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EFFECT OF MAGNETIC QUANTIZATION ON THE EINSTEIN RELATION IN n-CHANNEL INVERSION LAYERS ON KANE-TYPE SEMICONDUCTORS

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An attempt is made to derive a generalized expression for the Einstein relation of the diffusivity-mobility ratio of the carriers in n-channel inversion layers on Kane-type semiconductors under magnetic quantization by using the three-band Kane model. Besides, the oscillatory dependence of the same ratio on reciprocal magnetic field has also been theoretically investigated taking n-channel InAs as an example.

1. Introduction

It is well-known that the Einstein relation for the diffusivity-mobility ratio of the carriers in semiconductors (hereafter referred to as DMR) is a very useful one since one can determine the diffusivity from this relation by knowing the mobility and vice-versa. Besides, this is more accurate than any of the individual relation

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for diffusivity or mobility which are considered to be the two most videly used parameters for carrier transport in semiconductors. Furthermore, in recent years, the connection of DMR with the velocity auto-correlation function[1], the modification due to non-linear charge transport[2] and the different modifications of the DMR for degenerate semiconductors under various physical conditions have been extensively investigated in the literature[3-10]. Nevertheless, the DMR in n-channel inversion layers of Kane-type semiconductors under magnetic quantization has yet to be theoretically worked out for the more difficult case which occurs from the use of 3-band Kane model[12] since, in recent years, the inversion layers on non-parabolic semiconductors are being increasingly studied for their peculiar physical characteristics[11].

In what follows, we shall first derive the electron statistics for n-channel inversion layers on Kane-type semiconductors according to three-band Kane model by including the effects of spin and broadening, respectively. This will make our analysis a generalized one since one can obtain the corresponding results for n--channel inversion layers on parabolic semiconductors by equating the non-parabolicity parameter to zero. It may be stated in this contex that the various transport phenomena and the derivation of the expressions of many important physical parameters are based on the appropriate electron statistics in such materials. We shall then use the same statistics for the evaluation of the DMR in n-channel inversion layers on Kane-type semiconductors. We shall also derive the DMR in bulk specimens of Kane type semiconductors under magnetic quantization by including both spin and broadening effects for the purpose of comparison. Besides, we shall investigate theoretically the magnetic field dependence of the DMR, for bulk specimens of InAs and also taking n-channel InAs as examples.

2. *Theoretical background*

The density-of-states function of the electrons in $2D$ inversion layers on semiconductors in the presence of a quantizing magnetic field B along z direction can be expressed[11], taking into account the broadening of Landau levels, as

$$N_{2D}(E) = \frac{g_v\, e\, B}{2\,\pi\,\hbar}\left(\frac{2}{\pi}\right)^{1/2} \sum_{n=0}^{n_{max}} \frac{1}{\Gamma} \exp\left[-\frac{2}{\Gamma^2}(E - \varepsilon_{n,\pm})^2\right] \tag{1}$$

where g_v is the valley degeneracy, e is the electron charge, $\hbar = h/2\,\pi$, h is Planck's constant, $n\,(\equiv 0, 1, 2, \dots)$ is Landau quantum number, Γ is the width of broadened Landau levels, $\varepsilon_{n,\pm}$ are the unperturbed Landau energy levels including the effects of electron spin[11] and E is a band-structure dependent parameter. For three band Kane model the dispersion law under magnetic quantization can be expressed[12] as

$$\frac{\hbar^2\, k_z^2}{2\, m_0^*} = \gamma\,(\varepsilon) - \left(n + \frac{1}{2}\right) \hbar\, \omega_0 \pm \frac{1}{2}\, \mu_0\, B\, g\,(\varepsilon) \tag{2}$$

in which m_0^* is the band edge effective electron mass,

$$\gamma(\varepsilon) \equiv \frac{\varepsilon(\varepsilon + E_g)(\varepsilon + E_g + \Delta)\left(E_g + \frac{2}{3}\Delta\right)}{E_g(E_g + \Delta)\left(\varepsilon + E_g + \frac{2}{3}\Delta\right)},$$

E_g is the band gap, Δ is the spin-orbit splitting of the valence band,

$$\omega_0 \equiv \frac{eB}{m_0^*}, \quad g(\varepsilon) \equiv 2\left[1 + \left(1 - \frac{m_0}{m_0^*}\right)\Delta(3\varepsilon + 3E_g + 2\Delta)^{-1}\right],$$

m_0 is the free electron mass and $\mu_0 \equiv e\hbar/2m_0$.

