YU ISSN 0015—3206
FZKAAA 19 (2) 211 (1987)

DIBARYON STATES IN THE CHIRAL BAG MODEL

DAMIR BOSNAR

Physics Department, Faculty of Natural Science and Mathematics, University of Zagreb, 41000 Zagreb, Croatia, Yugoslavia

and

DUBRAVKO HORVAT

Department of Physics, Faculty of Electrical Engineering, University of Zagreb, 41000 Zagreb, Croatia, Yugoslavia

Received 12 February 1987

UDC 539.12

Original scientific paper

In the chiral bag model we calculate the masses of six quark states arising from the group theoretical decomposition of the underlying $SU(3)_F \otimes SU(3)_C \otimes SU(2)_I$ symmetry group. Restoration of the chiral symmetry gives rise to bigger binding energies and opens the question of variety of decay modes. A brief overview of decay modes is given.

1. *Introduction*

The existence of multiquark hadron states has attracted great interest during the last few years. Our current theory of hadrons allows for the existence of these »quark molecules«[1] bearing in mind that observed states are colourless. Since these states represent degrees of freedom of the QCD confining phase, one is forced to resort to phenomenological models. Jaffe[2,3] used the MIT bag model[4,5] to calculate various multiquark states, and discovered that a state, dubbed H, with $J^\pi = 0^+$, $I = 0$, $Y = S + B = 0$ had a mass $M_H = 2150$ MeV which is about 80 MeV below the threshold for strong decay into $\Lambda\Lambda$ ($m_\Lambda = 1115.6$ MeV), this being a consequence of recoupling the spins of the quarks in two Λ hyperons.

Also a baryon $B = 2$ state is recently rediscovered in the Skyrme model[6], in which baryons are described as solitons in an effective meson theory[7–13]. All the calculations in the nonlinear topological chiral model give masses lower than the bag model prediction. Also by considering a number of colours N it was shown within the same theory that for $N \to \infty$ only $B = 1$ states are stable[14], corresponding to mappings belonging to the third homotopy group, $\pi_3(\mathrm{SU}(N)) = Z$, whereas for $N = 3$ the multibaryon spectrum of the Skyrme and the quark model agree.

It is known that the MIT bag model breaks chiral symmetry, i. e. it is dynamically broken at the quark level so that solutions of the field equations derived from QCD do not respect the same symmetries as the fundamental Lagrangian. To restore the chiral symmetry one introduces mesons outside the bag and obtains the so called chiral bag model[15,16]. Further development is motivated by the fact that instabilities at small bag radius[17] can be cured by combining the Skyrme model with the chiral bag model (CBM) ending with the often called hybrid model[18,19].

In this article we will adopt the chiral bag model (CBM) to calculate masses of dibaryon states which arise as colourless six quark objects from the symmetry decomposition of the underlying general symmetry group.

In Sect. 2 the group theoretical analysis provides dibaryon spectrum and Sect. 3 describe our model. In Sect. 4 the mass spectrum of the dibaryon resonances is calculated and comparison with other calculations is given. Sect. 5 is devoted to discussion and concluding remarks.

2. *Colour singlet multiquark states*

It is well established that the internal quantum numbers of mesons correspond to quark-antiquark ($q\bar{q}$) states and those of baryons to three-quark states (qqq). Since multiquark states can also be formed, to rule out the existence of unobserved states like quarks, diquarks (qq), four-quark states, etc. one can postulate that all physical observables (currents, energy-momentum tensor, etc.) and all hadrons are colour singlets.

We consider the colourless six-quark states in the $s_{1/2}$ states, and we use the Pauli principle according to which a state should be totaly antisymmetric relative to the quark colour, isospin, spin and angular momentum. Assuming only three flavours the states can be classified according to $\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_F$ and $\mathrm{SU}(2)_J$ for the colour, flavour and for the angular momentum part.

In the development of the model the flavour symmetry will be broken to SU (2), i. e. we will assume that isospin is a good symmetry and ignore electromagnetic effects and u-d quark mass differences. Since SU (18) contains all the required groups[20] we decompose it at two levels, namely we introduce $\mathrm{SU}(6)_F$ symmetry group[21,22] as a subgroup of SU (18) whose 35 generators are given by[21]

$$\lambda_\alpha \otimes \underset{\sim}{1}_{(2)}; \quad \underset{\sim}{1}_{(3)} \otimes \sigma_i \quad (\alpha = 1, \ldots, 8; \quad i = 1, 2, 3)$$

$$\lambda_i \otimes \sigma_j \tag{2.1}$$

(see Appendix A) where $1_{(a)}$ is the identity operator on V ($a = 2, 3$), and a 6-dimensional space on which the group SU $(3)_F \otimes$ SU (2) acts irreducibly is formed by the tensor product

$$V_6 = V_2 \otimes V_3. \tag{2.2}$$

The space V_3 for instance is spanned by three states (u, d, s) forming representation $\underset{\sim}{3}$ for SU $(3)_F$. The space V_6 is irreducible under SU (3) $\times$ SU (2) and the group SU (3) $\times$ SU (2) is a maximal subgroup of the group SU (6). The reason for such a decomposition is quite physical since QCD is colour gauged and the product group SU $(6)_{FJ} \times$ SU $(3)_C$ is a subgroup of the unitary group SU (18).

Another decomposition arises when one has to break the SU $(3)_F$ symmetry, so that total spins $J_{s,n}$ of strange/nonstrange quarks have to appear as well as the hypercharge[20]:

$$\mathrm{SU}(6) \supset \mathrm{U}(1)_Y \otimes \mathrm{SU}(4)_{IJ_n} \otimes \mathrm{SU}(2)_{J_s} \tag{2.3}$$

where again

$$\mathrm{SU}(4)_{IJ_n} \supset \mathrm{SU}(2)_I \otimes \mathrm{SU}(2)_{J_n}. \tag{2.4}$$

Since we have to have a colour singlet which represented by the Young tableau has to be rectangular with 3 rows and 2 coloumns, in the decomposition of SU $(6)_{FJ}$ from Young tableaus we can see that the required representation is [490] (having 2 rows and three coloumns), i. e.

