YU ISSN 0015-3206
FZKAAA 19 (1) 111 (1987)

# OPTICAL POTENTIALS AND FORM FACTORS FOR ALPHA-PARTICLES AND $^{12}C$ IONS SCATTERED FROM $^{12}C$ AT INTERMEDIATE ENERGIES

SAYED A. E. KHALLAF* and HAROUN EL-SHEIKH

*Department of Physics, Faculty of Science, Aswan, Egypt*

Received 8 December 1986

UDC 539.128

Original scientific paper

Using high-energy double-folding formulae, the optical potentials and intrinsic form factors for alpha particles and $^{12}C$ ions scattered from $^{12}C$ nucleus at 1370 and 1016 MeV, respectively, are derived. The resulting potentials and form factors are compared with phenomenological and deformed Woods-Saxon potentials, respectively. The need to calculate cross sections using the present procedure is stressed.

## *1. Introduction*

The first order optical potential obtained by folding the free nucleon-nucleon $t$-matrix with the uncorrelated one-body density is successfully used in the analyses of $\sim$1 GeV proton scattering from spherical nuclei[1]. Recently, high-energy double-folding potentials were obtained by folding the energy dependent free nucleon-nucleon interaction with the densities of the projectile and target nuclei[2]. These double-folding potentials were used to calculate the eikonal phase shifts[3] and elastic differential, reaction and total cross sections for $^{12}C$-$^{12}C$ scattering at 204.2, 242.7 and 288.6 MeV[4]. The high-energy double-folding potentials were also used to describe the abrasion cross section for relativistic $^{20}Ne$ at 2.1 GeV/nucleon[5]. The results indicate that the high-energy double-folding potentials describe well the experimental data without consideration of neither the renormalization

* Present address: Department of Physics, Faculty of Science, Sana'a University, Sana'a, Y. A. R.

of the potentials nor the projectile break up effect. It is worth mentioning here that the double-folding potentials at low energies require either renormalization of the potentials[6] or break up consideration of the projectile[7] in order to describe adequately the experimental heavy ion data.

In the present work, the high-energy double-folding potential formula is used to calculate the optical potentials for alpha-particles and $^{12}C$ ions scattered from $^{12}C$ nucleus at 1370 and 1016 MeV, respectively. The transition form factors for the inelastic scattering of $\alpha$-particles and $^{12}C$ ions to the ($2^+$, 4.44 MeV) state in $^{12}C$ are also calculated to test the applicability of the high-energy double folding model to the transition form factor in this range of energy and mass numbers. The Tassie hydrodynamic model is used to derive the transition density for $^{12}C$ nucleus. The resulting potentials and form factors are compared with phenomenological and deformed Woods-Saxon potentials, respectively.

## 2. Theory

The central optical potential for nucleus-nucleus scattering is formulated as a high energy double folding result as follows[2-5]:

$$U(r) = A_P A_T \int \mathrm{d}\vec{z}\, \varrho_T(z) \int \mathrm{d}\vec{y}\, \varrho_P(\vec{r} + \vec{y} + \vec{z})\, t(E, y) \tag{1}$$

where $A_i$ and $\varrho_i$ are the mass numbers and the ground state single-particle nuclear densities for the colliding nuclei, respectively, $t(E, y)$ is the constituent-averaged two body amplitude, and $E$ is the nucleon energy in the two-body CM frame. $t(E, y)$ has been derived from the Fourier transform of the two-body scattering amplitudes[8]. The two-body scattering amplitudes of the following form[9]:

$$A_{pj}(q) = \frac{k_0\, \sigma_{pj}}{4\pi} (a_{pj} + \mathrm{i}) \exp(-1/2\, B_{pj}\, q^2) \tag{2}$$

$$(j = (p \text{ or } n)$$

yield:

$$t(E, y) = -\hbar \left(\frac{E}{m}\right)^{1/2} \frac{\sigma(a + \mathrm{i})}{(2\pi B)^{3/2}} \exp\left(\frac{-y^2}{2B}\right) \tag{3}$$

