47.83	0.17
$(3^+, 1)$	52.34	50.40	50.69	<0.01
$(4^+, 0)$	42.49	43.31	42.40	4.01

3^+ and 4^+ levels in ${}^4\text{He}$.

Negative parity levels

Negative parity levels are summarized in Tables 5 and 6. Our model predicts levels with $J^\pi = 0^- - 4^-$ to lie within the 49 MeV to 60 MeV energy region.

The shell-model calculations of Szydlik et al. predict no negative parity levels within the energy range of interest. R -matrix levels are predicted, but there are fewer negative parity levels than their positive parity counterparts. The impact of including channel contributions to the energy level position is similar to that noted for the positive parity levels. In addition, no level order shifts within a given J^π spectrum are introduced by including channel contributions to the level energy. Finally, level widths are similar to those of the positive parity levels.

TABLE 5.

(J^π, T)	Excitation energy (MeV)		R-matrix width (MeV)
	R-matrix (shell-model)	R-matrix (with channels)	
$(0^-, 0)$	53.32	57.11	2.63
$(0^-, 1)$	57.61	60.53	0.63
$(0^-, 1)$	59.96	60.66	0.38
$(1^-, 0)$	49.28	54.44	2.48
$(1^-, 1)$	56.28	60.72	0.86
$(2^-, 0)$	52.06	53.46	1.52
$(2^-, 1)$	52.82	58.87	0.05
$(2^-, 1)$	57.68	60.05	0.87
$(2^-, 0)$	58.51	60.68	3.38

0^- , 1^- and 2^- levels in ${}^4\text{He}$.

TABLE 6.

(J^π, T)	Excitation energy (MeV)		R-matrix width (MeV)
	R-matrix (shell-model)	R-matrix (with channels)	
$(3^-, 1)$	57.14	58.43	0.45
$(3^-, 1)$	57.62	58.52	0.22
$(3^-, 0)$	58.73	59.17	9.86
$(3^-, 0)$	59.45	59.20	3.56
$(4^-, 1)$	56.56	56.95	0.48
$(4^-, 0)$	57.11	64.98	3.04

3^- and 4^- levels in ${}^4\text{He}$.

4. Conclusions

R-matrix calculations predict that there are numerous $J^\pi = 0^+$, 1^+ , 2^+ , 3^+ , 4^+ , 0^- , 1^- , 2^- , 3^- and 4^- levels in the ${}^4\text{He}$ energy range between 33 MeV and 60 MeV excitation. Many of the levels are broad, greater than 3 MeV, and the levels tend to be overlapping. There are also many levels which are very narrow, 10 keV or less. Detection of the ${}^4\text{He}$ levels in the range $33 \text{ MeV} < E_x < 60 \text{ MeV}$ will be complicated by the fact that they are broad and overlapping.

Model comparisons with the perturbative treatment of Szydlik et al. indicate the perturbative levels are generally predicted to lie within 15% of the $4\hbar\omega$ shell-model levels. The $4\hbar\omega$ levels are also generally found to lie lower in energy than their perturbative counterparts. Although numerous negative parity levels are predicted between 49 MeV and 60 MeV, no perturbative negative parity levels

are predicted. Thus, the perturbative treatment appears to provide a more realistic description of the positive parity states.

The inclusion of channels can provide a significant change in the level position. Channel contributions can also change the relative level ordering and have a more significant impact on the levels above 33 MeV than those contributions had for the levels of Fiarman and Meyerhof.

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ENERGETSKA STANJA U ^4He OD 33 DO 60 MeV ENERGIJE UZBUDE

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Originalni znanstveni rad

Istražen je energetski spektar ^4He u području od 33 do 66 MeV energije uzbude koristeći poopćenu metodu R -matrice Lane-a i Robsona. Ti proračuni predviđaju brojna stanja angularnog momenta i pariteta 0^+ , 1^+ , 2^+ , 3^+ , 4^+ , 0^- , 1^- , 2^- , 3^- i 4^- . Stanja su uglavnom široka i preklapaju se, što otežava eksperimentalno istraživanje.