

DECOMPOSITION OF BOSON-FERMION GROUP CHAIN ASSOCIATED
WITH SYMMETRY/SUPERSYMMETRY IN THE CASE $j_\pi = \frac{3}{2}$, $j_\nu = \frac{3}{2}$,

$$SU^B(6) \subset SU^B(5)$$

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Quantum numbers associated with the boson-fermion-fermion group chain for odd proton and odd neutron in $j = \frac{3}{2}$ configurations with $U^B(5)$ boson core is explicitly constructed and the corresponding supersymmetric energy formula is compared to the parabolic rule.

1. Introduction

The approach to odd-even nuclei based on exploiting possible boson-fermion symmetries¹⁾, or more generally, supersymmetries²⁾ has received much attention. In particular, when the boson part of the system is described by one of the symmetry chains of the interacting boson model Hamiltonian³⁾ and the odd fermion is in a configuration with specific j values, the boson and fermion group chains have

been combined into a common boson-fermion group chain with spinor groups that transform simultaneously bosons and fermions^{1,4)}. These special solutions provide a simple framework to analyze and classify the levels in odd-even nuclei.

Recently, such an approach was extended to study odd-odd nuclei⁵⁻⁸⁾. In Ref. 6 we have investigated the case with odd proton and odd neutron in $j = \frac{3}{2}$ configurations coupled to SO (6) boson core, making an algebraic distinction between the particle-like and the hole-like unpaired nucleons in constructing Bose-Fermi symmetry schemes.

2. Group decomposition associated with $U^B(5)$ symmetry

In this paper we introduce the boson-fermion symmetry for odd-odd nuclei with odd proton and odd neutron in $j = 3/2$ configurations coupled to U (5) collective core. In this case, we combine the boson and fermion groups into a common group chain

$$\begin{array}{ccc}
 U^B(6) \otimes U^{F\pi}(4) \otimes U^{F\nu}(4) & & \\
 \downarrow & \searrow & \swarrow \\
 U^B(5) & & SU^{F\pi\nu}(4) \\
 \downarrow & & \downarrow \\
 SO^B(5) & & Sp^{F\pi\nu}(4) \\
 & \searrow & \swarrow \\
 & \text{Spin}^{BF\pi\nu}(5) & \\
 & \downarrow & \\
 & \text{Spin}^{BF\pi\nu}(3) & \\
 & \downarrow & \\
 & \text{Spin}^{BF\pi\nu}(2) &
 \end{array} \tag{1}$$

The generators of groups labelled by $F_{\pi\nu}$ are obtained by adding the generators of proton and neutron-fermion groups, and the generators of spinor groups labelled by $BF_{\pi\nu}$ are obtained by adding the generators of boson, proton-fermion and neutron-fermion groups in such a way that they close under the commutation relations of the algebras of the respective groups. The group chain (1) contains the total boson-fermion angular momentum group $\text{Spin}^{BF\pi\nu}(3)$ as a subgroup, corresponding to integer spins.

In the limit of group chain (1) the basis states are classified by the labels

$$|N, n_d, v, (\xi_1, \xi_2), (\tau_1, \tau_2), J, M_f\rangle \tag{2}$$

These quantum numbers characterize the irreducible representations (irreps) of the groups $U^B(6)$, $U^B(5)$, $SO^B(5)$, $Sp^{F\pi\nu}(4)$, $\text{Spin}^{BF\pi\nu}(5)$, $\text{Spin}^{BF\pi\nu}(3)$ and $\text{Spin}^{BF\pi\nu}(2)$, respectively. The quantum numbers $M_\pi = 1$ and $M_\nu = 1$ which label the irreps of $U^{F\pi}(4)$ and $U^{F\nu}(4)$, respectively, are omitted from the notation, because they produce only an overall shift in energy.

Let us determine the allowed values of the quantum numbers which appear in (2) for a given boson number N . Employing group-theoretical methods^{9,10)} we construct the representations of the subgroups in the chain (1) contained in their respective larger groups.