$$[490] \to (\mathrm{SU}(3)_F, \mathrm{SU}(2)_J) = (\underset{\sim}{1}, 1) \oplus (\underset{\sim}{8}, 3) \oplus (\underset{\sim}{8}, 5) (\underset{\sim}{10}, 3) \oplus (\overline{\underset{\sim}{10}}, 3) \oplus$$

$$\oplus (\underset{\sim}{27}, 1) \oplus (\underset{\sim}{27}, 5) \oplus (\overline{\underset{\sim}{10}}, 7) \oplus (\underset{\sim}{35}, 3) \oplus (\underset{\sim}{28}, 1).$$

Colour forces in a dynamical model (CBM in our calculation) will pick up the required decomposition and a set of operators for a given representation is needed to completely specify the given state. As a result hyperfine splitting with respect to the flavour ($N - \Lambda$) or spin ($N - \Delta$) occurs. So one has to calculate nonlinear operators — Casimir operators and their eigenvalues for symmetry groups given in the above decomposition (see Appendix A and Refs. 1, 2 and 21). For instance, for the unbroken symmetry group SU $(3)_c$ the F_c spin operator is given by $F_c^\alpha = 1/2\, \lambda^\alpha$ ($a = 1, \ldots, 8$) and by defining

$$C^2 = \sum_\alpha (F_c^\alpha)^2 \tag{2.5}$$

we can calculate C^2 from the simple formula

$$C^2 = \frac{1}{2}(p^2 + q^2 - pq + 3p) \tag{2.6}$$

when the dimension of representation $D(p, q)$ is given by

$$D(p, q) = \frac{1}{2}(p - q + 1)(p + 2)(q + 1). \tag{2.7}$$

Eigenvalues of the quadratic Casimir operator are given in Table 1.

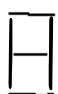
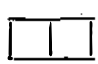
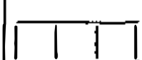
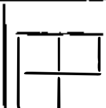
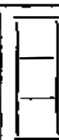
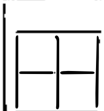
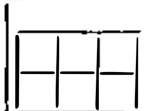
In the same way one calculates other Casimir operators and their eigenvalues. In Table 2 qudratic Casimir operators for SU (6) are given[23].

TABLE 1.

Dimension of representation		1	3	$\bar{3}$	6	$\tilde{6}$	8	10	$\overline{10}$	15	$\overline{15}$m	15s	15s	24	$\overline{24}$	27
Quarks indices	p	0	1	1	2	2	2	3	3	3	3	4	4	4	4	4
	q	0	0	1	0	2	1	0	3	1	2	0	4	1	3	2
$C^2(n)$		0	$\frac{4}{3}$		$\frac{10}{3}$		3	6		$\frac{16}{3}$		$\frac{28}{3}$		$\frac{25}{3}$		8

Eigenvalues of the quadratic Casimir operator for SU (3) (m — mixed, s — symmetric).

TABLE 2.

Dimension of representation	15	21	56	70	20	105	490
Young tableau							
$C^2(n)$	$\frac{14}{3}$	$\frac{20}{3}$	$\frac{45}{4}$	$\frac{33}{4}$	$\frac{21}{4}$	$\frac{32}{3}$	18

Young tableaus and eigenvalues of the quadratic Casimir for SU (6)[23].

In a QCD — based model one is forced to allow for colour forces mediated by gluons, and interactions of this kind will be represented by corresponding terms in a Lagrangian. So there will be corresponding contributions to the mass of (multiquark) states and for instance this will cause $N - \Delta$ splitting[5]. For the electrostatic interaction we have that a contribution is proportional to $\underset{\sim}{E}^\alpha \cdot \underset{\sim}{E}^\alpha \cdot (\alpha = 1, \dots 8)$ where

$$E_i^\alpha = F_{0i}^\alpha \tag{2.8}$$

is the colour electric field and a similar contribution comes from the colour-magnetic interaction with

$$B_i^\alpha = \frac{1}{2} \varepsilon_{ijk} F_{jk}^\alpha. \tag{2.9}$$

In a small domain inside the hadron, where colour dielectric constant and colour magnetic susceptibility are approximately equal to one ($\varepsilon = \mu = 1$) (this corresponding to the asymptotic freedom of QCD) the perturbation theoretic methods are applicable. The effect of the colour forces can be calculated from quantum mechanical methods by finding eigenvalues of corresponding operators. This brings us back to the group theoretic methods sketched at the beginning of this section since underlying symmetry group of the colour forces is contained in SU (18).

These lead to calculations of expressions of the type (calculated in Appendix A)

$$\sigma_i \sigma_j = \sum_k (\sigma_k)_i (\sigma_k)_j, \tag{2.10}$$

$$\lambda_i \lambda_j = \sum_\alpha (\lambda^\alpha)_i (\lambda^\alpha)_j \tag{2.11}$$

or

$$\sum (\lambda^c \sigma)_i (\lambda^c \sigma)_j. \tag{2.12}$$

If we sum the expression (2.10) over all quarks in a N — quark hadron we get[20]

$$\sum_{i>j} \sigma_i \sigma_j = 2 \underset{\sim}{J}^2 - \frac{3}{2} N \tag{2.13}$$

or (F reffers to flavour)

$$\sum_{i>j} \lambda_i^F \lambda_j^F = 2\, C^2 - 2\, N\, C^2 (3) = 2\, C^2 - \frac{8}{3} N \tag{2.14}$$

by using Table 1. In the same manner we get[20,24]

$$-\sum_{i>j} (\lambda^c \sigma)_i (\lambda^c \sigma)_j = N (N - 10) + \frac{4}{3} \underset{\sim}{J}^2 + 4\, C^2. \tag{2.15}$$

Performing restricted sums (strange quarks only, for instance) one gets similar expressions which are then incorporated in a particular operator acting at states of a dynamical model for hadrons. The actual form of operators requiring the above formalism is given in the next section together with appendices where additional details are given.