Putting $k_z = 0$ and $\varepsilon = \varepsilon_{n,\pm}$, from Eq. (2) we get

$$\gamma(\varepsilon_{n,\pm}) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \pm \frac{1}{2}\mu_0 B g(\varepsilon_{n,\pm}) = 0. \tag{3}$$

Combining Eq. (1) with the Fermi-Dirac occupation probability factor the generalized expression of the electron statistics under magnetic quantization in n-channel inversion layers on Kane-type semiconductors according to 3-band Kane model can be expressed as

$$n_0 = C k_B T \left(\frac{2}{\pi}\right)^{1/2} \sum_{n=0}^{n_{max}} \varrho_J(\Gamma)^{-1} \tag{4}$$

in which $C \equiv g_v eB/h$, k_B is the Boltzmann constant, T is the temperature,

$$\varrho_\tau \equiv [[\sum_{J=0}^{\infty}\Phi_+(J) + \sum_{J=0}^{\infty}\Phi_-(J)] - (\sqrt{\pi}/2a)\{1 - \mathrm{erf}(b)\}],$$

$$\Phi_\pm(J) \equiv [L_J(\sqrt{\pi})(2a)\exp(\delta_\pm^2)(1 \pm \mathrm{erf}(\delta_\pm))],$$

$$L_J \equiv (-1)^J \exp(-b^2), \; b \equiv \gamma^{-1}(\eta - \Theta_n), \; \gamma \equiv \Gamma(k_B T\sqrt{2})^{-1}, \; \eta \equiv E_F/k_B T,$$

E_F is the Fermi energy in the presence of magnetic quantization,

$$\Theta_n \equiv (k_B T)^{-1}(\varepsilon_{n,\pm}), \quad \delta_{,\pm} \equiv [q_\pm/2a], \quad q_\pm \equiv 2ab \equiv J, \; a \pm 1/\gamma$$

and erf denotes the error function. Thus, since the DMR of the electrons in semiconductors can be expressed[5] as

$$\frac{D}{\mu} = \frac{k_B T}{e} n_0 \bigg/ \frac{dn_0}{d\eta} \tag{5}$$

We can combine Eqs. (4) and (5) to obtain an expression of the same ratio in n-channel inversion layers on Kane-type semiconductors whose energy band structures are defined by three band Kane model as

$$\left(\frac{D}{\mu}\right)_B = \frac{k_B T}{e} [\sum_{n=0}^{n_{max}} \varrho_J (\Gamma)^{-1}] [\sum_{n=0}^{n_{max}} \zeta_J (\Gamma)^{-1}]^{-1} \tag{6}$$

where

$$\zeta_J \equiv [\exp(-b^2) + \sum_{J=0}^{\infty} \psi_+ (J) + \sum_{J=0}^{\infty} \psi_- (\tau)]$$

and

$$\psi_\pm (J) = [L_J (\sqrt{\pi}) \{1 \mp \mathrm{erf}(\delta_\pm)\} a^{-2} (\delta_\pm) - (a^{-2}) L_J -$$

$$- b (\sqrt{\pi}) \exp(\delta_\pm^2) L_J \{1 \mp \mathrm{erf}(\delta_\pm)\}].$$

It may be noted that the general forms of the electron statistics and the DMR, respectively, in n-channel inversion layers on parabolic semiconductors, including both spin and broadening effects, will be given by Eqs. (4) and (6) where

$$\varepsilon_{n,\pm} = \left(n + \frac{1}{2}\right) \hbar \omega_0 \pm \frac{1}{2} \mu_0 B g_0, \quad g_0 \equiv 2. \tag{7}$$

