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# THEORETICAL ANALYSIS OF THE GATE CAPACITANCE OF MOS STRUCTURES OF Si HAVING *p*-CHANNEL INVERSION LAYERS UNDER STRONG MAGNETIC QUANTIZATION

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An attempt is made to investigate theoretically the gate capacitance of MOS structures of Si having *p*-channel inversion layers in the presence of a quantizing magnetic field under weak electric field limit by formulating the appropriate modified 2*D* energy spectra of heavy, light and split-off holes, respectively. It is found that the gate capacitance exhibits spiky oscillations with changing magnetic field. This is in qualitative agreement with the experimental observations reported elsewhere for *n*-channel inversion layers on Si. It is further observed that the sharpness of the spikes increase with increasing magnetic field whereas the depths are found to decrease with increasing thickness of the insulating layer.

## 1. Introduction

In recent years there has been considerable interest in studying the semiconductor inversion layers which are formed at the surface of semiconductors in MOSFET devices under the influence of a sufficiently strong electric field applied perpendicular to the surface by means of a large gate bias[1]. In such layers, the carriers form a two-dimensional gas and are free to move parallel to the surface while their motion is quantized in a direction perpendicular to it leading to the formation of electric sub-bands. This quantized motion generates many special features of MOSFET devices. One such important feature in the capacitance of MOS structures[1-5] and it has been experimentally observed that its dependence on the surface electric field becomes oscillatory under magnetic quantization[2]. The fact that the gate capacitance can easily be controlled by varying the gate voltage is of much importance from the standpoint of technical considerations. However, all the studies reported so far on the gate capacitance were concerned with $n$-channel inversion layers on semiconductors having various types of bands structures[1-5]. In fact, a model expression of the gate capacitance in $p$-channel inversion layers on Si has yet to be theoretically worked out for the more difficult case owing to the presence of a quantizing magnetic field.

In what follows, we shall first derive an expression of the total hole concentration per unit area in $p$-channel inversion layers on Si under magnetic quantization by considering the appropriate modified energy spectra of heavy, light and split-off holes, respectively. We shall then formulate the gate capacitance with the proper use of the hole statistics. Besides, we shall investigate theoretically the effect of a quantizing magnetic field on the same capacitance under weak electric field limit since the condition for such limits is usually satisfied under the range of surface fields normally used in inversion layers.

## 2. Theoretical background

In MOS-structures, the gate capacitance is formed by the series combination of the fixed insulator capacitance $C_{ins}$ due to insulating layer and the semiconductor surface capacitance $C_{sc}$ due to space charge layer which can be controlled by varying the gate voltage. Thus the gate capacitance per unit area is given[1] by

$$C_g^{-1} = C_{ins}^{-1} + C_{sc}^{-1} \tag{1}$$

where $C_{ins} = \dfrac{\varepsilon_{ins}}{d_{ins}}$, $\varepsilon_{ins}$ and $d_{ins}$ are, respectively, the permittivity and the thickness of the insulating layer,

$$C_{sc} = e \frac{\mathrm{d}p}{\mathrm{d}V_0} = |e| \frac{\mathrm{d}p}{\mathrm{d}V_g} \left[1 - |e| \frac{d_{ins}}{\varepsilon_{ins}} \cdot \frac{\mathrm{d}p}{\mathrm{d}V_g}\right]^{-1} \tag{2}$$

$$V_0 = [V_g - (ep\, d_{ins})\, \varepsilon_{ins}^{-1}] \tag{3}$$

where $p$ is total surface concentration of the carriers, $V_0$ is the surface potential and $V_g$ is the gate voltage. It appears, therefore, that the derivation of the magneto-gate capacitance of MOS structures of Si having $p$-channel inversion layers requires an expression of the total surface concentration of the carriers per unit area under magnetic quantization as a function of gate voltage which in turn is determined by the corresponding $2D$ dispersion relation of the carriers.