3. Bag model : MIT and CBM

The MIT bag model energy of a static spherical cavity approximation is

$$E(R) = E_V + E_0 + E_Q + E_M + E_E \tag{3.1}$$

where the volume term is

$$E_V = \frac{4\pi}{3} B R^3. \tag{3.2}$$

The zero point energy term is

$$E_0 = -Z_0/R \tag{3.3}$$

and the quark kinetic energy contribution is

$$E_Q = \frac{N_0\, \omega_0 (R)}{R} + \frac{N_S\, \omega_s (m_s R)}{R} \tag{3.4}$$

where $N_{0,S}$ is a number of nonstrange/strange quarks. ω's are solutions to the linear boundary condition imposed by the reason of confinement

$$-\mathrm{i}\, \hat{r}\, \underset{\sim}{\alpha}\, \psi = \beta\, \psi \quad (r = R) \tag{3.5}$$

It should be pointed out that (3.5) is not the only possible BC which quantizes energy, but for instance the so called bilinear BC leads to calculational difficulties[25,26].

As mentioned before perturbative gluon interactions are expressed by the colour electric (E_E) and the colour magnetic (E_M) contributions, and they are given by[5,24]

$$E_E = b\, \varepsilon \tag{3.6}$$

and

$$E_M = -\sum_{i>j} \frac{\alpha_s}{R} M (m_i R, m_j R)\, (F^c \underset{\sim}{\sigma})_i\, (F^c \underset{\sim}{\sigma})_j. \tag{3.7}$$

In our calculations we will neglect the E_E contribution. The M — function in (3.7) is determined by three parameters (given in Ref. 5 or 24).

Because the linear BC (3.5) breaks chiral symmetry[26] a new BC must be introduced as well as a field residing in the bag exterior which restores chiral symmetry. The new BC for the SU (2) case is

$$-\,\mathrm{i}\, \hat{r}\, \underset{\sim}{\gamma}\, \psi = \mathrm{e}^{\mathrm{i}\, \underset{\sim}{\tau}\, \underset{\sim}{\varphi}_M\, \gamma_5 / f_M}\, \psi \tag{3.8}$$

and the field φ_M satisfies the Klein-Gordon equation for $r > R_{BAG}$. $f_M = 0.093$ GeV is the pion decay constant. So nonlinearities in (3.8) are neglected and φ_M is treated as an elementary pion field.

From the continuity of the isovector axial current $\underset{\sim}{A}$ a solution for φ_M is found, and an extension of the quark core is made by assigning $\bar{q}q$ (flavour) content to a meson field[27]. Therefore we enlarge the symmetry group to SU $(2)_F$ and include meson nonet into consideration.

The meson field with a mass μ is given by

$$\underset{\sim}{\varphi}^M = D_0^M \frac{\mathrm{e}^{-\mu r}}{\mu\, r} (\chi_m^+ \underset{\sim}{\lambda}\, \chi_n)\, (d_m\, b_n + b_m^+\, d_n^+) + D_1^M \left(\frac{1}{\mu\, r} + \frac{1}{\mu^2 r^2} \right) \mathrm{e}^{-\mu r}\, [\chi_p^+\, (\underset{\sim}{\sigma}\, \hat{r})\, \underset{\sim}{\lambda}\, \chi_q] \cdot$$
$$\cdot\, (b_p^+\, b_q + d_p\, d_q^+) \tag{3.9}$$

where $D^M_{0,1}$ are constants determined from the continuity of the axial current, and $b\,(d)$ are quark (antiquark) annihilation operators.

Mesonic contribution to energy (3.1) of a hadron is given by the operator

$$E_p(R) = \hat{E}_p(R) \cdot \tilde{\Sigma} \tag{3.10}$$

where $\hat{E}_p(R)$ is in somewhat different notation given in Ref. 27 and

$$\tilde{\Sigma} = \Sigma \langle H \mid (\underset{\sim}{\sigma}_i \underset{\sim}{\sigma}_j)(\underset{\sim}{\tau}_i \underset{\sim}{\tau}_j) \mid H \rangle \tag{3.11}$$

should be evaluated between hadron states $|H\rangle$.

The boundary condition (3.8) leads, when developed up to linear term in the meson field, to a transcendental equation whose solutions (for $m_{u,d} = 0$ and $m_s \neq 0$) enter in the kinetic energy term (3.4).

Therefore, we end up with five terms for the hadron masses[7] ((3.1) + (3.10)) which depend on four parameters: the bag constant B, the colour coupling constant a_s, the zero point energy parameter Z_0, and the strange quark mass m_s. From the stability condition

$$\frac{\partial E}{\partial R} = 0 \tag{3.12}$$

R is determined for a given set of the four parameters which are adjusted to reproduce baryon masses.

In Table 3 we present four sets of fitting parameters together with respective baryon masses, as well as some other fits found in literature to facilitate comparison. Our fits are calculated in such a way that we can investigate stability of calculations with respect to small fluctuations of the model parameters.

4. *Dibaryon masses in the CBM*

Until now we have prepared almost all ingredients for the calculations of dibaryon masses. What is left to calculate is the magnetic energy part (3.7) which can be expressed in terms of Casimir operators discussed in Sec. 2. (and Appendix A). Also, we have to calculate the pionic contribution (3.11) when SU (3) group is taken to be the flavour symmetry group (see Appendix B).

Here we quote the results only and more details can be found in appendix.

We have applied our model to the dibaryon spectrum starting from $S = 0$ ($Y = -2$) states down to $S = -2$ ($Y = 0$) states and special attention was paid to the H — dibaryon. So in doing summation in (3.7) sometimes only nonstrange quarks have to be taken into account ($S = 0$), but mostly summing over nonstrange and strange quarks has to be done.

For the first case ($Y = -2$) we have (N_n equals a number of the nonstrange quarks)

$$\sum_{n_1 > n_2} (\lambda^c \sigma)_1 (\lambda^c \sigma)_2 = \frac{3}{4} N_n^2 - N_n - C_4 + \frac{4}{3} C_2(\sigma, n) + 4\, C_2(\tau, n) \tag{4.1}$$

TABLE 3.