where $q$ is the momentum transfer, $m$ is the nucleon mass, $\sigma$ is the average NN total cross section, $a$ is the average of the ratio of the real to imaginary parts of the NN forward scattering amplitude and $B$ is the average slope parameter. The parameters $\sigma$, $a$ and $B$ are defined as follows[10]:

$$\left.\begin{aligned} \sigma &= 1/2\,(\sigma_{pp} + \sigma_{pn}) \\ a &= (a_{pp}\sigma_{pp} + a_{pn}\sigma_{pn})/(\sigma_{pp} + \sigma_{pn}) \\ B &= 1/2\,(B_{pp} + B_{pn}). \end{aligned}\right\} \tag{4}$$

The ground state single particle densities for $^{12}C$ and $\alpha$-particles are chosen in the following forms[4,11]:

$$\varrho_{12C}(r) = \varrho_0 \left(1 + \frac{\nu}{a^2} r^2\right) \exp(-r^2/a^2) \tag{5}$$

with

$$a = 1.692\,\text{fm and } \nu = 1.082$$

and

$$\varrho_\alpha(r) = \left(\frac{4}{3\pi}\alpha_0\right)^{3/2} \exp\left(-\frac{4}{3}\alpha_0 r^2\right) \tag{6}$$

with

$$\alpha_0 = 0.415\,\text{fm}^{-2}.$$

The constant $\varrho_0$ is determined by the normalization condition:

$$\int \varrho_{12C}(r)\,\mathrm{d}\vec{r} = 1. \tag{7}$$

For the inelastic transition of multipolarity $L$, the form factor for a nucleus--nucleus inelastic scattering may be written as[12]:

$$A_L(r) = \beta_L R\, F_L(r) \tag{8}$$

where $\beta_L R$ is the deformation length and $F_L(r)$ is the intrinsic form factor. The approach of deforming the target density directly in a folding model has been employed to obtain the inelastic form factor with considerable success at low energies. This approach is an extension of the double-folding model of the optical potential for elastic scattering[12]. In the present work, the high energy double-folding model has been extended to obtain the intrinsic form factor for a nucleus--nucleus inelastic scattering of the form:

$$F_L(r) = A_P A_T \int \mathrm{d}\vec{z}\, \varrho_{tr}(z) \int \mathrm{d}\vec{y}\, \varrho_P(\vec{r} + \vec{y} + \vec{z})\, t(E, y) \tag{9}$$

where $\varrho_{tr}(z)$ is the transition single-particle density for the target nucleus. Here, the radial dependence of the transition density $\varrho_{tr}(z)$ for the $2^+$ excitation of $^{12}C$ is chosen in the Tassie form, i. e.

$$\varrho_{tr}(z) = C_L z^{L-1} \frac{\mathrm{d}\varrho_T(z)}{\mathrm{d}z} \tag{10}$$

where $\varrho_T(z)$ is the ground state single-particle density for the target nucleus. The normalizing constant $C_L$ is determined by assuming that the proton transition density is $(Z_T/A_T)$ times the mass transition density and then choosing $C_L$ to give the measured value of $B(E2)$ for $^{12}C$ nucleus[13] i. e.

$$\int_0^\infty A_T \varrho_{tr.}(z)\, z^{L+2}\, \mathrm{d}z = \frac{A_T}{Z_T e} (B\,(EL))^{1/2} \tag{11}$$

where $Z_T$ is the charge number of the target nucleus.

## 3. *Results and discussion*

### *a. The optical potentials*

Using the high energy double folding formula given by expression (1), the central real and imaginary parts of the potentials for 1370 MeV alpha-particles and 1016 MeV $^{12}C$ ions scattered from $^{12}C$ nucleus are calculated. The single-particle nuclear densities for $^{12}C$ ions and $\alpha$-particle are given by expressions (5) and (6), respectively. The averaged two-body amplitude $t\,(E, y)$ given by expression (3) has been derived from the Fourier transform of the two-body scattering amplitudes given by expression (2). The parameters $\sigma$, $a$ and $B$ are derived from Eqs. (4) with the parameters given in Table 1. The 100 MeV and 325 MeV proton-nucleon amplitudes given in Table 1 are used in the present work for the 1016 MeV $^{12}C$ ions ($\sim$85 MeV/nucleon) and 1370 MeV alpha-particles (342.5 MeV/nucleon), respectively.