The quantum numbers n_d and v in a standard way label irreps of $U^B(5)$ and $SO^B(5)$ contained in the irreps of $[N]$ of $U^B(6)$

$$n_d = 0, 1, 2, \dots, N \tag{3}$$

$$v = n_d, n_d - 2, \dots, 1 \text{ or } 0. \tag{4}$$

In the product $U^{F\pi}(4) \otimes U^{F\nu}(4)$ there arise symmetric and antisymmetric irreps of $SU^{F\pi\nu}(4)$. In the next step we get the $Sp^{F\pi\nu}(4)$ content (given here in the standard notation of $Spin(5) \approx Sp(4)$)

$$\begin{aligned} (1, 1, 0)^a_{SU^{F\pi\nu}(4)} \oplus (2, 0, 0)^s_{SU^{F\pi\nu}(4)} &\rightarrow [(0, 0)_{Sp^{F\pi\nu}(4)} \oplus \\ &\oplus (1, 0)_{Sp^{F\pi\nu}(4)}]_a \oplus [(1, 1)_{Sp^{F\pi\nu}(4)}]_s \end{aligned} \tag{5}$$

with the allowed values of the $Sp(4)$ labels given by the r.h.s. of relation (5). The notations s and a stand for symmetric and antisymmetric irreps, respectively.

The quantum numbers (τ_1, τ_2) , which label the irreps of $Spin^{BF\pi\nu}(5)$, arise in the product $SO^B(5) \otimes Sp^{F\pi\nu}(4)$:

$$\begin{aligned} (v, 0)_{SO^B(5)} \oplus \{[(0, 0) \oplus (1, 0)]_a \oplus [(1, 1)]_s\}_{Sp^{F\pi\nu}(4)} &= \\ = \{[(v, 0) \oplus (v + 1, 0) \oplus (v, 1) \oplus (v - 1, 0)]_a \oplus [(v + 1, 1) \oplus \\ \oplus (v, 1) \oplus \Theta_{v,1}(v, 0) \oplus (v - 1, 1)]_s\}_{Spin^{BF\pi\nu}(4)}, \end{aligned} \tag{6}$$

with $\Theta_{m,1} = \begin{cases} 1 & \text{for } m \begin{cases} > 1 \\ < 1 \end{cases} \end{cases}$. In the next step, the two types of $Spin^{BF\pi\nu}(5)$ irreps which appear on the r. h. s. of relation (8) are decomposed into $Spin^{BF\pi\nu}(3)$ contents:

$$(k, 1)_{Spin^{BF\pi\nu}(5)} = \sum_{l=1}^{2k+1} a_l [J = l], \tag{7}$$

$$\begin{aligned} (k, 0)_{Spin^{BF\pi\nu}(5)} &= \sum_{l=2}^k [J = 2l] \oplus \sum_{l=2}^{k-2} [J = 2l + 1] \oplus b(k) \otimes c(k), \\ &\text{for } k < 8, \end{aligned} \tag{8}$$

with

$$a_l = E\left(\frac{l+1}{2}\right) \left\{ 1 - E\left(\frac{l}{k+1}\right) \right\} + \left\{ 1 + E\left(k - \frac{l-1}{2}\right) \right\} E\left(\frac{l}{k+1}\right) - \delta_{l,k+1} \tag{9}$$

$$b(k) = \begin{cases} [J = 2(k-3)], & k \geq 6 \\ 0, & k < 6 \end{cases} \quad (10)$$

$$c(k) = \begin{cases} [J = 3] \oplus [J = 0], & k = 0 \pmod{3} \\ [J = 2], & k = 1, 2 \pmod{3}. \end{cases} \quad (11)$$

$[J]$ denotes irrep of $\text{Spin}^{BFnv}(3)$ with angular momentum J and E is the integer-part function. Since the decomposition of irreps of $\text{Spin}(5)$ into those of $\text{Spin}(3)$ is not unique, in this step an extra label n_d can be introduced, similarly as for half-integer spin representations in the treatment of the odd-even nuclei.