Incidentally, the energy spectra of the heavy, light and split-off holes in bulk specimens of $p$-type Si in the absence of any quantization can, respectively, be expressed[6] as

$$E_1 = (A - B)\, k^2 \tag{4}$$

and

$$E_{2,3} = \frac{Bk^2}{2} + Ak^2 - \frac{\Delta}{2} \pm \left[B^2k^4 + \left(\frac{Bk^2}{2} + \frac{\Delta}{2}\right)^2\right]^{1/2} \tag{5}$$

$A = -4.28\,(\hbar^2/2m_0)$, $B = -0.75\,(\hbar^2/m_0)$ and they represent some combinations of light and heavy hole masses[6], $\Delta$ is the magnitude of the spin-orbit splitting at $k = 0$ and the indices 1, 2, 3 designate heavy hole, light-hole and split-off bands, correspondingly. Thus, using Eqs. (4) and (5) and applying the generalized quantization rule as given elsewhere[7], the modified dispersion relations of the heavy, light and split-off holes in $p$-channel inversion layers on Si can, respectively, be written, under the weak electric field limit, as

$$G\varepsilon_1 = \frac{\hbar^2 k_s^2}{2m_0} + [\hbar\, G\, e\, F_s\, (2m_0)^{-1/2}]^{2/3} \cdot S(i_1) \tag{6}$$

$$a\varepsilon_2 + \beta + [P\varepsilon_2^2 + Q\varepsilon_2 + R]^{1/2} = \frac{\hbar^2 k_s^2}{2m_0} + [\hbar e F_s/\sqrt{2m_0}]^{2/3} \times$$

$$\times \left[a + \frac{Q + 2P\varepsilon_2}{\sqrt{P\varepsilon_2^2 + Q\varepsilon_2 + R}}\right]^{2/3} \cdot [S(i_2)] \tag{7}$$

and

$$a\varepsilon_3 + \beta - \sqrt{P\varepsilon_3^2 + Q\varepsilon_3 + R} = \frac{\hbar^2 k_s^2}{2m_0} + [\hbar e F_s/\sqrt{2m_0}]^{2/3} \times$$

$$\times \left[a - \frac{Q + 2P\varepsilon_3}{\sqrt{P\varepsilon_3^2 + Q\varepsilon_3 + R}}\right]^{2/3} \cdot [S(i_3)] \tag{8}$$

in which $G = \hbar^2 (A - B)^{-1} (2m_0)^{-1}$, $\hbar = h/2\pi$, $h$ is Planck's constant, $m_0$ is the free electron mass, $\varepsilon_1$ is the energy of the heavy holes as measured from the edge of the valence band at the surface, $k_s^2 = k_x^2 + k_y^2$, $i_1$ $(= 0, 1, 2, \ldots)$ is the electric sub-band index for heavy holes, $F_s$ is the surface electric field, $S(i_1)$ are the zeros of the Airy function $[A_i(-i_1) = 0]$, $a = g'(B + 2A)$, $g' = \zeta/\Theta$, $\zeta = \hbar^2/2m_0$, $\Theta = \left[A^2 + AB - \frac{B^2}{4}\right]$, $\varepsilon_2$ is the energy of the holes light as measured from the edge of the valence bands at the surface, $\beta = \Delta (A + B) g'$, $P = 2 (\zeta B/\Theta)^2$, $Q = (g')^2 \cdot [2AB\Delta + 3\Delta B^2]$, $R = [g']^2 [\Delta (A + B)]^2$, $i_2$ is the electric sub-band index for the light holes, $\varepsilon_3$ is the energy of the split-off holes as measured from the edge of the valence band at the surface and $i_3$ is the electric-sub band index of the split-off holes. It may be noted in this context that under the substitutions, $\Delta = 0$, $m_0 = m^*$, $G = 1$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = E$, $i_1 = i_2 = i_3 = i$ and $\frac{B}{2} + A \pm \frac{B}{2}\sqrt{5} = \frac{\hbar^2}{2m^*}$ the equations (6) to (8) assume the well-known form[1]

$$E = \frac{\hbar^2 k_s^2}{2m^*} + [\hbar e F_s/\sqrt{2m^*}]^{2/3} \cdot S(i). \tag{8a}$$

