YU ISSN 0015-3206
FZKAAA 19 (3) 311 (1987)

THE POLARIZATION STATE OF THE LIGHT TRANSMITTED BY A LIQUID LAYER BETWEEN TWO TRANSPARENT WALLS

JOSIP MOSER* and LJILJANA JANIĆIJEVIĆ

Fizički institut, Prirodno-matematički fakultet, P. O. Box 162, 91 000 Skopje

Received 9 December 1986

Revised manuscript received 1 June 1987

UDC 535.51

Original scientific paper

The state of polarization of the monochromatic incident light with linear polarization, transmitted by a liquid layer which is situated between two parallel walls of transparent isotropic material, has been studied theoretically. It is shown that the transmitted light is elliptically polarized. The general solution is specialised to cases of normal incidence, empty container and the liquid layer itself.

1. *Introduction*

The monochromatic light of linear polarization transmitted by an isotropic plate, changes its state of polarization, as it is shown in Refs. 1 and 2. When a liquid layer instead of a plate is used, we meet with the problem of keeping it in a container and we need an information about the influence of its walls on the state of polarization of the light transmitted through it. The aim of this article is to treat this problem theoretically.

2. *Theory*

Liquid of refractive index n_2 forms a layer of thickness d_2 between the walls of the container made of transparent isotropic matherial of refractive index n_1. The thickness of both walls is d_1.

* 41 020 Zagreb, Balokovićeva 41

Let the direction of the linear polarization of the incident light be determined by the azimuthal angle ψ_0, while its angle of incidence is a_0. Without affecting the final results, we shall take that the amplitude of the incident light is equal to one. We resolve the light vectors into two components, one perpendicular (subscript n) and one parallel (subscript p) to the plane of incidence. For the incident light these components are

$$E_{0p} = \cos\psi_0 \qquad E_{0n} = \sin\psi_0. \tag{1}$$

Propagating through the container walls and the liquid layer, the light passes four boundary surfaces. If we denote the complex amplitude of the transmitted light by E, the transmission coefficient is given by

$$t = \frac{E}{E_0}. \tag{2}$$

On the other hand, according to Wolter[3], the transmission coefficient for the four boundary surfaces case, is defined by

$$t = \frac{16 g_4 g_3 g_2 g_1}{N}. \tag{3}$$

where the g_j — quantities ($j = 1, 2, 3, 4$) are functions of the incident angle and refractive index of the corresponding medium and are differently defined for the perpendicular and parallel vector components (see Eqs. (7)). When the incident and outgoing media are equal, all values in Wolter formula indexed by 4 and 0 are identical, as well as those having indexes 3 and 1, since they reffer to the container walls media. Thus

$$t = \frac{16 g_0 g_1^2 g_2}{N} \tag{4}$$

where

$$N = (g_0 + g_1)\{(g_1 + g_2)[(g_2 + g_1)(g_1 + g_0)\,e^{i\varphi_1} + (g_2 - g_1)(g_1 - g_0)\,e^{-i\varphi_1}]\,e^{i\varphi_2} +$$
$$+ (g_1 - g_2)[(g_2 - g_1)(g_1 + g_0)\,e^{i\varphi_1} + (g_2 + g_1)(g_1 - g_0)\,e^{-i\varphi_1}]\,e^{i\varphi_2}\}\,e^{i\varphi_1} +$$
$$+ (g_0 - g_1)\{(g_1 - g_2)[(g_2 + g_1)(g_1 + g_0)\,e^{i\varphi_1} + (g_2 - g_1)(g_1 - g_0)\,e^{-i\varphi_1}]\,e^{i\varphi_2} +$$
$$+ (g_1 + g_2)[(g_2 - g_1)(g_1 + g_0)\,e^{i\varphi_1} + (g_2 + g_1)(g_1 - g_0)\,e^{-i\varphi_1}]\,e^{-i\varphi_2}\}\,e^{-i\varphi_1}. \tag{5}$$

Denoting by a_l ($l = 1, 2$) the propagation angles of the light through the corresponding media, we use the following notation for the angles φ_l in (5)

$$\varphi_l = \frac{2\pi}{\lambda} n_l d_l \cos a_l. \tag{6}$$

The g_l — quantities in expressions (3), (4) and (5) have diferent values for the perpendicular and parallel vector components

$$g_{lp} = \frac{\cos a_l}{n_l} \quad \text{and} \quad g_{ln} = n_l \cos a_l. \tag{7}$$