3. The energy formula and supersymmetry

Assuming the boson-fermion symmetry associated with the group chain (1), the Hamiltonian can be written in terms of the Casimir operators of the group chain (1), leading to the energy formula

$$E = E_0 + \varepsilon n_d + \alpha n_d (n_d + 4) + 2 \beta v (v + 3) + 2 \delta [\xi_1 (\xi_1 + 3) + \xi_2 (\xi_2 + 1)] + 2 \beta' [\tau_1 (\tau_1 + 3) + \tau_2 (\tau_2 + 1)] + 2 \gamma J (J + 1). \quad (12)$$

The term E_0 contains terms linear and quadratic in N, M_n, M_v and is irrelevant for the considerations of a particular odd-odd nucleus with fixed values of N, M_n and M_v . The allowed values of the labels $n_d, v, (\xi_1, \xi_2), (\tau_1, \tau_2), (J)$ are given by (3)—(11). The parameters $\varepsilon, \alpha, \beta, \beta', \gamma$ are defined in such a way as to resemble in the energy formula for odd-even nuclei in Ref. 4.

In Table 1 we present the quantum numbers which characterize the irreps associated with the group chain (1).

TABLE 1.

$U^B(6)$	$U^B(5)$	$SO^B(5)$	$Sp^F(4)$	$\text{Spin}^{BF}(5)$	J		
(1,0,0,0,0)	(1,0,0,0,0)	(1,0)	(0,0)	(1,0)	2		
			(1,0)	(2,0)	4+2		
				(1,1)	3+1		
				(0,0)	0		
				(1,1)	(2,1)	5+4+3+2+1	
				(1,1)	(1,1)	3+1	
				(1,0)	(1,0)	2	
				(0,0,0,0,0)	(0,0)	(0,0)	0
					(1,0)	(1,0)	2
					(1,1)	(1,1)	3+1
(2,0,0,0,0)	(2,0,0,0,0)	(2,0)	(0,0)	(2,0)	4+2		
			(1,0)	(3,0)	6+4+3+0		
			(2,1)	(2,1)	5+4+3+2+1		
			(1,0)	(1,0)	2		

		(1,1)	(3,1)	7+6+2·5+4+2·3+2+1
			(2,1)	5+4+3+2+1
			(2,0)	4+2
			(1,1)	3+1
	(0,0)	(0,0)	(0,0)	0
		(1,0)	(1,0)	2
		(1,1)	(1,1)	3+1
(1,0,0,0,0)	(1,0)	(0,0)	(1,0)	2
		(1,0)	(2,0)	4+2
			(1,1)	3+1
			(0,0)	0
		(1,1)	(2,1)	5+4+3+2+1
			(1,0)	2
(0,0,0,0,0)	(0,0)	(0,0)	(0,0)	0
		(1,0)	(1,0)	2
		(1,1)	(1,1)	3+1
(3,0,0,0,0)	(3,0)	(0,0)	(3,0)	6+4+3+0
		(1,0)	(4,0)	8+6+5+4+2
			(3,1)	7+6+2·5+4+2·3+2+1
			(2,0)	4+2
		(1,1)	(4,1)	9+8+2·7+2·6+2·5+2·5+2·4+ +2·3+2+1
			(3,1)	7+6+2·5+4+2·3+2+1
			(3,0)	6+4+3+0
			(2,1)	5+4+3+2+1
	(1,0)	(0,0)	(1,0)	2
		(1,0)	(2,0)	4+2
			(1,1)	3+1
			(0,0)	0
		(1,1)	(2,1)	5+4+3+2+1
			(1,1)	3+1
			(1,0)	2
(2,0,0,0,0)	(2,0)	(0,0)	(2,0)	4+2
		(1,0)	(3,0)	6+4+2+0
			(2,1)	5+4+3+2+1
			(1,0)	2
		(1,1)	(3,1)	7+6+2·5+4+2·3+2+1
			(2,1)	5+4+3+2+1
			(2,0)	4+2
			(1,1)	3+1
	(0,0)	(0,0)	(0,0)	0
		(1,0)	(1,0)	2
		(1,1)	(1,1)	3+1
(1,0,0,0,0)	(1,0)	(0,0)	(1,0)	2
		(1,0)	(2,0)	4+2
			(1,1)	3+1
			(0,0)	0
		(1,1)	(2,1)	5+4+3+2+1
			(1,1)	3+1
			(1,0)	2
(0,0,0,0,0)	(0,0)	(0,0)	(0,0)	0
		(1,0)	(1,0)	2
		(1,1)	(1,1)	3+1