The use of Eq. (2) leads to the expressions of the electron statistics and the DMR, respectively, under magnetic quantization in bulk specimens of Kane-type semiconductors whose energy-band structures are defined by three band Kane model as

$$n_0 = \frac{e B}{2 \pi^2 \hbar} \sum_{n=0}^{n_{max}} [G_\pm (n, E_F, \Gamma) + y_\pm (n, E_F, \Gamma)] \tag{8}$$

and

$$\left(\frac{D}{\mu}\right)_B = \frac{1}{e} \left[\frac{\sum_{n=0}^{n_{max}} \{G_\pm (n, E_F, \Gamma) + y_\pm (n, E_F, \Gamma)\}}{\sum_{n=0}^{n_{max}} \{G'_\pm (n, E_F, \Gamma) + y'_\pm (n, E_F, \Gamma)\}} \right] \tag{9}$$

where

$$G_\pm (n, E_F, \Gamma) \equiv (\sqrt{2})^{-1} [C_\pm (n, E_F, \Gamma) + \{C_\pm^2 (n, E_F, \Gamma) - D_\pm^2 (n, E_F, \Gamma)\}^{1/2}]^{1/2},$$

$$C_\pm (n, E_F, \Gamma) \equiv \left[\gamma_1 (E_F, \Gamma) - \left(n + \frac{1}{2}\right) \hbar \omega_0 \pm \frac{1}{2} \mu_0 B g_1 (E_F, \Gamma)\right],$$

$$\gamma_1 (E_F, \Gamma) \equiv \left(E_g + \frac{2}{3} \Delta\right) \left[E_g (E_g + \Delta) \left\{\Gamma^2 + \left(E_F + E_g + \frac{1}{3} \Delta\right)^2\right\}\right]^{-1} \times$$

$$\times \left[(E_F^2 - \Gamma^2 + E_g^2 + E_g \Delta + 3 E_g E_F) \left\{ E_F \left(E_F + E_g + \frac{2}{3} \Delta \right) + \Gamma^2 \right\} - \right.$$

$$\left. - \Gamma^2 (2 E_F + 3 E_g) \left(E_g + \frac{2}{3} \Delta \right) \right],$$

$$g_1 (E_F, \Gamma) \equiv 2 \left[1 + \left\{ (3 E_F + 3 E_g + 2 \Delta)^2 + 9 \Gamma^2 \right\}^{-1} \left(1 - \frac{m_0}{m_0^*} \right) \Delta (3 E_F + \right.$$

$$\left. + 3 E_g + 2 \Delta) \right],$$

$$D_\pm (E_F, \Gamma) \equiv \left[\gamma_2 (E_F, \Gamma) \pm \frac{1}{2} g_2 (E_F, \Gamma) \mu_0 B \right],$$

$$\gamma_2 (E_F, \Gamma) \equiv \left[[E_g (E_g + \Delta) \left\{ \Gamma^2 + \left(E_F + E_g + \frac{2}{3} \Delta \right)^2 \right\} \right]^{-1} \Gamma \left[(E_g^2 - \Gamma^2 + E_g^2 + \right.$$

$$\left. + E_g \Delta + 3 E_F E_g) \left(E_g + \frac{2}{3} \Delta \right) + (3 E_g + 2 E_F) \left\{ E_F \left(E_F + E_g + \frac{2}{3} \Delta \right) + \Gamma^2 \right\} \right],$$

$$g_2 (E_F, \Gamma) \equiv -6 \Gamma \left[1 + \left(1 - \frac{m_0}{m_0^*} \right) \left\{ (3 E_F + 3 E_g + 2 \Delta)^2 + 9 \Gamma^2 \right\}^{-1} \right],$$

$$y_\pm (n, E_F, \Gamma) \equiv \sum_{r=1}^{s} 2 (k_B T)^{2r} (1 - 2^{1-2r}) \zeta (2r) \frac{d^{2r}}{d E_F^{2r}} [G_\pm (n, E_F, \Gamma)],$$

r is the set of real positive integers, $\zeta (2r)$ is the zeta function of order $2r$ and we have used the notation $\frac{d}{dE_F} [f (E_F)] \equiv f' (E_F)$ where $f (E_F)$ is any differentiable function of E_F.