TABLE 1.

| | Energy (MeV) | $\sigma_{pj}$ (fm$^2$) | $\alpha_{pj}$ | $B_{pj}$ (fm$^2$) |
|---|---|---|---|---|
| p + p | 100 | 3.32 | 1.87 | 0.66 |
| p + n | 100 | 7.27 | 1.00 | 0.36 |
| p + p | 325 | 2.45 | 0.45 | 0.26 |
| p + n | 325 | 3.61 | 0.16 | 0.36 |

Parameters of the proton-nucleon amplitudes (see Eq. (2)) taken from Ref. 16.

Figs. 1 and 2 display the real and imaginary parts of the resulting optical potentials for alpha particles and $^{12}C$ ions, respectively, in comparison with the real and imaginary parts of the phenomenological potentials given in Table 2. Corrections due to relativistic kinematics are taken into account by multiplying the potential strengths $V_0$ and $W_I$ of the phenomenological potentials given in Table 2, by $\gamma_{rel}$ defined by[14]:

$$\gamma_{rel} = \frac{k^2}{E_0^2 - m^2} \frac{E_0}{M} \tag{12}$$

where $k$ is the wave number in the relativistic CM momentum, $E_0$ is the total energy in the CM system, $m_1$ is the mass of the incident particle and $M$ is the reduced mass. Here, $\gamma_{rel}$ equals 1.202 and 1.0 for the $\alpha$-particles and $^{12}C$ projectiles, respectively.

TABLE 2.

| Reaction | Energy (MeV) | $V_0$ (MeV) | $R_0$ (fm) | $a_0$ (fm) | $W_I$ (MeV) | $R_I$ (fm) | $a_I$ (fm) | References |
|---|---|---|---|---|---|---|---|---|
| $\alpha + {}^{12}C$ | 1370 | 65.58 | 0.733 | 0.638 | 229.8 | 0.733 | 0.683 | 14 |
| ${}^{12}C + {}^{12}C$ | 1.16 | 80 | 0.8 | 0.74 | 42.0 | 0.9 | 0.73 | 17 |

The phenomenological optical potential parameters shown in Figs. 1—4. They are parametrized in the form:

$$U(r) = -V_0 \left[1 + \exp\left(\frac{r - R_0}{a_0}\right)\right]^{-1} - \mathrm{i}W_I \left[1 + \exp\left(\frac{r - R_I}{a_I}\right)\right]^{-1}$$

where

$$R_i = r_i \left[A_T^{1/3} + A_P^{1/3}\right] \text{ for } {}^{12}C + {}^{12}C \text{ reaction}$$

and

$$R_i = r_i A_T^{1/3} \text{ for } \alpha + {}^{12}C \text{ reaction.}$$

*b. The intrinsic form factors*

The high-energy double-folding formula given by expression (9) has been used to calculate the real and imaginary parts of the transition form factors for the inelastic scattering of $\alpha$-particles and ${}^{12}C$ ions at 1370 and 1016 MeV, respectively, to the $2^+$ 4.44 MeV state in ${}^{12}C$. The transition density has been deduced from the ground state density for ${}^{12}C$ nucleus given by expression (5) using the Tassie hydrodynamic model. Expression (11) has been used to determine the constant $C_L$ (see expression (10)) where the proton part of the transition density would yield the measured value of $B(E2)$. The choice of $B(E2) = 42\, e^2$ fm$^4$ for ${}^{12}C$-nucleus[13,15] yields $C_2 = 0.44757$ fm$^{-1}$. The averaged two-body amplitude $t(E, y)$ and parameters $\sigma$, $\alpha$ and $B$ are the same as in the elastic scattering case.