Let us consider the supergroup structure associated with the group chain (1).
In this case

$$U^B(6) \otimes U^{F\pi}(4) \otimes U^{F\nu}(4) \subset U^B(6) \otimes U^F(8). \quad (13)$$

Alternatively, as shown in Ref. 11, it is possible to construct the generators of the symplectic group $Usp(2n)$, giving rise to the decomposition

$$U^B(6) \otimes U^F(8) \supset U^B(6) \otimes Sp^F(8). \quad (14)$$

The appealing aspect of this chain is that it emphasizes the residual interaction between the unpaired fermions¹¹⁾.

In this paper we consider the supergroup embedding associated with (13)

$$U^B(6) \otimes U^F(8) \supset U(6/8). \quad (15)$$

As an example, in the framework of such a supersymmetry, the nuclei ^{64}Zn , ^{63}Cu and ^{62}Cu correspond to the members $(N=4, M_\pi=0, M_\nu=0)$, $(N=3, M_\pi=1, M_\nu=0)$ and $(N=2, M_\pi=1, M_\nu=1)$ of the $\{\mathcal{N}_0=4\}$ supersymmetry multiplet, respectively.

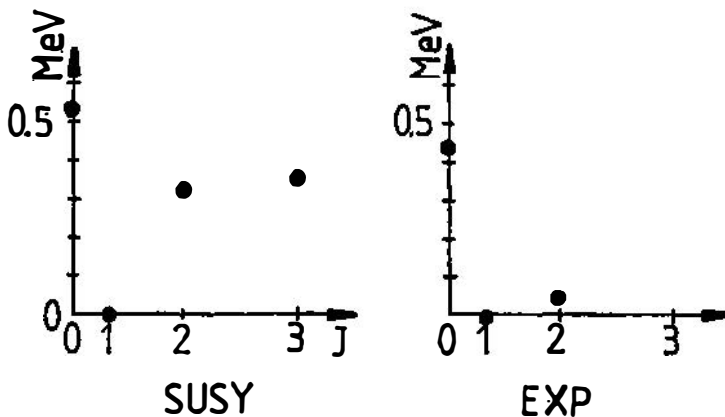


Fig. 1 Supersymmetry prediction for the analog of $\pi p_{3/2} \nu p_{3/2}$ multiplet and the available data in ^{62}Cu .

Assuming such a supersymmetry, we can describe the odd-odd nucleus ^{62}Cu by using the boson-fermion parametrization for odd-even nucleus ^{63}Cu from Ref. 4. In this way we obtain the lowest quadruplet, associated with the $(j_\pi = 3/2, j_\nu = 3/2)$ multiplet in the conventional approach. The result is presented in Fig. 1a, and compared to the available experimental data in Fig. 1b. As seen, the pattern resembles a rather distorted open-up parabola, predicted in the standard particle-phonon model (parabolic rule)¹²⁾ for the particle-hole case. The relation between boson-fermion group structure and the parabolic rule requires further consideration.

In conclusion, we note that the boson-fermion Hamiltonian for odd-odd nuclei (IBFFM) has been recently diagonalized for several realistic cases^{13, 14)}. The conditions under which the IBFFM reduces to dynamical symmetries is presently under investigation, in analogy to the simpler case of odd-even system (IBFM)¹⁵⁾.

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DEKOMPOZICIJA BOZONSKO-FERMIONSKOG GRUPNOG LANCA
PRIDRUŽENOG SIMETRIJI/SUPERSIMETRIJI U SLUČAJU

$$j_{\pi} = \frac{3}{2}, \quad j_{\nu} = \frac{3}{2}, \quad U^B(6) \subset U^B(5)$$

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Konstruirani su kvantni brojevi pridruženi bozonsko-fermionskom grupnom lancu za neparni proton i neparni neutron u konfiguraciji $j = \frac{3}{2}$ sa $U^B(5)$ srednjicom i odgovarajuća supersimetrijska energijska formula je uspoređena s paraboličnim pravilom.