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LETTER TO THE EDITOR

SU (6) UNIFICATION OF FUNDAMENTAL INTERACTIONS IN A FERMION-SCALAR MODEL

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A fermion-scalar model with $SU(3)_{HC} \times SU(3)_C \times U(1)_Q$ as unbroken gauge symmetry is constructed. The unification of the hypercolour, colour and electromagnetic interactions is realized within SU (6) symmetry with the unification scale $\sim 10^{15}$ GeV. Only three generations of the quarks and leptons can be accomodated in this scheme.

During last few years many authors have speculated about the internal structure of the leptons, quarks and the intermediate bosons W and Z (see Ref. 1 for example). The simplest of existing models is the fermion-scalar model in which all composites are made of only two kinds of preons whose spins are 1/2 and 0[2-7]. The problem with these models is that they do not satisfy the naturality condition[8] which argues against the presence of the scalar preons in the lagrangian. This difficulty can be overcomed by the double confinement mechanism[8] in which the scalar preons are treated as composites of two even more fundamental fermions, described by another gauge group with the much higher confinement scale.

The standard Salam-Weinberg theory of the weak interactions has been very successful in describing the physics in the low-energy region. An alternative to this model is the idea that all basic low-energy interactions in nature are mediated by

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massless elementary gauge bosons of unbroken gauge symmetries. In this approach the W and Z are not gauge bosons but they have some composite structure and the weak interactions of quarks and leptons are considered as residual forces obeying a global SU (2) symmetry. The W and Z play the same role as $\varrho^{\pm}$ and ϱ^{0} in hadron physics. Quarks and leptons must be SU (2) singlets. This condition is satisfied for the right-handed fermions, which are structureless in this picture, and the left-handed fermions are made of the fermion and scalar dublets forming singlet states[5].

In addition to the successful reproduction of the weak interactions phenomenology, in a recent paper of Maalampi and Pulido[10] an attempt has been made to unify binding (hypercolour) force of preons with the colour and electromagnetic forces. They have applied the supersymmetric SU (5) unified theory.

In order to obtain the preon confinement and the hypercolour coupling constant stronger than colour coupling constant they have introduced four additional pairs od decuplets. The total number of preons is quite large. The unification scale they have obtained is rather low ($\sim 10^{11}$ GeV), but the proton decay is suppressed by an additional factor coming from compositeness.

The use of SU $(4)_{HC}$ as a hypercolour force straightforwardly satisfies the condition that the hypercolour force is stronger than the colour force at lower energies. The unification of fundamental forces in these models usually gives the unification scale larger than the Planck mass scale[11,12].

In the unification scheme proposed by Marinescu and Schmidt[13] the technicolour extension of the standard model has been used within quite complicated SU(8) unification. The unification point was found far in the postplanckian energy region.

In this note we construct a model with SU $(3)_{HC}$ as the hypercolour symmetry and we unify the fundamental interactions within SU (6) symmetry.

We avoid the use of the additional preonic states. The physical states of the quarks and leptons are now SU $(3)_{HC}$ singlets, the right-handed fermions being structureless and the left-handed quarks and leptons being composed of SU $(3)_{HC}$ preon triplets. Similarly as in the SU $(2)_{HC}$ model the hypercolour interaction becomes strong at the scale around $\Lambda_{HC} \sim 300$ GeV or few hundred GeV, since no evidence of compositeness has been seen.

The model we propose contains two fermionic preons and two scalar ones for each generation. The weak bosons are composed of two scalar preons and the left-handed quarks and leptons are made of one scalar and one fermionic preons.

The preonic assignments of the spin, hypercolour, colour and electric charges are respectively:

$$x\,(1/2, 3, 3, 0) \qquad y\,(1/2, 3, 1, -2/3)$$
$$\alpha\,(0, \bar{3}, 1, -1/3) \quad \beta\,(0, \bar{3}, 1, 2/3). \tag{1}$$

The composition of the first generation of left-handed quarks and leptons is:

$$u_L \sim \beta x \quad d_L \sim \alpha x$$
$$e_L^- \sim \alpha y \quad \nu_L^- \sim \beta y. \tag{2}$$

The weak bosons are composed of two scalar preons:

$$W_\mu^+ \sim \beta^* D_\mu \alpha$$

$$W_\mu^- \sim \alpha^* D_\mu \beta$$

$$Z_\mu^0 \sim (\beta^* D_\mu \beta - \alpha^* D_\mu \alpha) \tag{3}$$

where D_μ is $SU(3)_{HC} \times U(1)_Q$ covariant derivative.

The right-handed quarks and leptons do not couple to the weak bosons and they can not be constructed from the same preons as left-handed ones and therefore they are either pointlike or built from different preons with the corresponding bosons much heavier than W and Z.

The second and third generations of quarks and leptons are made of different preonic flavours with the same hypercolour, colour and electric charge assigments.

