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LETTER TO THE EDITOR

GRADED SCHWINGER REALIZATION FOR DYNAMICAL SUPERSYMMETRY

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and

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We have constructed Schwinger realization for SU(6/4) supergroup, associated with coupling $j = \frac{3}{2}$ fermion to the SU(6) boson core.

Using holomorphically parametrized generalized coherent state and the explicit form of root vectors, a single deductive inference is recently presented[1)] for Schwinger realization (Interacting boson model-IBM[2)]), Holstein-Primakoff realization (Truncated quadrupole phonon model-TQM[3)]) and Dyson realization (Finite quadrupole phonon model-FQM).

The Schwinger realization of SU(6) quadrupole collective algebra reads

$$\hat{q}_\mu^{SR} = s\, d_\mu^+ + d_{\bar{\mu}} s \tag{1}$$

$$\hat{p}_\mu^{SR} = \mathrm{i}\,(s d_{\bar{\mu}}^+ - s^+ d_\mu) \tag{2}$$

$$\mathrm{i}\,[\hat{q}_\mu, \hat{p}_{\mu'}]^{SR} = d_\mu^+ d_{\mu'}^+ + d_{\bar{\mu}'}^+ d_{\bar{u}} - 2\delta_{\mu\mu'} s^+ s \tag{3}$$

$$[\hat{q}_\mu, \hat{q}_{\mu'}]^{SR} = d_\mu^+ d_{\bar{\mu}'} - d_{\mu'}^+ d_{\bar{\mu}}. \tag{4}$$

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Here $s^\dagger$ and $d_\mu^\dagger$ are creation operators of s and d bosons, respectively,

$$\tilde{d}_\mu^\dagger = (-)^\mu d_{-\mu}^\dagger \quad \text{and} \quad N = \sum_\mu b_\mu^\dagger b_\mu .$$

In the present note we present an extension of Schwinger realization to the U(6/4) supergroup, which corresponds to $j = 3/2$ fermion coupled to O(6) boson core[4-6].

The SU(6/4) graded algebra is generated by a set of 99 operators in canonical form, $\{S_p^q, p, q = 1, \ldots, 10\}$, with standard commutation/anticommutation relations for the Lie bracket $[S_p^q, S_r^s]$ (Ref. 7). Indices in the range $n = 1, \ldots, 6$ and $6 + \tau = 7, \ldots, 10$ ($\tau = 1, \ldots, 4$) are referred to as even and odd, respectively. The corresponding grading is $\deg(n) = 0$ and $\deg(6 + \tau) = 1$, respectively. Generators S_p^q are referred to as even and odd for $\deg(p) + \deg(q) = 0{,}2$ and 1, respectively.

In order to obtain a convenient form for microscopic mapping, we transform the generators $\{S_p^q\}$ into a new set of generators which explicitly separates the SU(6) $\oplus$ SU(4) subalgebra, with the appearance of the 6×6 and 4×4 transformation matrices of the Cartan-Weyl canonical form associated with SU(6) and SU(4) algebra, respectively. In the next step we introduce the 1×10 one-row matrix $\zeta^\dagger$ for the supermultiplet

$$\zeta^\dagger = s^\dagger d_{-2}^\dagger\ d_{-1}^\dagger\ d_0^\dagger\ d_1^\dagger\ d_2^\dagger\ a_{-3/2}^\dagger\ a_{-1/2}^\dagger\ a_{1/2}^\dagger\ a_{3/2}^\dagger . \tag{5}$$

Here $s^\dagger$ and $d_\mu^\dagger$ denote creation operators of s-boson and d-boson angular with momentum z-component $\mu = 0, \pm 1, \pm 2$, respectively, and $a_m^\dagger$ the creation operator of $j = 3/2$ fermion with angular momentum z-components $m = \pm 1/2, \pm 3/2$.

In this way we obtain

$$E_\alpha = \frac{1}{\sqrt{12}} s^\dagger d_\mu, \qquad \alpha = 1, \ldots, 5; \quad \mu = 0, \pm 1, \pm 2 \tag{6}$$

$$E_\alpha = \frac{1}{\sqrt{12}} d_\mu^\dagger d_\nu, \qquad \alpha = 6, \ldots, 15; \quad \mu, \nu = 0, \pm 1, \pm 2 \tag{7}$$

$$H_1 = \frac{1}{2\sqrt{6}} (s^\dagger s - d_{-2}\, d_{-2}) \tag{8}$$

$$H_2 = \frac{1}{6\sqrt{2}} (s^\dagger s + d_{-2}^\dagger\, d_{-2} - 2 d_{-1}^\dagger\, d_{-1}) \tag{9}$$

$$H_3 = \frac{1}{12} (s^\dagger s + d_{-2}^\dagger\, d_{-2} + d_{-1}^\dagger\, d_{-1} - 3 d_0^\dagger d_0) \tag{10}$$