		F.1	Our fits *F.2*	*F.3*	*F.4*	Ref. 5 *F.5*	Ref. 24 *F.6*	Ref. 28 *F.7*
$B^{1/4}$		.1296	.133	.144	.146	.116	.151	.163
Z_0		.4	.8	1.0	1.2	.84	1.31	1.98
α_c		.7	.5	.6	.5	.56	.35	.42
m_s		.278	.286	.301	.309	.206	.218	.232
m_N	Fit	.937	.937	.938	.938	.938	.939	.939
	exp	.938						
m_Λ	Fit	1.082	1.084	1.082	1.084	1.105	1.132	1.116
	exp	1.116						
m_Σ	Fit	1.178	1.170	1.182	1.177	1.144	1.209	1.167
	exp	1.189						
m_Ξ	Fit	1.321	1.321	1.321	1.321	1.289	1.361	1.312
	exp	1.321						
m_Δ	Fit	1.2995	1.215	1.2891	1.238	1.233	1.232	1.232
	exp	1.236						
m_Ω	Fit	1.7842	1.751	1.858	1.842	1.672	1.672	1.672
	exp	1.672						
$\bar{R}$		5.71	5.46	4.92	4.86	4.96	5.12	4.56

Different sets of bag parameters: CBM (*F.1—F.4*) our fits, *F.5* — MIT bag model, *F.6,7* — MIT bag + pions model.
$\bar{R}$ is an average radius (in GeV^{-1}).

or when Casimirs are considered we find

$$\sum_{n_1>n_2} (\lambda^c\sigma)_1 (\lambda^c\sigma)_2 = \frac{4}{3} N_n^2 - 8 N_n + \frac{4}{3} \underset{\sim}{J}_n^2 + 4 \underset{\sim}{I}^2 \tag{4.2}$$

where J_n is spin of the nonstrange quarks.

For the strange sector the nonstrange (N_n), strange (N_s) and the total (N) number of quarks is connected with the hypercharge by

$$N_n = \frac{2}{3} N + Y; \quad N_s = \frac{1}{3} N - Y \tag{4.3}$$

so

$$\sum_{n_1>n_2} (\lambda^c\sigma)_1 (\lambda^c\sigma)_2 = \frac{3}{4} N_n^2 - N_n + \frac{4}{3} \underset{\sim}{J}_n^2 + 4 \underset{\sim}{I}^2 + 2 \underset{\sim}{J}_s^2 +$$

$$+\frac{1}{4}Y^2-\frac{1}{6}N(N-18)-\frac{4}{3}(N-9)Y \tag{4.4}$$

where $\underset{\sim}{J}_s$ is spin of the strange quarks.

These two expressions ((4.2) and (4.4)) are to be introduced in (3.7) for a given Y. In the strange sector similar calculation for sum over all and sum over strange quarks has to be done (Ref. 20 and Appendix A).

In the very similar way, the matrix element (3.11) is calculated when the SU (3) group is taken to be the flavour-symmetry group (Appendix B). By taking sum over the total of N quarks

$$\sum_{i,j=1}^{N}(\lambda_i^F\underset{\sim}{\sigma}_i)(\lambda_j^F\underset{\sim}{\sigma}_j)=-2N^2+36N-\frac{8}{3}\underset{\sim}{J}-4C_3(F) \tag{4.5}$$

only eigenvalues of the quadratic Casimir operators (see for instance Table 1) have to be calculated between diagonal states.

TABLE 4.

I	3	2	1	1	0	0
J	0	1	2	0	3	1
F	28	35	27	27	$\overline{10}$	$\overline{10}$
M_2	48	$\frac{80}{3}$	16	8	16	$\frac{8}{3}$
Σ	36	52	52	76	36	76
M/R (FIT 1)	2.23	2.12	2.08	2.02	2.10	2.00
	7.61	7.42	7.32	7.23	7.45	7.20
M/R (FIT 2)	2.18	2.09	2.06	2.00	2.08	1.99
	7.28	7.13	7.04	6.97	7.16	6.95
M/R (FIT 3)	2.34	2.23	2.19	2.12	2.22	2.10
	6.75	6.60	6.51	6.46	6.62	6.44
M/R (FIT 4)	2.31	2.21	2.17	2.11	2.20	2.09
	6.62	6.49	6.40	6.36	6.51	6.34
M/R Ref. 24	2.69	2.46	2.36	2.24	2.38	2.18
	6.78	6.61	6.52	6.45	6.52	6.41

$Y = 2$ ($S = 0$) sector F is the SU (3) representation and $M2$ is calculated from (4.2). Masses are given in MeV and R's in MeV^{-1}. Σ is calculated from (3.11) (see Ref. 20), M_1, M_2, M_3 are eigenvalues of magnetic energy operators (2.15), (A.26) and (A.12), respective.

TABLE 5.

I	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	3/2				1/2							
J	0	1	2	0	0	3	1				2				1			
J_s	1/2	1/2	1/2	1/2	1/2	1/2	$\frac{1}{2}$				$\frac{1}{2}$				$\frac{1}{2}$			
J_n	1/2	1/2	3/2	1/2	1/2	5/2	3/2		1/2		$\frac{5}{2}$		$\frac{3}{2}$		3/2		1/2	
F	28	35	27	27	27	$\overline{10}$	10	35	10	35	8	27	8	27	$\overline{10}$	8	$\overline{10}$	8
Σ	72	$\frac{272}{3}$	96	112	112	88	$\frac{344}{3}$	$\frac{272}{3}$	$\frac{344}{3}$	$\frac{272}{3}$	116	96	116	96	$\frac{344}{3}$	$\frac{380}{3}$	$\frac{344}{3}$	$\frac{380}{3}$
M_1	48	$\frac{80}{3}$	72	8	8	16	8/3	80/3	8/3	80/3	-4	72	-4	72	$\frac{8}{3}$	$-\frac{28}{3}$	$\frac{8}{3}$	$-\frac{28}{3}$
M_2	32	32	16	12	0	$\frac{32}{3}$	16		12	12	$\frac{32}{3}$		4		4		0	

$Y = 1$ $(S = -1)$ sector. $J_{s/n}$ is the total spin of strange/nonstrange quarks. Σ is calculated in Apendix B.

In Table 4 we present the appropriate quantum numbers and matrix elements for the nonstrange sector, as well as the masses for different fits. Also, results from literature are given.

This nonstrange sector (apart from stability properties) is interesting when one considers low energy π-nucleus scattering and possible resonant channels which could give a hint of how the results are to be compared in the experiments.