Incidentally, following Lifshitz and Pitaevskii[8] the modification of the Eqs. (6), (7) and (8) in the presence of a quantizing magnetic field $H$ along the direction parallel to the electric field (i. e. $z$-direction) can, respectively, be expressed as

$$G\varepsilon_{1,n_1} = \left(n_1 + \frac{1}{2}\right) \hbar\omega_0 + [\hbar G e F_s (2m_0)^{-1}]^{2/3} S(i_1) \tag{9}$$

$$\alpha\varepsilon_{2,n_2} + \beta + \sqrt{P\varepsilon_{2,n_2}^2 + Q\varepsilon_{2,n_2} + R} = \left(n_2 + \frac{1}{2}\right) \hbar\omega_0 + S(i_2) [\hbar e F_s/\sqrt{2m_0}]^{2/3} \cdot \left[\alpha + \frac{Q + 2P\varepsilon_{2,n_2}}{\{P\varepsilon_{2,n_2}^2 + Q\varepsilon_{2,n_2} + R\}^{1/2}}\right]^{2/3} \tag{10}$$

and

$$\alpha\varepsilon_{3,n_3} + \beta - \sqrt{P\varepsilon_{3,n_3}^2 + Q\varepsilon_{3,n_3} + R} = \left(n_3 + \frac{1}{2}\right) \hbar\omega_0 + S(i_3) [\hbar e F_s/\sqrt{2m_0}]^{2/3} \cdot \left[\alpha - \frac{Q + 2P\varepsilon_{3,n_3}}{\{P\varepsilon_{3,n_3}^2 + Q\varepsilon_{3,n_3} + R\}^{1/2}}\right]^{2/3} \tag{11}$$

where $\omega_0 = \frac{eH}{m_0}$ and $\varepsilon_{1,n1}$, $\varepsilon_{2,n2}$ and $\varepsilon_{3,n3}$ are the energies of the heavy, light and split-off holes under magnetic quantization as measured from the edge of the valence bands at the surface, respectively. Besides, $n_1$, $n_2$ and $n_3$ are the Landau numbers of the heavy, light and the split-off holes, respectively. Using Eqs. (9) to (11), the total density-of-states function can be expressed as

$$\varrho(\varepsilon) = \frac{eH}{\pi\hbar}\left[\sum_{i_1=0}^{i_{1max}} \sum_{n_1=0}^{n_{1max}} \delta'(\varepsilon - \varepsilon_{1,n1}) + \sum_{i_2=0}^{i_{2max}} \sum_{n_2=0}^{n_{2max}} \delta'(\varepsilon - \varepsilon_{2,n2}) + \sum_{i_3=0}^{i_{3max}} \sum_{n_3=0}^{n_{3max}} \delta'(\varepsilon - \varepsilon_{3,n3})\right] \tag{12}$$

where $\delta'$ is the Dirac's delta function. Combining Eq. (12) with the Fermi-Dirac occupation probability factor the total hole concentration per unit area can be written as

$$p = \frac{eH}{\pi\hbar}\left[\sum_{i_1=0}^{i_{1max}}\sum_{n_1=0}^{n_{1max}} F_{-1}(\eta_1) + \sum_{i_2=0}^{i_{2max}}\sum_{n_2=0}^{n_{2max}} F_{-1}(\eta_2) + \sum_{i_3=0}^{i_{3max}}\sum_{n_3=0}^{n_{3max}} F_{-1}(\eta_3)\right] \tag{13}$$

where $F_j(n_l)$ is the Fermi-Dirac integral which can be written as[9]