By rearangement of the terms in (5) we get

$$\begin{aligned} N - 16 g_2 g_1^2 g_0 \cos 2\varphi_1 \cos \varphi_2 - 8 g_0 g_1 (g_2^2 + g_1^2) \sin 2\varphi_1 \sin \varphi_2 + \\ + \mathrm{i} \, [8 g_2 g_1 (g_1^2 + g_0^2) \sin 2\varphi_1 \cos \varphi_2 + 4 (g_2^2 + g_1^2)(g_1^2 + g_0^2) \cos 2\varphi_1 \sin \varphi_2 + \\ + 4 (g_2^2 - g_1^2)(g_1^2 - g_0^2) \sin \varphi_2 \end{aligned} \tag{8}$$

which introduced in (4) yields

$$\begin{aligned} t^{-1} = \cos 2\varphi_1 \cos \varphi_2 - \frac{1}{2}\left(\frac{g_2}{g_1} + \frac{g_1}{g_2}\right) \sin 2\varphi_1 \sin \varphi_2 + \\ + \mathrm{i} \frac{1}{2}\left[\left(\frac{g_1}{g_0} + \frac{g_0}{g_1}\right) \sin 2\varphi_1 \cos \varphi_2 + \frac{1}{4}\left(\frac{g_2}{g_1} + \frac{g_1}{g_2}\right)\left(\frac{g_1}{g_0} + \frac{g_0}{g_1}\right) \sin 2\varphi_1 \sin \varphi_2 + \\ + \frac{1}{4}\left(\frac{g_2}{g_1} - \frac{g_1}{g_2}\right)\left(\frac{g_1}{g_0} - \frac{g_0}{g_1}\right) \sin \varphi_2\right]. \end{aligned} \tag{9}$$

Using the shorter notations

$$\begin{aligned} R = \frac{1}{2}\left(\frac{g_2}{g_1} + \frac{g_1}{g_2}\right) \quad R' = \frac{1}{2}\left(\frac{g_2}{g_1} - \frac{g_1}{g_2}\right) \\ S = \frac{1}{2}\left(\frac{g_1}{g_0} + \frac{g_0}{g_1}\right) \quad S' = \frac{1}{2}\left(\frac{g_1}{g_0} - \frac{g_0}{g_1}\right) \end{aligned} \tag{10}$$

we can write

$$\begin{aligned} t^{-1} = \cos 2\varphi_1 \cos \varphi_2 - R \sin 2\varphi_1 \sin \varphi_2 + \mathrm{i} \, (S \sin 2\varphi_1 \cos \varphi_2 + \\ + RS \cos 2\varphi_1 \sin \varphi_2 + R'S' \sin \varphi_2) \end{aligned} \tag{11}$$

or simply

$$t^{-1} = K + \mathrm{i} L \tag{12}$$

where

$$\begin{aligned} K = \cos 2\varphi_1 \cos \varphi_2 - R \sin 2\varphi_1 \sin \varphi_2 \\ L = S \sin 2\varphi_1 \cos \varphi_2 + RS \cos 2\varphi_1 \sin \varphi_2. \end{aligned} \tag{13}$$

Therefore, according to (2), (1) and (12), we find that

$$E_p = \frac{\cos \psi_0}{K_p + \mathrm{i} L_p} = \frac{(K_p - \mathrm{i} L_p) \cos \psi_0}{K_p^2 + L_p^2}$$
$$E_n = \frac{\sin \psi_0}{K_n + \mathrm{i} L_n} = \frac{(K_n - \mathrm{i} L_n) \sin \psi_0}{K_n^2 + L_n^2} \tag{14}$$

the components of the transmitted light vectors are complex. This indicates that the transmitted light is elliptically polarized. Let us determine the parameters of this polarization.

3. The parameters of the transmitted light polarization

The elliptical polarization of the light is characterised by two parameters. These are the azimuth ψ of the major axis of the vibrating ellipse and the ellipticity ϑ which is defined by the ratio

$$\mathrm{tg}\, \vartheta = \frac{b}{a} \tag{15}$$

with a and b being the semiaxes of the vibrating ellipse.

These two parameters are connected to the light vector components by the relations[4]

$$\mathrm{tg}\, 2\psi = \mathrm{tg}\, 2\xi \cos \delta$$
$$\sin 2\vartheta = \sin 2\xi \sin \delta \tag{16}$$

in which ξ is defined by

$$\mathrm{tg}\, \xi = \frac{A_n}{A_d} \tag{17}$$

while

$$\delta = \delta_p - \delta_n. \tag{18}$$

A_n and A_p are the real amplitudes of the light vector components, while δ_n and δ_p are the corresponding phases.