Figs. 3 and 4 display the resulting intrinsic form factors $F_2(r)$ compared with the deformed optical potentials $\frac{\mathrm{d}U(r)}{\mathrm{d}r}$ where $U(r)$ are the phenomenological optical pontentials given in Table 2. The depths $V_0$ and $W_I$ of the phenomenological potentials of Table 2 are multiplied by the relativistic correction factors $\gamma_{rel}$ defined by Eq. (12).

Figs. 1 and 2 show good agreement between the high-energy double-folding optical potentials predicted by expression (1) and the phenomenological potentials for 1370 MeV alpha particles and 1016 MeV ${}^{12}C$ ions scattered from the ${}^{12}C$ nucleus. Figs. 3 and 4 show also that the intrinsic form factors for the inelastic scattering of 1370 MeV alpha particles and 1016 MeV ${}^{12}C$ ions scattered from the ${}^{12}C$ nucleus calculated by expression (9) are in good agreement with the deformed optical potentials. These fits obtained without renormalization of the high-energy double-folding potentials are in the important surface region. In the interior region the differences are quite large. It may be useful to mention here that it was early[12] shown that the elastic and inelastic heavy ion reactions at lower energies are sensitive to the radial region around the strong absorption radius and that the interior regions of the potentials and form factors where the largest differences occur are not significant to the calculations of the cross sections.

Finally, it seems that the high energy double folding procedure presented here is applicable for the transition form factors of 1370 MeV alpha paıticles and 1016 MeV $^{12}C$ ions inelastically scattered to the fiıst excited state (2+, 4.44 MeV) in $^{12}C$. The need to calculate the cross sections using the present procedure is stressed.

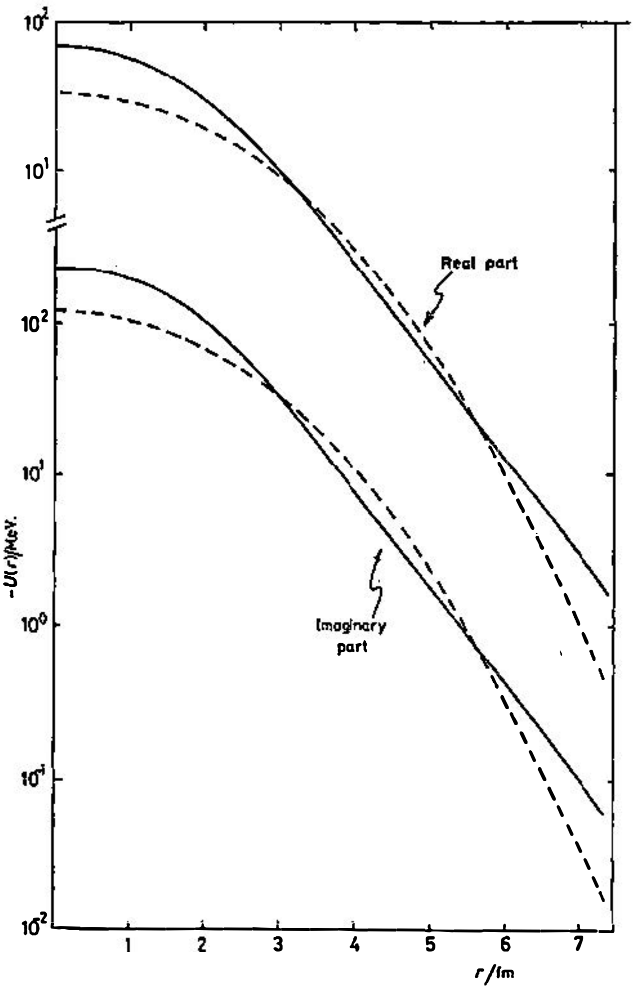


Fig. 1. The real and imaginary parts of the optical potentials for the elastic scatteıing of $\alpha$-particles from $^{12}C$ at 1370 MeV. The solid curves represent the phenomenological potentials given in Table 2 and the dashed curves represent the predictions of the high-energy double-folding model given by Eq (1).