In Table 5 we give the $Y = 1$ ($S = 1$) spectrum. Multiplicity of states arise from nonstrange-strange spin couplings which yield the total spin $\underset{\sim}{J}$. Some of these states are selected and masses are calculated. The choice of the quantum numbers and representations is such that results from literature can be compared with ours. The results are presented in Table 6.

TABLE 6.

I	1/2	1/2
J	2	2
J_n	3/2	1/2
F	8	8
Σ	116	380/3
M/R (FIT 1)	2.00	1.95
	6.18	6.18
M/R (FIT 3)	2.05	2.00
	5.63	5.51
M/R Ref. 24	2.27	2.20
	6.33	6.28

$Y = 1$ ($S = -1$) sector, for 8 representation (see also caption of Table 4).

Tables 7 give the results for $Y = 0$ (S $= -2$) sector. H-dibaryon ($I = 0$, $\underset{\sim}{J} = 0$) has become very popular lately especially in connection with the explanation of the Cygnus X-3 events[29-31] in terms of radioactivity in strange quark matter[32,33]. One should notice J-F degeneracy (for a given $J_S = 0$ for instance there are two F's, 1 and 27) which is lifted by $M1$ interaction. The mass spectrum is presented in Table 8.

5. *Discussion and conclusion*

In this paper we calculated masses within the chiral bag model developed to restore the chiral symmetry which was broken in the MIT bag model.

TABLE 7a.

I	1															
J	1															
J_s	1				1				1				0			
J_n	1				0				2				1			
F	10	10	8	35	10	10	8	35	10	10	8	35	10	10	8	35
Σ	$\frac{342}{3}$	$\frac{344}{3}$	$\frac{380}{3}$	$\frac{277}{3}$	$\frac{344}{3}$	$\frac{344}{3}$	$\frac{380}{3}$	$\frac{372}{3}$	$\frac{344}{3}$	$\frac{344}{3}$	$\frac{380}{3}$	$\frac{272}{3}$	$\frac{347}{3}$	$\frac{347}{3}$	$\frac{380}{3}$	$\frac{272}{3}$
M_1	$\frac{8}{3}$	$\frac{8}{3}$	$-\frac{28}{3}$	$\frac{80}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	$-\frac{28}{3}$	$\frac{80}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	$-\frac{28}{3}$	$\frac{80}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	$-\frac{28}{3}$	$\frac{80}{3}$
M_2	8/3				0				8				$\frac{20}{3}$			
M_3	8/3				8/3				8/3				4			

Quantum numbers and matrix elements for $Y = 0$ ($S = -2$) sector.
States with $I = 0$, $J = 0$ are H-dibaryon.

TABLE 7b.

I	1		2	0	0				1	2				1	2	0			
J	2		2	1	0				0	0				3	1	2			
J_s	1		1	1	0		1		1	0		1		1	1	0		1	
J_n	1		1	1	0		1		1	0		1		2	1	2		1	
F	27	28	27	8	27	1	27	1	27	27	28	27	28	10	35	8	27	8	27
Σ	96	116	96	$\frac{380}{3}$	112	144	112	144	112	112	72	112	72	88	$\frac{272}{3}$	116	96	116	96
M_1	72	−4	72	$-\frac{28}{3}$	8	−24	8	−24	8	8	48	8	48	16	$\frac{80}{3}$	−4	72	−4	72
M_2	8/3		56/3	−16/3	−4		−16/3		8/3	20		56/3		8	$\frac{56}{3}$			−26/3	
M_3	8/3		8/3	8/3	4		8/3		8/3	4		8/3		8/3	8/3	4		8/3	

Quantum numbers and matrix elements for $Y = 0$ $(S = -2)$ sector.

TABLE 8.

J	0	0	0	0
J_n	0	0	1	1
J_s	0	0	1	1
F	1	27	1	27
M/R (FIT 1)	2.00	2.10	1.99	2.10
	5.45	6.49	5.47	6.48

H — dibaryon ($Y = 0 = B + S$) (see caption of Table 4).

Starting from the symmetry considerations motivated by quantum number which describe the six quark states, and dictated by the Pauli principle we were able to construct a spectrum of dibaryon states in the nonstrange ($Y = B + S = 2$) and the strange ($Y = 0, 1$) sector.

In the nonstrange sector from Table 4 it can be seen that our model consistently produced dibaryons with lower masses in the range from 10 to 20 percent. This is rather important sector since it can play a role in the nucleon-nucleon scattering at low energies[34], where many precise experimental data are available.

In the strange sector with $S = -1$ there is no flavour singlet state (as in $S = 0$)[35] but $S = -1$ is closer to the strong decay threshold which for ΛN is at 2055 MeV and for ΣN at 2130 MeV. These states have very small widths[34] and there are some experimental signals near the ΣN threshold.

Our model for the $S = -1$ sector gives masses which are, differing from the MIT-bag-model-based calculations[24], lower for about 10% (Table 6) than the MIT bag calculations and this brings us below strong the decay threshold, within the $\Delta S = 1$ weak decay of an $S = -1$ dibaryon. One can proceede then via a well known route by taking H_{eff} and calculate widths in a model, or one can take a phenomenological amplitude of a form[30]

$$\mathcal{M} = G\, m_M\, (A - B\, \gamma_5) \tag{5.1}$$

where m_M, and A and B can be inferred from a $\Delta S = 1$ process with the required selection rules in terms of isospin assignments.

For the $Y = 0$ sector when consulting Tables 7 and 8 we can see that again masses of $S = -2$ dibaryons are smaller then the masses calculated in the MIT bag[20,24] or in the topological soliton model[36]. This simply means that in CBM model H-dibaryon is more deeply bound with respect to a strong decay, and that one should look for more favourable (weak) decays with $|\Delta S| = 1$ or $|\Delta S| = 2$.

We have also checked that the possibility of the strange-nonstrange spin couplings which give the total spin 0 (Table 7) when $J_{n/s} = 0$ and when $J_{n/s} \neq 0$ does not bring practically any difference (less then 1 %). These results are given in Table 9.

TABLE 9.