Expressing the complex light vector amplitudes as

$$E_p = A_p (\cos \delta_p + \mathrm{i} \sin \delta_p) \qquad E_n = A_n (\cos \delta_n + \mathrm{i} \sin \delta_n) \tag{19}$$

and comparing them with expressions (14) we find

$$A_p \cos \delta_p = \frac{K_p \cos \psi_0}{K_p^2 + L_p^2} \qquad A_n \cos \delta_n = \frac{K_n \sin \psi_0}{K_n^2 + L_n^2}$$
$$A_p \sin \delta_p = -\frac{L_p \cos \psi_0}{K_p^2 + L_p^2} \qquad A_n \sin \delta_n = -\frac{L_n \sin \psi_0}{K_n^2 + L_n^2}. \tag{20}$$

The introduction of (20) in the parameter expressions (16) yields

$$\operatorname{tg} 2\psi = \frac{2A_n A_p}{A_p^2 - A_n^2} (\cos \delta_p \cos \delta_n + \sin \delta_p \sin \delta_n)$$
$$\sin 2\vartheta = \frac{2A_n A_p}{A_p^2 + A_n^2} (\sin\delta_p \cos \delta_n - \cos \delta_p \sin \delta_n) \tag{21}$$

or

$$\operatorname{tg} 2\psi = \frac{2 (K_p K_n + L_p L_n) \operatorname{tg} \psi_0}{K_n^2 + L_n^2 - (K_p^2 + L_p^2) \operatorname{tg}^2 \psi_0} \tag{22}$$

$$\sin 2\vartheta = \frac{2 (K_p L_n - L_p K_n) \operatorname{tg} \psi_0}{K_n^2 + L_n^2 + (K_p^2 + L_p^2) \operatorname{tg}^2 \psi_0}. \tag{23}$$

By expressions (22) and (23) the principal parameters of the elliptical polarisation of the transmitted light are determined.

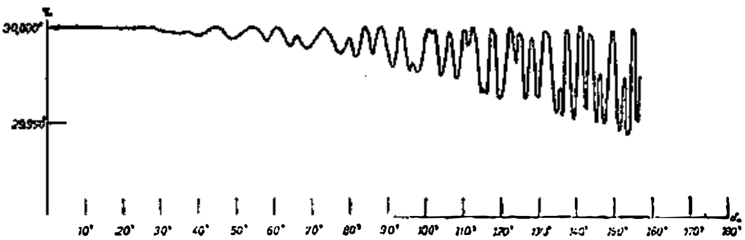

Fig. 1. The azimuth of the light transmitted by the glass container of water as function of the incident angle.

In order to get an impression about the azimuth ψ versus the incident angle α_0 dependence, we did a numerical calculation for the case of a water layer of thickness $d_2 = 10$ mm and refractive index $n_2 = 1.333$ in a glass container of refractive index $n_1 = 1.515$ whose walls are one milimeter thick i. e. $d_1 = 1$ mm. The results of this calculation are presented by the graph on Fig. 1. It is evident that the orientation of the vibrating ellipse is highly sensitive on the changes of the value of the incident angle α_0. The azimuth ψ versus the incident angle is an oscillating function with changeable amplitude and period.

4. Specialization of the theoretical results

a) The walls are of different thickness

The preceding results reffer to liquid layer situated between walls of equal thickness. When this it not the case, but $d_3 \neq d_1$, according to expressions (6) it follows that $\varphi_1 \neq \varphi_3$, but $g_1 = g_3$, since the walls are made of same isotropic material. This introduced in (4) yields an expression like (12), but now

$$K = \cos(\varphi_1 + \varphi_3)\cos\varphi_2 - R\sin(\varphi_1 + \varphi_3)\sin\varphi_2$$

$$L = S\sin(\varphi_1 + \varphi_3)\cos\varphi_2 + RS\cos(\varphi_1 + \varphi_3)\sin\varphi_2 + R'S'\cos(\varphi_1 - \varphi_3)\sin\varphi_2. \tag{24}$$

The steps which should follow the theory are the same as in the preceding section.

b) Normal incidence of the light

Of special interest is the case when the incident light propagates in the direction which is normal to the outer surface of the container. Than, all propagation angles are zero and

$$\cos a_l = 1 \qquad l = 1, 2.$$

According to (7) and (10) it follows that

$$R_p = R_n; \qquad S_p = S_n; \qquad R'_p = -R'_n; \qquad S'_p = -S'_n \tag{25}$$

and

$$K_p = K_n \qquad L_p = L_n. \tag{26}$$

Therefore the polarization parameters for the case of normal incidence are determined by

$$\operatorname{tg} 2\psi = \frac{2\operatorname{tg}\psi_0}{1 - \operatorname{tg}^2\psi_0} = \operatorname{tg} 2\psi_0; \qquad \sin 2\vartheta = 0 \tag{27}$$

or

$$\psi = \psi_0 \quad \text{and} \quad \vartheta = 0. \tag{28}$$

The interpretation of the result is that the transmitted light in the case of normal incidence is with linear polarization, having an azimuth equal to that of the incident light.