For $E_g \to \infty$, as for parabolic energy bands the forms of the above equations remain unchanged where

$$G_\pm (n, E_F, \Gamma) \equiv (\sqrt{2})^{-1} [Z_\pm (n, E_F) + \{Z_F^2 (n, E_F) - \Gamma^2\}^{1/2}]^{1/2}$$

in which

$$Z_\pm (n, E_F) \equiv \left[E_F - \left(n + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} g_0 \mu_0 B \right].$$

In the absence of spin and broadening effects together with the condition $E_F << E_g$ Eqs. (8) and (9) assume the well-known forms[6]

$$n_0 = N_c \Theta_0 \sum_{n=0}^{n_{max}} \frac{1}{\sqrt{a_0}} \left[\left(1 + \frac{3}{2} a\, b_0 \right) F_{-\frac{1}{2}}(\eta') + \frac{3}{4} a\, k_B\, T F_{\frac{1}{2}}(\eta') \right] \quad (10)$$

and

$$\left(\frac{D}{\mu}\right)_B = \frac{k_B T}{e} \left[\frac{\sum_{n=0}^{n_{max}} \frac{1}{\sqrt{a_0}} \left[\left(1 + \frac{3}{2} a\, b_0 \right) F_{-\frac{1}{2}}(\eta') + \frac{3}{4} a\, k_B\, T F_{\frac{1}{2}}(\eta') \right]}{\sum_{0=n}^{n_{max}} \frac{1}{\sqrt{a_0}} \left[\left(1 + \frac{3}{2} a\, b_0 \right) F_{-\frac{3}{2}}(\eta') + \frac{3}{4} a k_B T F_{-\frac{1}{2}}(\eta') \right]} \right] \quad (11)$$

where

$$N_c \equiv (2\,(2\,\pi\, m_0^*\, k_B\, T/h^2), \quad a = 1/E_g, \quad b_0 \equiv \left(n + \frac{1}{2} \right) \hbar\, \omega_0 / a_0,$$

$$a_0 \equiv \left[1 + a \left(n + \frac{1}{2} \right) \hbar\, \omega_0 \right], \quad \eta' \equiv \frac{E_F - b_0}{k_B\, T}$$

and $F_j(\eta')$ is the Fermi-Dirac integral of order j as defined by Blakemore[16].

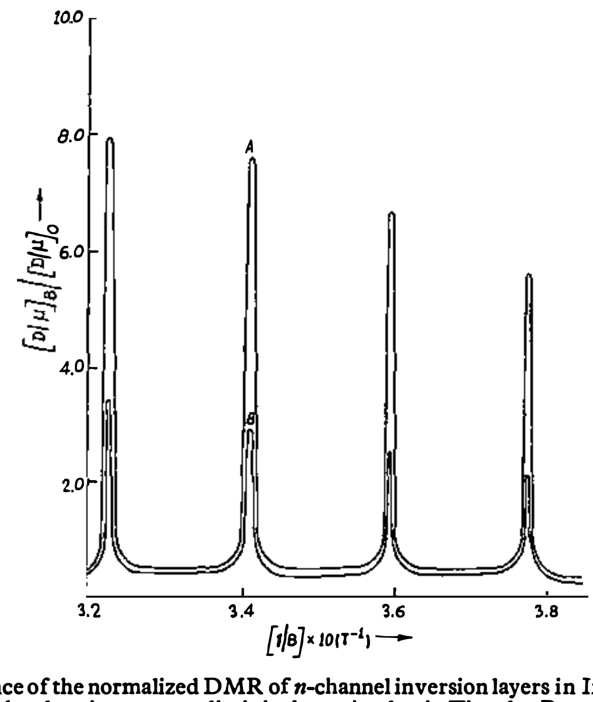

Fig. 1. Dependence of the normalized DMR of n-channel inversion layers in InAs on a quantizing magnetic field in the electric quantum limit is shown in plot A. The plot B corresponds to n-channel inversion layers on parabolic semiconductors.