$M_F^{(J,Y)}$ \ FIT	1	2	3
$M_8^{(1,0)}$	2.07	2.06	2.13
$M_8^{(2,0)}$	2.12	2.10	2.18
$M_{10}^{(1,0)}$	2.14	2.12	2.21
$M_{10}^{(3,0)}$	2.27	2.23	2.33
$M_{27(0,0)}^{(0,0)}$	2.10	2.14	2.17
$M_{27(1,1)}^{(0,0)}$	2.10	2.08	2.17
$M_{1(0,0)}^{(0,0)}$	2.00	2.00	2.06
$M_{1(1,1)}^{(0,0)}$	2.00	2.00	2.06

$Y = 0$ sector for different representations (F) and for $(J_s, J_n) = (0, 0)$ or $(J_s, J_n) = (1, 1)$. The ast two rows are results for H — dibaryon.

Another interesting comparison can be made by involving the calculation in Ref. 36 where the following relations (»sum rules«) arise from their model:

$$M_8^{(1,0)} - M_H = 302 \text{ MeV}$$

$$M_8^{(2,0)} - M_H = 349 \text{ MeV}$$

$$M_{10}^{(1,0)} - M_H = 581 \text{ MeV}$$

$$M_{10}^{(3,0)} - M_H = 698 \text{ MeV}$$

where notation gives the total J, Y and the representation (F): $M_F^{(J,Y)}$. Also $M_H \sim$ $\sim M_{F=A}^{(0,0)}$.

In our model the above relations give on the right hand side the following values (for the respective fits as in Table 9) (in MeV):

FIT1	FIT2	FIT3	AVERAGE
68.4	62.9	70.1	67.1
117.1	105.2	120.2	114.2
144.1	126.3	144.7	138.4
265.0	234.5	270.9	256.8

which shows that differences are smaller for different representations, i. e. states are more deeply bound and closer to the lowest H-dibaryon state. When a comparison is made it should be pointed out that for a given (J, F) there is always more then one state differing in isospin (Table 7). The states are mostly chosen in such a way that there was no splitting in the spin of strange quarks, although other choices would lead to the same conclusions.

According to our results one should look for a second order weak decay of H-dibaryon since the $|\Delta S| = 1$ threshold (in $N + \Lambda$) is 2.055 MeV and the average mass from the three fits is 2.02 MeV. This kind of decay is considered in Ref. 30 in a simplified model of V-A theory, as well as in Ref. 29 where calculation is done by looking at the box diagram for the transition $ss \to dd$.

One reliable method for calculations of masses are the lattice calculations. One of the latest results[37] gives that $m_H > 2m_\Lambda$, which is in a complete disagreement with all the calculations cited or did in this paper.

Apparently the existence of H-dibaryon together with its decay modes is still an open question which has not been resolved by recent experimental searches[38] aimed at energies below the $\Lambda\Lambda$ threshold including the possible mass range which s the result of our calculation for $Y = 0$ sector.

Appendix A

Here we spell out the major steps in the calculation of the colour magnetic energy.

In order to calculate the colour magnetic contribution to barion masses we have to consider spin and isospin states of three quarks separately. In the calculation of the expectation value of the colour magnetic operator between dibaryon states the colour and colour spin tensor operators have to be expressed in terms of quadratic Casimir operators of SU $(2)_F$, SU $(3)_c$ and SU (2) . Then the summation over quarks is done according to a baryon flavour contents: a) all quarks, b) strange quarks and c) nonstrange quarks.

a) *The sum over all quarks*

From the Pauli exclusion principle

$$P^F_{ij} P^C_{ij} P^J_{ij} = -1 \tag{A1}$$

where the permutation operators are

$$\begin{aligned} P^J_{ij} &= \frac{1}{2}\left(\frac{2}{3} + \sigma_i \sigma_j\right) \quad (\text{for SU}(2)_J) \\ P^F_{ij} &= \frac{1}{3} + \frac{1}{2}\lambda^F_i \lambda^F_j \quad (\text{for SU}(3)_F \\ P^C_{ij} &= \frac{1}{3} + \frac{1}{2}\lambda^C_i \lambda^C_j \quad (\text{for SU}(3)_c) \end{aligned} \tag{A2}$$

we get

$$-(\lambda^c\sigma)_i(\lambda^c\sigma)_j = 1 + \frac{1}{3}\sigma_i\sigma_j + \frac{3}{2}\lambda_i\lambda_j - \frac{1}{2}(\lambda^F\sigma)_i(\lambda^F\sigma)_j. \tag{A.3}$$

By introducing 35 SU (6) generators A_i (2.1) we can write

$$2\sum_{i>j} A_i A_j = \sum_{i>j}\left[\frac{1}{3}\sigma_i\sigma_j + \frac{1}{2}\lambda_i^F\lambda_j^F + \frac{1}{2}(\lambda^F\sigma)_i(\lambda^F\sigma)_j\right] \tag{A.4}$$

and from (A.3) we get

$$-\sum_{i>j}(\lambda^c\sigma)_i(\lambda^c\sigma)_j = \sum_{i>j}(1 - 2A_iA_j + \frac{2}{3}\sigma_i\sigma_j + 2\lambda_i^F\lambda_j^F). \tag{A.5}$$

From the normalization condition for Casimirs (for SU (6) for example)

$$\sum_{a=1}^{35}(A_a)^2 = \frac{35}{6} \tag{A.6}$$

and from their eigenvalues[21,22] ((2.6) and Table 1) we obtain

$$\sum_{i>j}\sigma_i\sigma_j = 2C_2 - \frac{3}{2}N \tag{A.7}$$

$$\sum_{i>j}\lambda_i^F\lambda_j^F = 2C_3 - \frac{8}{3}N$$

$$2\sum_{i>j}(\sum_a (A_a)_i(A_a)_j) = C_6 - \frac{35}{6}N = \frac{1}{2}N\left(\frac{19}{3} - N\right) \tag{A.8}$$

and together with (A5) we finally get (2.15).