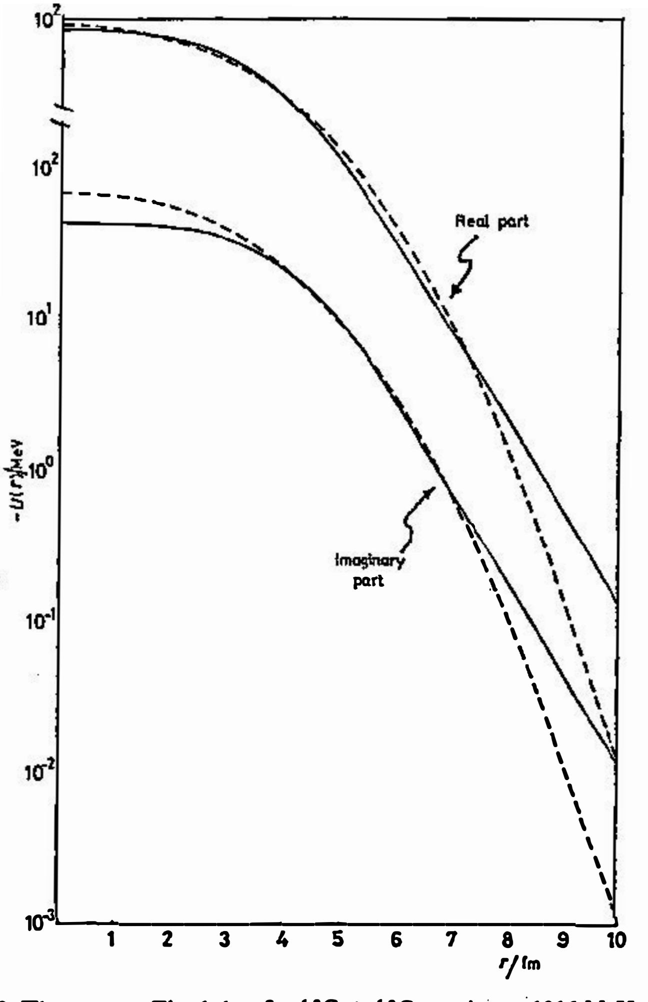


Fig. 2. The same as Fig. 1, but for $^{12}C + ^{12}C$ reaction at 1016 MeV.