The masses of the fermions are provided by the Yukawa coupling terms like:

$$\lambda_1 \Lambda_{HC} \bar{u}_R u_L \text{ and } \lambda_2 \Lambda_{HC} \bar{d}_R d_L \tag{4}$$

where for the u and d quarks $m_u \simeq \lambda_1 \Lambda_{HC}$ and $m_d \simeq \lambda_2 \Lambda_{HC}$. In this scheme we assume, as usual, that the chiral symmetry present at the preon level can persist through the presumed confinement[6]. The chiral symmetry is broken by the Yukawa coupling of preons and as in the standard model Yukawa coupling constants are small (e. g. for the electron it is 10^{-6}).

The minimal unifying group containing $SU(3)_{HC} \times SU(3)_C \times U(1)_Q$ is $SU(6)$. Similarly to the situation in the ordinary *Grand unifying theories*[14,15] we will consider only the totally antisymmetric representation of SU (6). The possible representations and their decomposition under the $SU(3)_{HC} \times SU(3)_C \times U(1)_Q$ are the following:

$$\square = (\square, \cdot) + (\cdot, \square)$$
$$6 \quad (3, 1, Q_1) + (1, 3, Q_2)$$

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} = (\begin{smallmatrix}\square\\\square\end{smallmatrix}, \cdot) + (\square, \square) + (\cdot, \begin{smallmatrix}\square\\\square\end{smallmatrix}) \tag{5}$$
$$15 \quad (\bar{3}, 1, 2Q_1) + (3, 3, Q_1 + Q_2) + (1, \bar{3}, 2Q_2)$$

$$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} = (\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}, \cdot) + (\begin{smallmatrix}\square\\\square\end{smallmatrix}, \square) + (\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}) + (\cdot, \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix})$$
$$20 \quad (1, 1, 3Q_1) + (\bar{3}, 3, 2Q_1 + Q_2) + (3, \bar{3}, Q_1 + 2Q_2) + (1, 1, 3Q_2).$$

Since Q should be a generator of SU (6) it must satisfy $\mathrm{Tr}\, Q = 0$ which implies:

$$Q_1 = -Q_2. \tag{6}$$

With this request we have no possibility of assigning the almost standard choice of electric charges $Q_x = 1/6$, $Q_y = -1/2$, $Q_\alpha = 1/2$, $Q_\beta = -1/2$ and we are forced to take the assignments as given in Eq. (1). (The same choice of electric charges has been made in the Greenberg et al. model[2].)

For the accomodation of the first generation we explore the representation 15, 15*, 6*:

$$\begin{aligned} &15: \quad \beta\,(0, \overline{3}, 1, 2/3) + x\,(1/2, 3, 3, 0) + u_R^c\,(1/2, 1, \overline{3}, -2/3) \\ &15^*: \; y\,(1/2, 3, 1, -2/3) \\ &6^*: \quad \alpha\,(0, \overline{3}, 1, -1/3) + d_R^c\,(1/2, 1, \overline{3}, 1/2). \end{aligned} \tag{7}$$

The rest of the representation 15 can be treated as Higgs colour triplet states. The second and third generations are constructed from $x_{1,2}$, $y_{1,2}$, $\alpha_{1,2}$ and $\beta_{1,2}$ preons. The SU $(3)_{HC}$ singlet components of 15 and 6* have quantum numbers of right-handed quarks. The lepton states can be accomodated into the representation 20 of the same group.

The hypercolour force should be stronger than colour force at the hypercolour scale what implies:

$$a_{HC}\,(\Lambda_{HC}) > a_C\,(\Lambda_{HC}). \tag{8}$$

It is obvious that this condition is fulfilled for the hypercolour group larger than SU (3), e. g. SU (4)[10,11]. We investigate whether SU (3) can be a hypercolour symmetry. In order to study this request we employ the one-loop approximation for the running coupling constants:

$$\frac{1}{a_{HC}\,(\mu)} = \frac{1}{a_u\,(M_x)} - \frac{b_{HC}}{6\pi} \ln \frac{M_y}{\mu} \tag{9.a}$$

$$\frac{1}{a_C\,(\mu)} = \frac{1}{a_u\,(M_x)} - \frac{b_C}{6\pi} \ln \frac{M_x}{\mu}. \tag{9.b}$$

$$\frac{3}{4}\,\frac{1}{a_Q\,(\mu)} = \frac{1}{a_u\,(M_x)} - \frac{b_Q}{6\pi} \ln \frac{M_x}{\mu}. \tag{9.c}$$

where 3/4 comes from the normalization of the electric charge and $a_u\,(M_x)$ is the coupling constant at the unification energy M_x, and the coefficients b are calculated using the well-known result for the SU (N) groups:

$$b_N = 11\,N - 4\,T\,(R_f) - T\,(R_s) \tag{10}$$

where the subscripts f and s refer to fermion and scalar preons.