b) *The sum over strange quarks*

In the similar fashion as in *a*), from

$$P_{ij}^{J_z} P_{ij}^{C} = -1 \tag{A.9}$$

where now

$$P_{ij}^{J_z} = \frac{1}{2}(1 + \sigma_i\sigma_j) \tag{A.10}$$

we get

$$-(\lambda^c\sigma)_1(\lambda^c\sigma)_2 = 3 - \frac{1}{3}\sigma_1\sigma_2 \tag{A.11}$$

and from the definition of Casimirs we obtain

$$-\sum_{s_1>s_2}(\lambda^c\sigma)_1\,(\lambda^c\sigma)_2=\frac{3}{2}N_s^2-N_s-\frac{2}{3}C_2. \tag{A.12}$$

c) *The sum over nonstrange quarks*

From the antisymmetry of permutation operators

$$P_{12}^{J_n}\,P_{12}^{I}\,P_{12}^{C}=-1 \tag{A.13}$$

where now (see also (A.2) and (A.10))

$$P_1^I=\frac{1}{2}(1+\tau_1\,\tau_2)$$

we get

$$(\lambda^c\sigma)_1\,(\lambda^c\sigma)_2=\frac{3}{2}+\frac{3}{2}\tau_1\,\tau_2+\frac{1}{6}\sigma_1\sigma_2-\frac{1}{2}(\tau\sigma)_1\,(\tau\sigma)_2. \tag{A.14}$$

By introducing 15 SU (4, I, I_n) generators B_b in the same way as we did in (2.1) for SU (6), and from

$$\mathrm{Tr}\,B_b^2=1 \tag{A.15}$$

we get

$$2\sum_{n_1>n_2}B_1\,B_2=C_4-\frac{15}{4}N \tag{A.16}$$

where we used the quadratic Casimir operator

$$C_4\,(4)=\sum_{b=1}^{15}(B_b)^2=\frac{15}{4}. \tag{A.17}$$

Together with

$$\sum_{n_1>n_2}\sigma_1\sigma_2=2\,C_2\,(n)-\frac{3}{2}N_n \tag{A.18}$$

$$\sum_{n_1>n_2}\tau_1\,\tau_2=2\,I^2-\frac{3}{2}N_n$$

one finds

$$-\sum_{n_1>n_2}(\lambda^c\sigma)_1\,(\lambda^c\sigma)_2=\frac{3}{4}N_n^2-N_n-C_4+\frac{4}{3}C_2\,(\alpha,n)+4\,C_2\,(\tau,n). \tag{A.19}$$

The next step is to express C_4 in terms of quadratic Casimir of SU (2). So

$$\sum_{n_1>n_2} \lambda_1^c \lambda_2^c = -\frac{7}{12} N_n^2 + \frac{13}{3} N_n - C_4 = 2\, C_3\,(C, n) - \frac{8}{3} N_n \qquad \text{(A.20)}$$

and

$$\sum_{s_1>s_2} \lambda_1^c \lambda_2^c = 2\, C_3\,(C, s) - \frac{8}{3} N_s \qquad \text{(A.21)}$$

where $C_3\,(C, n)$ ($C_3\,(C, s)$) is quadratic Casimir for SU $(3)_c$ group for nonstrange (strange) quarks only. We know that

$$C_3\,(C, n) = C_3\,(C, s) \qquad \text{(A.22)}$$

since the colour irreducible representations must be complex conjugate of each other.

For instance in the $Y = 2$ region $C_3\,(C, n) = 0$ (colour singlet condition) and

$$C_4 = \frac{7}{12}\, 12\, N_n\,(12 - N_n) \qquad \text{(A.23)}$$

so

$$-\sum_{n_1>n_2} (\lambda^c \sigma)_1\,(\lambda^c \sigma)_2 = \frac{4}{3} N_n^2 - 8\, N_n + \frac{4}{3} J_n^2 + 4\, I^2. \qquad \text{(A.24)}$$

In the $S \neq 0$ region we have

$$N_n = \frac{2}{3} N + Y \quad N_s = \frac{1}{3} N - Y$$

and one finds

$$C_4 = 2\, J_s^2 + \frac{1}{4} Y^2 - \frac{1}{6} N\,(N - 18) + \frac{4}{3}(N - 9)\, Y \qquad \text{(A.25)}$$

and

$$-\sum_{n_1>n_2} (\lambda^c \sigma)_1\,(\lambda^c \sigma)_2 = \frac{3}{4} N_n^2 - N_n + \frac{4}{3} J_n^2 + 4\, I^2 + 2\, J_s^2 + \frac{1}{4} Y^2 - $$

$$- \frac{1}{6} N\,(N - 18) - \frac{4}{3}(N - 9)\, Y. \qquad \text{(A.26)}$$

Appendix B

In this appendix we calculate the spin-flavour operator (3.11) in terms of quadratic Casimir operators of SU $(3)_F$ and SU (2) . We use again the Pauli principle as in Appendix A and by using results from App. A we get

$$(\lambda_i^F \underset{\sim}{\sigma}_i)\,(\lambda_j^F \underset{\sim}{\sigma}_j) = -2 - 2\, \lambda_i^c \lambda_j^c - \frac{2}{3} \underset{\sim}{\sigma}_i \underset{\sim}{\sigma}_j - \lambda_i^F \lambda_j^F \quad (i \neq j) \qquad \text{(B.1)}$$

and from the following expression

$$\sum_{a=1}^{8} (\lambda^a \underset{\sim}{\sigma})_i (\lambda^a \underset{\sim}{\sigma})_i = 16 \tag{B.2}$$

we have for $i = j$

$$(\lambda_i^{aF} \underset{\sim}{\sigma}_i)(\lambda_i^{cF} \underset{\sim}{\sigma}_i) = -20. \tag{B.3}$$

Since the above expression appears N times in summation over i, j in (B.1) we have to substract it N times and complete the summation by adding (B.2) N times. Hence we obtain

$$\sum_{i,j=1} (\lambda_i^F \underset{\sim}{\sigma}_i)(\lambda_j^F \underset{\sim}{\sigma}_j) = -2N^2 + 36\, N - \frac{8}{3} \underset{\sim}{J}^2 - 4\, C_3\,(F) \tag{B.4}$$

where J is the total spin of six quarks and $C_3\,(F)$ is quadratic Casimir operator for $SU\,(3)_F$. The expectation value of this operator contains eigenvalues of quadratic Casimirs which are constants for a given irreducible representation.

Acknowledgement

We acknowledge the support from the U. S. National Science Foundation (contract 681) and JFP—683.