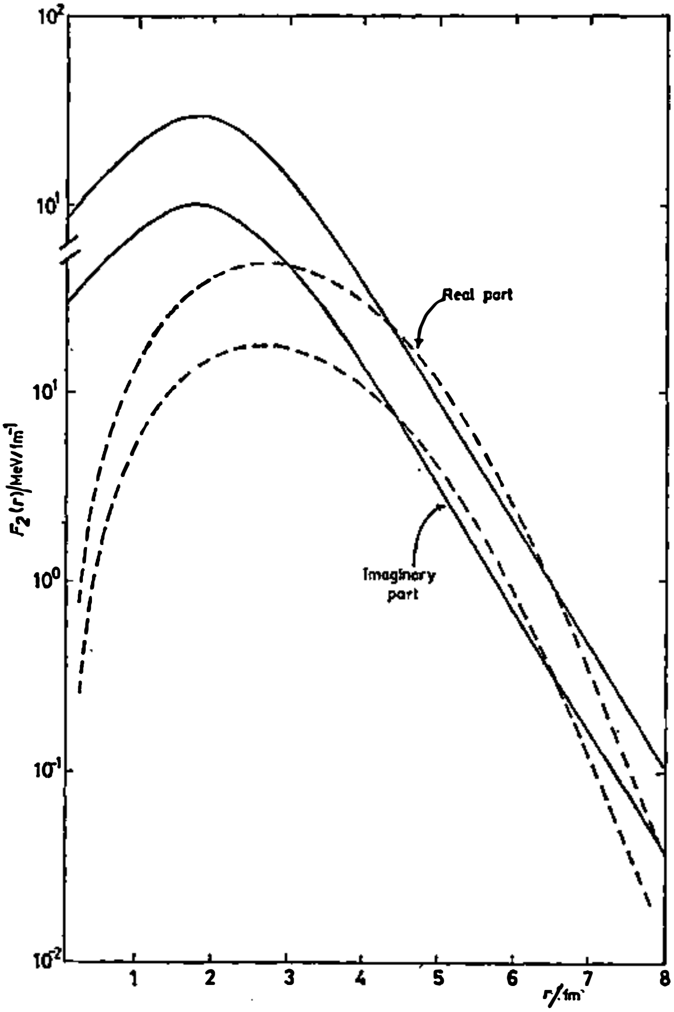


Fig. 3. The real and imaginary parts of the transition intrinsic form factors for the inelastically scattered $\alpha$-particles to the $2^+$, 4.44 MeV state in $^{12}C$ at 1370 MeV. The solid curves represent the deformed optical potentials $\left(\frac{dU(r)}{dr}\right)$ with the parameters given in Table 2. The dashed curves represent the high-energy double-folding calculations using expression (9).

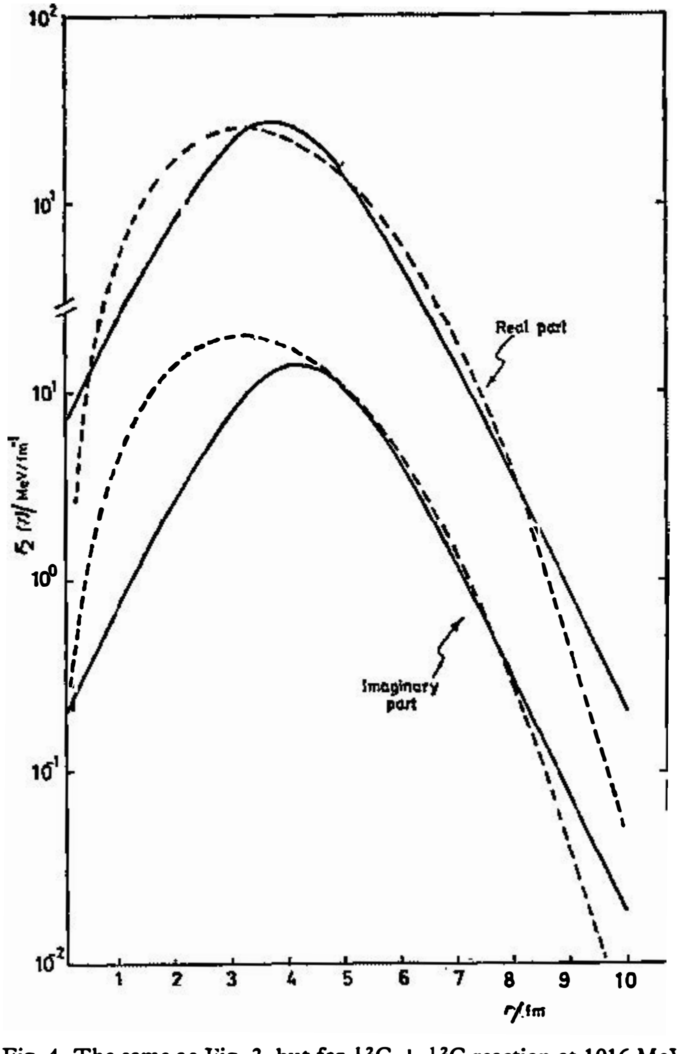


Fig. 4. The same as Fig. 3, but for $^{12}C + {}^{12}C$ reaction at 1016 MeV.