$$H_4 = \frac{1}{4\sqrt{15}} (s^\dagger s + d_{-2}^\dagger\, d_{-2} + d_{-1}^\dagger\, d_{-1} + d_0^\dagger\, d_0 - 4 d_1^\dagger\, d_1) \tag{11}$$

$$H_5 = \frac{1}{6\sqrt{10}} (s^\dagger s + d_{-2}^\dagger\, d_{-2} + d_{-1}^\dagger\, d_{-1} + d_0^\dagger\, d_0 + d_1^\dagger\, d_1 - 5 d_2^\dagger\, d_{-2}) \tag{12}$$

$$E_{-\alpha} = (E_\alpha)^\dagger \tag{13}$$

$$e_\beta = \frac{1}{\sqrt{8}} a_\sigma^\dagger a_\tau \qquad \beta = 1, \ldots, 6; \quad \sigma, \tau = 1, \ldots, 4 \tag{14}$$

$$h_1 = \frac{1}{4}(a_1^\dagger a_1 - a_2^\dagger a_2) \tag{15}$$

$$h_2 = \frac{1}{4\sqrt{3}}(a_1^\dagger a_1 + a_2^\dagger a_2 - 2a_3^\dagger a_3) \tag{16}$$

$$h_3 = \frac{1}{4\sqrt{6}}(a_1^\dagger a_1 + a_2^\dagger a_2 + a_3^\dagger a_3 - 3a_4^\dagger a_4) \tag{17}$$

$$e_{-\beta} = (e_\beta)^\dagger \tag{18}$$

$$\mathscr{H}_9 = -2 \sum_{m=1}^{6} b_m^\dagger b_m - 3 \sum_{\sigma=1}^{4} a_\sigma^\dagger a_\sigma = -2\,(s^\dagger s + \sum_{\mu=-2}^{2} d_\mu^\dagger d_\mu) - 3 \sum_{\sigma=1}^{4} a_\sigma^\dagger a_\sigma \tag{19}$$

$$O_n^{6+\tau} = b_n^\dagger a_\tau \tag{20}$$

$$O_{6+\tau}^n = a_\tau^\dagger b_n \tag{21}$$

where the GSR labels are omitted. The operators $\{b_m^\dagger; m = 1, \ldots, 6\}$ and $\{a_\sigma^\dagger; \sigma = 1, \ldots, 4\}$ correspond to the boson creation operators $\{s^\dagger, d_\mu^\dagger; \mu = -2, \ldots, 2\}$ and fermion creation operators $\{a_m, m = \pm 1/2, \pm 3/2\}$, respectively.

The operators (6)—(13) present graded Schwinger realization (GSR) for generators of the SU(6) subalgebra $\{G_A\}$ and the operators (14)—(19) with GSR of the SU(4) subalgebra $\{g_\lambda\}$ and (20), (21) are odd generators $\{O\}$ of the SU(6/4) superalgebra.

The generators (6)—(21), to be referred to as IBFM/PTQM form of SU(6/4) generators, are appropriate for investigation of SUSY's.

We see that the IMB/TQM Hamiltonian[2,3] is SO(3)-invariant built from the even generators of SU(6/4).

On the other hand, the IBFM/PTQM Hamiltonian[8] is an SO(3)-scalar built from both even and odd generators of SU(6/4). Dynamical and monopole boson-fermion interactions are built from even generators of SU(6/4), and the exchange interaction from odd generators.

Generally, H_{IBFM} is rotational invariant constructed from the generators of SU(6/M) superalgebra in the form of GSR, i. e. its inherent feature is SU(6/M) supersymmetric algebraic structure.

On the other hand, the IBFM form (6)—(21) in GSR realization provides a suitable basis to establish microscopic foundation of the nuclear SU(6/4) SUSY. This enables us to infer the conditions which lead to an approximate closure of the SU(6/4) graded algebra from the microscopic point of view and extract an algebraic structure of SU(6/4) graded algebra from microscopic Hamiltonian.

As to the microscopic approach along this line, we propose mapping employing Random Phase Approximation in the framework of QCA, by enforcing a set of non-linear conditions on the microscopic structure of RPA operators[9]. This yields microscopically the enforced SU(6) subalgebra of SU(6/4) superalgebra. On the other hand, bifermion operators, corresponding to valence-shell quasiparticles should be mapped in a straightforward way onto SU(4) subalgebra (14)—(18). The microscopic origin of odd generators $\{O_1\}$ is more complex. In this case the mapping, consistent with that for even generators, implies the microscopic operators $Q^+_\mu \alpha_\tau$ and $\alpha^+_\tau Q_\mu$. The analysis of commutation/anticommutation relations reveals that such microscopic analogs of odd generators can be obtained under additional SU(6/4) enforcing conditions, which are presently under investigation.

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GRADIRANA SCHWINGEROVA REALIZACIJA ZA DINAMIČKU SUPERSIMETRIJU

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Konstruirana je Schwingerova realizacija za SU(6/4) supergrupu, koja je pridružena vezanju fermiona $j = \frac{3}{2}$ i SU(6) bozonske srednjice.

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