References

1) D. Flamm and F. Schoberl, *Introduction to the Quark Model of Elementary Particles* (Gordon and Breach. New York, 1982);
2) R. L. Jaffe, Phys. Rev. Lett. **38** (1977) 195;
R. L. Jaffe, Phys. Rev. D **15** (1977) 267, 281;
3) R. L. Jaffe, in *Point-like Structures Inside and Outside Hadrons*, Proceedings of the Seventeenth International School of Subnuclear Physics, Erice, Italy, 1979, ed. by A. Zichichi (Plenum, New York, 1982);
4) A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. **D9** (1974) 3471;
5) T. De Grand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. **D12** (1975) 2060;
6) T. H. R. Skyrme, Proc. Roy. Soc. (London) **A260** (1961) 127; Nucl. Phys. **31** (1962) 556;
7) N. K. Pak and H. C. Tze, Ann. Phys. (NY) **117** (1979) 164;
A. P. Balachandran. V. P. Nair. S. G. Rajeev and A. Stern, Phys. Rev. Lett. **49** (1982) 1124 and Phys. Rev. **D27** (1983) 1153;
8) See also *Solitons in Nuclear and Elementary Particle Physics*. Proceedings fo the Lewes Workshop, ed. by A. Chodos. E. Hadjimichael and C. Tze (World Scientific. Singapore 1984);
9) D. Klabučar, Phys. Lett. **149B** (1984) 31;
10) A. P. Balachandran, A. Barducci, F. Lizzi, G. J. Rodgers and A. Stern, Phys. Rev. Lett. **52** (1984) 887;
11) U. G. Meissner and U. B. Kaulfuss, Phys. Rev. **C30** (1984) 2058;
12) E. Sorace and M. Tarlini, Phys. Rev. **D33** (1986) 253;
13) U. G. Meissner, Phys. Lett. **166B** (1986) 169;
14) H. Gomm, F. Lizzi and G. Sparano, Syracuse Univ. Preprint SU-4222-298 (1984);
15) A. Chodos and C. H. Thorn, Phys. Rev. **D12** (1975) 2733;

16) G. E. Brown and M. Rho, Phys. Lett. **82B** (1979) 177; G. E. Brown, M. Rho and V. Vento, Phys. Lett. **84B** (1979) 383;
17) V. Vento, M. Rho, E. M. Nyman, J. H. Jun and G. E. Brown, Nucl. Phys. **A345** (1980) 413;
18) P. J. Mulders, Phys. Rev. **D30** (1984) 1073;
19) I. Zahed, U. G. Meissner and A. Wirzba, Phys. Lett. **140B** (1984) 117;
M. Rho, A. S. Goldhaber and G. E. Brown, Phys. Rev. Lett. **51** (1983) 747;
L. Vepstas, A. D. Jackson and A. S. Goldhaber, Phys. Lett. **140B** (1984) 280;
G. E. Brown, A. D. Jackson, M. Rho and V. Vento, Phys. Lett. **140B** (1984) 285;
20) A. T. M. Aerts, P. J. G. Mulders and J. J. de Swart, Phys. Rev. **D17** (1978) 260;
21) H. Bacry, *Lectures on Group Theory and Particle Theory*, Gordon and Breach (London,1977);
22) D. B. Lichtenberg, *Unitary Symmetry and Elementary Particles*, Academic Press (NewYork 1978);
23) J. L. Rosner, Phys. Rev. **D33** (1986) 2043;
24) P. J. Mulders and A. W. Thomas, J. Phys. **G9** (1983) 1159;
25) See for instance A. W. Thomas, Advances in Nucl. Phys. **13** (1983) 1;
26) F. Myhrer, Int. Rev. Nucl. Phys. **1** (1984) 326;
27) Many details and formulae can be found in D. Horvat and D. Tadić, Z. Phys. C — Particles and Fields **31** (1986) 311;
28) H. Takashita, A. Hosaka and H. Toki, Prog. Theor. Phys. **75** (1986) 890;
29) J. F. Donoghue, E. Golowich and B. R. Holstein, Phys. lett. **174B** (1986) 441;
30) I. B. Khriplovich and E. V. Shuryak, Sov. J. Nucl. Phys. **43** (1986) 858;
31) J. F. Donoghue, E. Golowich and B. R. Holstein, Phys. Rev. **D34** (1986) 3434;
32) M. S. Berger and R. L. Jaffe, MIT Preprint, CTP-1360 (August 1986);
33) P. D. Barnes, Plenary Talk at the Second Conf. on the Intersections Between Particle and Nuclear Physics, Lake Louise, Canada, 1986., Carnegie Mellon University Perprint, CMU —DOE/ER/03244 — 125;
L. Heller and J. A. Tjon, Los Alamos Preprint, LA-UR-86-2708;
K. Saito, Tohoku College Preprint, TCP-01 (1984);
34) An excellent review of experimental searches and data is: M. P. Locher, M. E. Sainio and A. Švarc, SIN Preprint, PR-85-12 (October 1985) (Submitted to Adv. in Nucl. Phys.);
35) A. T. M. Aerts and C. B. Dover, Phys. Lett. **146B** (1984) 95;
36) A. P. Balachandram, F. Lizzi, V. G. J. Rodgers and A. Stern, Nucl. Phys. **B256** (1985) 525;
37) P. K. Mackenzie and H. B. Thacker, Talk at the Workshop *Lattice Gauge Theory — a Challenge in Large Scale Computing*, Wuppertal, West Germany (1985);
38) RISK Collaboration, Ann. Physik Leipzig **43** (1986) 407.

DIBARIONSKA STANJA U KIRALNOM MODELU VREĆE

DAMIR BOSNAR

Fizički zavod, Prirodoslovno-matematički fakultet Sveučilišta u Zagrebu, 41000 Zagreb

i

DUBRAVKO HORVAT

Zavod za fiziku, Elektrotehnički fakultet Sveučilišta u Zagrebu, 41000 Zagreb

UDK 539.12

Originalni znanstveni rad

U kiralnom modelu vreće izračunate su mase stanja od šest kvarkova, dobivenih grupno-teorijskom dekompozicijom $SU(3)_F \otimes SU(3)_C \otimes SU(2)_J$ grupe simetrija. Uspostavljanjem kiralne simetrije povećava se energija vezanja i otvara se mogućnost različitih načina raspada. Dan je kratak pregled vrsta raspada.