References


1) L. Ray, W. R. Coker and G. W. Hoffmann, Phys. Rev. **C18** (1978) 2641;
2) J. W. Wilson and L. W. Townsend, Can. J. Phys. **59** (1981) 1569;
L. W. Townsend, J. W. Wilson and H. B. Bidasaria, Can. J. Phys. **60** (1982) 1514;
3) L. W. Townsend, H. B. Bidasaria and J. W. Wilson, Can. Phys. **61** (1983) 867;
4) H. B. Bidasaria, L. W. Townsend and J. W. Wilson, J. Phys. G. Nucl. Phys. **9** (1983) L 17;
5) L. W. Townsend, Can. J. Phys. **61** (1983) 93;

6) G. R. Satchler and W. G. Love, Phys. Rep. **55** (1979) 183 and references therein; S. A. E. Khallaf, A. L. Elatter and M. El-Azab Farid, Fizika **15** (1983) 125 and references therein;
7) Y. Sakuragi, M. Yahiro and M. Kaminura, Prog. Theor. Phys. **70** (1983) 1047;
8) D. Slanina and H. McManus, Nucl. Phys. **A116** (1968) 271;
9) L. Ray, G. M. Hoffmann, M. Barlett, J. McGill, J. Amann, G. Adams, G. Pauletta, M. Gazzaly and G. S. Blanpied, Phys. Rev. **C23** (1981) 828;
10) A. Y. Abdul Magd, M. H. Simbel and M. Y. Farag, Can. J. Phys. **58** (1980) 1230;
11) S. A. E. Khallaf and H. M. Ali Ahmed, Fizika **17** (1985) 169;
12) C. B. Dover, P. J. Moffa and J. P. Vary, Phys. Lett. **56B** (1975) 4; P. J. Moffa, C. B. Dover and J. P. Vary, Phys. Rev. **C16** (1977) 1857;
13) F. E. Bertrand, G. R. Satchler, D. J. Horen, J. R. Wu, A. D. Bacher, G. T. Emery, W. P. Jones, D. W. Miller and A. Van der Woude, Phys. Rev. **C22** (1980) 1832; E. E. Gross, J. R. Beene, K. A. Erb, M. P. Fewell, D. Shapira, M. J. Rhoades-Brown and G. R. Satchler, Nucl. Phys. **A401** (1983) 362;
14) A. Ingemarsson and O. Jonsson, Nucl. Phys. **A388** (1982) 644;
15) R. G. Stokstad, R. M. Wieland, G. R. Satchler, C. B. Fulmer, D. C. Hansley, S. Raman, L. D. Rickertsen, A. H. Snell and P. H. Stelson, Phys. Rev. **C20** (1979) 655;
16) L. Ray, Phys. Rev. **C20** (1979) 1857;
17) M. Buenerd, P. Martin, R. Bertholet, C. Guet, M. Maurel, J. Mougey, H. Nifenecker, J. Pinston, P. Perrin, F. Schussler, J. Julien, J. P. Bondorf, L. Carlen, H. A. Gustafsson, B. Jakobsson, T. Johansson, P. Kristiansson, O. B. Nielsen, A. Oskarsson, I. Otterlund, H. Ryde. B. Schroder and G. Tibell, Phys. Rev. **C26** (1979) 1299.

## OPTIČKI POTENCIJALI I FAKTORI FORME ZA RASPRŠENJE $\alpha$-ČESTICA i $^{12}$C IONA SREDNJIH ENERGIJA NA $^{12}$C

SAYED A. E. KHALLAF* i HAROUN EL-SHEIKH

*Department of Physics, Faculty of Science, Aswan, Egypt*

UDK 539.128

Originalni znanstveni rad

Upotrebom visokoenergetskog izraza dvostruke konvolucije izvedeni su optički potencijali i faktori forme za raspršenje 1370 MeV alfa čestica i 1016 MeV $^{12}$C iona na $^{12}$C. Dobiveni potencijali i form-faktori uspoređeni su s fenomenološkim i deformiranim Woods-Saxon potencijalima. Istaknuta je potreba računanja udarnih presjeka upotrebom prikazane procedure.

Printed by the Grafički Zavod Hrvatske, Zagreb