$$F_j(\eta_l) = \frac{1}{\sqrt{j+1}}\int_0^{\infty} \frac{\varrho_0^j \, d\varrho_0}{1+\exp(\varrho_0 - \eta_l)}$$

in which $j$ is the set of real numbers, $\varrho_0$ is the dummy variable of integration, $\eta_l = (K_B T)^{-1}\,[E_F' - \varepsilon_{l,nl}]$, $l = 1, 2$ and 3, $K_B$ is the Boltzmann constant, $T$ is the temperature and $E_F'$ is the Fermi energy in the presence of magnetic quantization as measured from edge of the valence band at the surface in the absence of any quantization and is given by

$$E_F' = eV_g - pe^2\frac{d_{ins}}{\varepsilon_{ins}} - E_f \tag{14}$$

in which $E_f$ is the energy separation between the Fermi level and the edge of the valence band in the bulk of the $n$-type substrate material in the presence of magnetic quantization. Thus, combining Eqs. (1), (2), (3), (13) and (14) the gate capacitance of MOS structures of Si having $p$-channel inversion layers under weak electric field limit can be expressed, in the presence of a quantizing magnetic field as

$$C_{gw}(H) = e\Omega\left[\sum_{i_1=0}^{i_{1max}}\sum_{n_1=0}^{n_{1max}} F_{-2}(\eta_1) + \sum_{i_2=0}^{i_{2max}}\sum_{n_2=0}^{n_{2max}} F_{-2}(\eta_2) + \right.$$
$$\left. + \sum_{i_3=0}^{i_{3max}}\sum_{n_3=0}^{n_{3max}} F_{-2}(\eta_3)\right]\left[1 + \sum_{i_1=0}^{i_{1max}}\sum_{n_1=0}^{n_{1max}} f(1) + \right.$$
$$\left. + \sum_{i_2=0}^{i_{2max}}\sum_{n_2=0}^{n_{2max}} f(2) + \sum_{i_3=0}^{i_{3max}}\sum_{n_3=0}^{n_{3max}} f(3)\right]^{-1} \tag{15}$$

where

$$\Omega = e^2 H(\pi\hbar K_B T)^{-1}, \; f(l) = \omega_l F_{-2}(\eta_l),$$

$$\omega_l = e^2 d_{ins}\varepsilon_{ins} + C_l, \; C_1 = \frac{2}{3} G^{-1}\left[\hbar^2 eG/\varepsilon_{sc}\sqrt{2m_0}^{\,2/3}\right]\cdot S(i_1)\,p^{-1/3},$$

$\varepsilon_{sc}$ is the semiconductor permittivity,

$$C_2 = \frac{2}{3} p^{-2/3} \psi_+ (2) [a + \left\{P\varepsilon_{2,n_2} + \frac{Q}{2}\right\}\{P\varepsilon^2_{2,n_2} +$$

$$+ Q\varepsilon_{2,n_2} + R\}^{-1/2} - \frac{2}{3} S(i_2) \{\hbar e F_s / \sqrt{2m_0}\}^{2/3} \psi_+ (-1) \cdot$$

$$\cdot \{2P\{P\varepsilon^2_{2,n_2} + Q\varepsilon_{2,n_2} + R\}^{-1/2} - P(Q + 2P\varepsilon_{2,n_2})^2 \cdot$$

$$\cdot (P\varepsilon^2_{2,n_2} + Q\varepsilon_{2,n_2} + R)^{-3/2}\}],$$

$$\psi_+ (t) = [a + (Q + 2P\varepsilon_{2,n_2}) \{P\varepsilon^2_{2,n_2} + Q\varepsilon_{2,n_2} + R\}^{-1/2}]^{t/3},$$