The U $(1)_Q$ electromagnetic coefficient b_Q is given by:

$$b_Q = -4\,\mathrm{Tr}\,Q_f^2 - \mathrm{Tr}\,Q_s^2 \tag{11}$$

and Q_f and Q_s denote the electric charges of fermion and scalar preons, respectively. The factors $G(R)$ are determined from:

$$T(R)\,\delta_{\alpha\beta} = \mathrm{Tr}\,(T_a\,T_b) \tag{12}$$

T_a being the generators of the representation R. If R is a fundamental representation of the group:

$$2\,T(R) = n \tag{13}$$

where n is the number of the fundamental N-plets.

The condition (8) implies $b_{HC} > b_C$ or:

$$4\,T(R_f^{HC}) - T(R_s^{HC}) < 4\,T(R_f^C) - T(R_s^C). \tag{14}$$

Using the formulas (10) and (11) we find $b_{HC} = 2$, $b_C = 1$ and $b_Q = -53/3$. The condition (8) is obviously satisfied.

It is very interesting that no more than three generations can meet the condition (8).

In order to determine the unification scale M_x we fix $a_Q = 0.00784$ and $a_C = 0.101$ at the energy scale of 100 GeV[10]. We find:

$$M_x = 3.44 \times 10^{14}\ \mathrm{GeV} \qquad a_u = 0.0689 \tag{15}$$

$$a_{HC}(100) = 0.188 \qquad a_{HC}(500) = 0.171.$$

The dependence of the coupling constants on the energy scale is shown in Fig. 1.

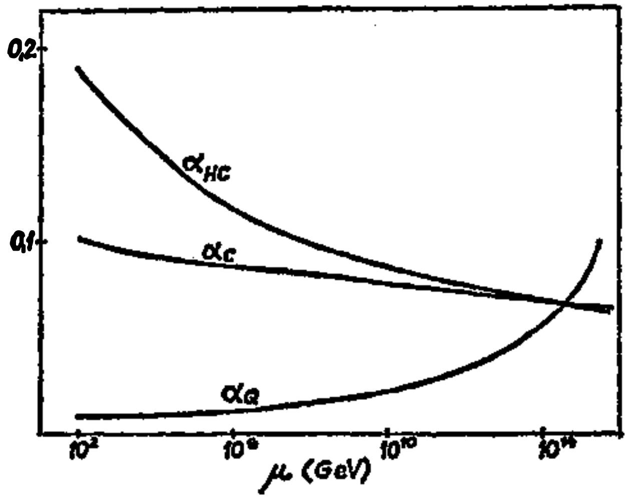

Fig. 1. The dependence of the coupling constants on energy scale.

The unification scale value makes the main proton decay mode (the same one as in the ordinary GUT[14,16]) suppressed by $1/\Lambda_{HC}$ coming from compositeness.

We have examined the possible variations of this model: e. g. we have constructed the model where scalar preon belongs to the representation 3 of the $SU(3)_C$ and also the model where 3 and $\bar{3}$ hypercolour states are interchanged and in no case the condition (8) can be met.

Also the same condition can not be satisfied with the following, otherwise possible model:

$$6: \quad a\,(1/2, 3, 1, 1/3) + d_R\,(1/2, 1, 3, -1/3)$$

$$6^*: \quad y\,(0, \bar{3}, 1, -1/3)$$

$$15^*: \beta\,(1/2, 3, 1, -2/3) + u_R\,(1/2, 1, 3, 2/3)$$

$$20: \quad e_R^+\,(1/2, 1, 1, 1) + x\,(0, 3, \bar{3}, 1/3)$$

where somewhat different electric charges of preons are assigned. The decomposition of the left-handed quarks and leptons is given by:

$$u_L \sim a\,x,\ a_L \sim \beta\,x,\ \nu_e \sim ay \text{ and } e^- \sim \beta y.$$

In conclusion we point out that the small number of preons —12 with hypercolour realized as $SU(3)_{HC}$ symmetry unified within SU (6) symmetry seems more appealing than $SU(2)_{HC}$ symmetry with $(\overline{10} + 10) \times 4$ and $(10 + \bar{5}) \times 3$ supermultiplets necessary within SU (5) supersymmetric unification scheme[10].

The unique features of the proposed model: the acceptable unification scale and the limit on the number of generations which can be adjusted into the unification scheme make this model quite attractive.

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SU (6) UJEDINJENJE FUNDAMENTALNIH INTERAKCIJA U FERMION-SKALAR MODELU

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Fermionsko-skalarni model sa $SU(3)_{HC} \times SU(3)_C \times U(1)_Q$ kao nenarušenom gradijentnom simetrijom je konstruisan. Unifikacija hiperkolora, kolora i elektromagnetne interakcije realizirana je na energijama 10^{15} GeV. Jedino se tri generacije kvarkova i leptona mogu smjestiti u ovakvu shemu.