3. Results and discussions

Taking n-channel InAs as an example and using Eqs. (2) and (4) the appropriate equations together with the parameters[5] $m_0^* = 0.026\ m_0$, $g_v = 1$, $\Delta = 0.43$ eV, $E_g = 0.36$ eV, $\Gamma = 5 \times 10^{-4}$ eV, $T = 4.2$ K and $F_s = 5.6 \times 10^6$ V/m, we have plotted the normalized DMR versus $1/B$ as shown in Fig. 1 in which the plot B corresponds to the same dependence for n-channel inversion layers on parabolic energy bands. In Fig. 2 we have plotted the normalized DMR versus $1/B$ for bulk specimens of n-InAs in which the plot B corresponds to parabolic energy band. It appears from the both the figures that the DMR is an oscillatory function of the magnetic field. The oscillatory dependence is due to the crossing over the Fermi level by the magnetic sub-bands in steps resulting in a successive reduction in the number of occupied Landau levels as the magnetic field is increased and it may be noted that the origin of the oscillation in DMR is the same as that of the Shubnikov-de Haas oscillations. Though the DMR exhibits an oscillatory magnetic field dependence for bulk semiconductors, the magnitude of oscillatory spikes are much sharper for the $2D$ electron gases in inversion layers on non-parabolic semiconductors even in the presence of broadening. The peaks in both the figures would increase in number with decrease in amplitude if the spin-splitting term were included in the numerical computations. We must note that equation (5) is true for the special case of gradual channel approximation as stated in the literature[7,8]. Besides, we have neglected the magnetic field -dependence of Γ and considered also the electric

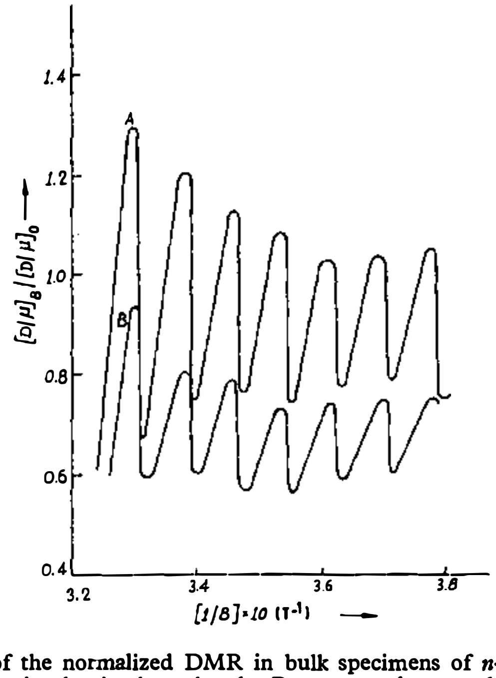

Fig. 2. Dependence of the normalized DMR in bulk specimens of n-InAs on a quantizing magnetic field is shown in plot A where the plot B corresponds to parabolic energy bands.

quantum limit. However, since most of the carriers occupy the lowest electric sub-band at low temperatures[13] for which the effects of magnetic quantization becomes pronounced, it is sufficiently accurate for such temperatures to consider occupation of only the lowest electric sub-band. Incidentally, it may be noted that since the performance of the semiconductor devices at the device terminals and the speed of operation of modern switching semiconductor devices are significantly influenced by the degree of carrier degeneracy[14–15], the simplest way of analysing semiconductor devices would be to use the expression for the DMR which in turn enables us to express the above features of them in terms of carrier concentration. Moreover, since the available noise power is directly proportional to the DMR as discussed elsewhere[4], the experimental results on the thermal noise of Kane-type semiconductors will provide an experimental check for the prediction on the above ratio and also a technique for probing the band structure in degenerate semiconductors. Besides, the general features of the effect of magnetic quantization on the DMR as discussed here would also be valid for most of the small gap semiconductors. Finally it may be noted that though in a more rigorous treatment the many body effects, the hot-electron effects and the formation of band-tails should be considered along with a self-consistent procedure, this simplified analysis exhibits the major qualitative features of the DMR in n-channel inversion layers on Kane-type semiconductors under magnetic quantization with reasonable accuracy.

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UTJECAJ KVANTIZACIJE MAGNETSKIM POLJEM NA EINSTEINOVU RELACIJE U *n*-KANALNOM INVERZNOM SLOJU U POLUVODIČIMA KANE-OVOG TIPA

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Koristeći trozonski Kaneov model izvedena je generalizirana Einstenova relacija za omjer difuzione konstante i pokretljivosti elektrona u *n*-kanalnom inverznom sloju u poluvodičima Kaneovog tipa. Na primjeru *n*-kanalnog poluvodiča InAs teorijski je ispitana oscilatorna zavisnost Einsteinove relacije o recipročnoj vrijednosti magnetskog polja.