$t$ is the set of real numbers,

$$\psi_- (t) = [a - Q + 2P\varepsilon_{3,n_3}) \{P\varepsilon^2_{3,n_3} + Q\varepsilon_{3,n_3} + R\}^{-1/2}]^{t/3}$$

and

$$C_3 = \frac{2}{3} p^{-2/3} \psi_- (2) [a + \left\{P\varepsilon_{3,n_3} + \frac{Q}{2}\right\} \{P\varepsilon_{3,n_3} +$$

$$+ Q\varepsilon_{3,n_3} + R\}^{-1/2} - \frac{2}{3} S(i_3) \{\hbar e F_s / \sqrt{2m_0}\}^{2/3} \psi_- (-1)$$

$$\{2P\{P\varepsilon^2_{3,n_3} + Q\varepsilon_{3,n_3} + R\}^{-1/2} - P(Q + 2P\varepsilon_{3,n_3})^2$$

$$(P\varepsilon^2_{3,n_3} + Q\varepsilon_{3,n_3} + R)^{-3/2}\}].$$

## *3. Results and discussion*

Using Eqs. (13) and (15) and taking the parameters[1,6] $\varepsilon_{ins} = 3.8\varepsilon_0$, $\varepsilon_{sc} = 11.8\varepsilon_0$, $A = -4.28\hbar^2/2m_0$, $B = -0.75\hbar^2/m_0$, $\Delta = 0.03$ eV, and $F_s = 5.6 \times 10^5$ V/m we have determined the dependence of $c_{g\omega}(H)/c_{g\omega}(0)$ on the reciprocal magnetic field at 4.2 K corresponding to three different values of the insulating layer for $p$-channel inversion layers on Si in the weak electric field limit, as shown in Fig. 1. The period of oscillations can be expressed by $\Delta'\left(\frac{1}{H}\right) = \frac{e}{\pi \hbar p}$ as is found graphically to be 0.9 and 0.897 $T^{-1}$, as calculated from the relation for the period corresponding to a surface concentration of $5.4 \times 10^{14}$ m$^{-2}$ induced at the surface by the field considered. The depths of the spikes increase with increasing magnetic field and decrease with increasing thickness of the insulating layer. The oscillatory nature of the magnetic field dependence of the normalized gate capaci-

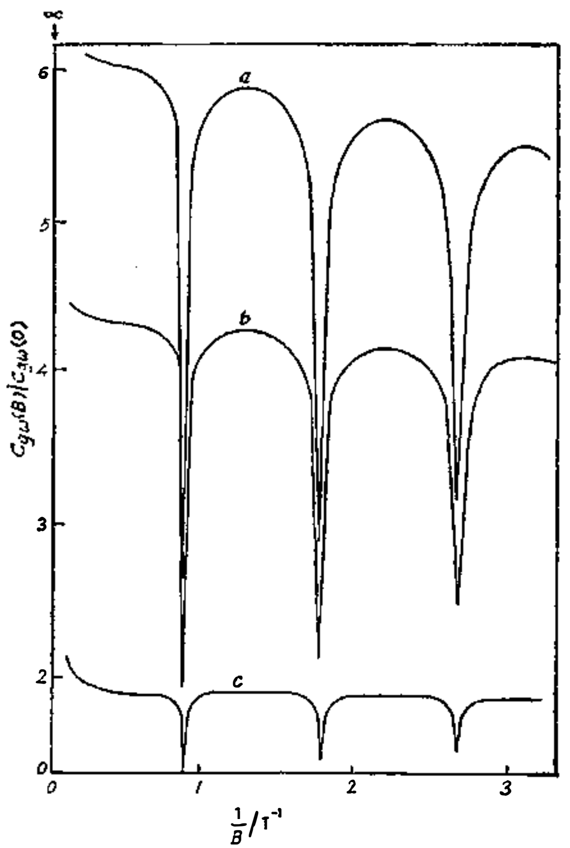


Fig. 1. The dependence of $c_{g\omega}(H)/c_{g\omega}(0)$ on the reciprocal quantizing magnetic field corresponding to three different values of the insulating layer for $p$-channel inversion layers on Si at 4.2 K. $a = 10\,\mu m$, $b = 20\,\mu m$ and $c = 30\,\mu m$.

tance is found to be somewhat similar to that observed[3] experimentally in MOS structures of Si having $n$-channel inversion layers. In the absence of available data on the gate capacitance of MOS structures of $p$-channel inversion layers on Si under magnetic quantization, we can only refer to the above agreement to indicate indirectly the validity of our theoretical calculations. This behaviour is expected since with varying magnetic field, each time a Landau level crosses the Fermi level, a change is reflected in the capacitance through the redistribution of the carriers among the Landau levels. It may be noted that the $3D$ quantization leads to discrete energy levels somewhat like atomic energy levels and consequently, produces very sharp changes in the oscillatory gate capacitances as shown in the figure. The collision broadening effects which have been neglected here would, really, reduce the sharpness of the spikes. Even then, the sharpness would be much greater than that observed for oscillatory phenomena based on the same physics in $3D$ electron gases of semiconductors. This follows from the inherent feature of the $3D$ quantization of the $2D$ electron gas dealt with here. Under such quantization, there remains no free electron state in between any two successive Landau levels unlike that found for 3*D* electron gases of semiconductors under 2*D* quantization in the *k*-space in the presence of a quantizing field. Consequently, the crossing of the Fermi level by the Landau levels under 3*D* quantization would have much greater impact on the redistribution of the electrons amongst the available states, as compared to that found for 2*D* quantization. It is basically this impact which results in the increased sharpness of the oscillatory spikes. The neglect of the collision broadening does it, therefore, not render meaningless to stress this point. It may further be noted that though the spin-splittings of the Landau levels are usually large in nonparabolic semiconductors, these have not taken into consideration in the present calculation. The spin effects would simply increase the number of spikes splitting each of the spikes into two, and reduce their amplitudes. In a calculation of the present type, the amplitudes are really arbitrary and depend strictly on the resolution of computation. Thus, for simplicity, the influences of the spin-splitting term, the broadening of Landau levels formation of band tails, the many-body interactions and the hot-electron effects have been neglected without losing much information on the oscillatory features of the magnetogate capacitance.

It may however be stated that, as far as the determination of the effective mass under the degenerate electron distribution at the surface is concerned, measurement of magneto-capacitance as compared to that of conductivity or cyclotron resonance would not be more advantageous regarding the experimental facilities required or accuracies achieved. Nevertheless, it is felt that the theoretical investigation presented here on the basis of triangular potential well approximation exhibits the basic qualitative features of magneto-gate capacitance with reasonable accuracy and would be of much significance as the interest on gate capacitance has been growing very much in recent years from the point of view of technical applications together with the exploration of other fundamental aspects of semiconductor surfaces in MOS structures. It may finally be noted that the basic purpose of the present work is not solely to demonstrate the effect of magnetic quantization on the gate capacitance, but also to formulate the modified dispersion relations of the carriers under magnetic quantization in *p*-channel inversion layers of Si in the weak electric field limit since the various transport phenomena and the derivation of the expressions for many important physical parameters of semiconductor devices are based on the corresponding energy spectra in such materials.

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# TEORIJSKA ANALIZA KAPACITETA UPRAVLJAČKE ELEKTRODE MOS STRUKTURA Si SA *p*-KANALNIM INVERZIONIM SLOJEVIMA U UVJETIMA JAKE MAGNETSKE KVANTIZACIJE

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Originalni znanstveni rad

Istraživan je kapacitet upravljačke elektrode MOS strukture Si sa *p*-kanalnim inverzionim slojevima u prisustvu kvantizirajućeg magnetskog polja i u granici slabog električnog polja. Teorijski su formulirani modificirani 2*D* energetski spektri teških, lakih i odijeljenih šupljina. Nađeno je da kapacitet vrata pokazuje oštre oscilacije sa promjenom magnetskog polja što je u suglasnosti sa eksperimentom.