c) The light is transmitted by an empty container

Let us now treat the special case when the container is empty. Putting

$$n_2 = 1 \qquad g_2 = g_0$$

since

$$R = S \quad \text{and} \quad R' = S'$$

relations (13) turn into

$$\begin{aligned} K &= \cos 2\varphi_1 \cos \varphi_2 - S \sin 2\varphi_1 \sin \varphi_2 \\ L &= S \sin 2\varphi_1 \cos \varphi_2 + S^2 \cos 2\varphi_1 \sin \varphi_2 - S'^2 \sin \varphi_2. \end{aligned} \qquad (29)$$

These values introduced in expressions (22) and (23) determine the parameters of the transmitted light polarization. On Fig. 2 we present the results of our calculations for this case. As it is seen, the transmitted light azimuth ψ versus the incident angle α_0 approves as an oscillating function with oscillatory modulated amplitude.

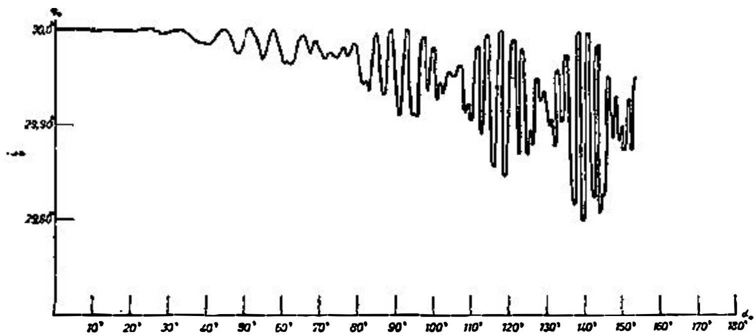

Fig. 2. The azimuth of the light transmitted by an empty container as a function of the incident angle.

d) The light is transmitted by the liquid layer itself

Finally, for the sake of control, let us specialize our results, when there are not any walls of the container, but there is a rigid liquid instead. It means that the transmission goes through a plate of refractive index n_2 and thickness d_2. Therefore we should take that $d_1 = d_3 = 0$, which gives $\varphi_1 = \varphi_3 = 0$ and $g_1 = g_3 = g_0$. From expressions (10) we also have

$$S = 1 \quad \text{and} \quad S' = 0$$

which simplifies the formulae (13) into

$$\begin{aligned} K &= \cos \varphi_2 \\ L_{p,n} &= R_{p,n} \sin \varphi_2. \end{aligned} \qquad (30)$$

From (10) it is also evident that

$$R = \frac{1}{2}\left(\frac{g_2}{g_0} + \frac{g_0}{g_2}\right)$$

and thus, with (7) in mind

$$R_p = \frac{1}{2}\left(\frac{\cos a_2}{n_2 \cos a_0} + \frac{n_2 \cos a_0}{\cos a_2}\right)$$

$$R_n = \frac{1}{2}\left(\frac{n_2 \cos a_2}{\cos a_0} + \frac{\cos a_0}{n_2 \cos a_2}\right). \tag{31}$$

All this introduced in (22) and (23) gives

$$\mathrm{tg}\, 2\psi = \frac{2\,(1 + R_p R_n\, \mathrm{tg}^2\, \varphi_2)\, \mathrm{tg}\, \psi_0}{1 + R_n^2\, \mathrm{tg}^2\, \varphi_2 - (1 + R_p^2\, \mathrm{tg}^2\, \varphi_2)\, \mathrm{tg}^2\, \psi_0} \tag{32}$$

$$\sin 2\vartheta = \frac{2\,(R_n - R_p)\, \mathrm{tg}\, \varphi_2\, \mathrm{tg}\, \psi_0}{1 + R_n^2\, \mathrm{tg}^2\, \varphi_2 + (1 + R_p^2\, \mathrm{tg}^2\, \varphi_2)\, \mathrm{tg}^2\, \psi_0}. \tag{33}$$

These are the known relations that determine the polarization state of the transmitted light by an isotropic plate[1,2]. In case of an inclined incidence it is elliptical, while at normal incidence it is linear.

References

1) K. Zander, H. Melle and J. Moser, PTB-Bericht. Opt. (1983) 14;
2) K. Zander, J. Moser and H. Melle, Optik **70** (1985) 6;
3) H. Wolter, Hdb. der Physik, Ed XXIV, Springer, p-p 472, (1956);
4) M. Born, E. Wolf, *Principles of Optics*, Perg. Press, Oxford, p-p 27, (1970).

STANJE POLARIZACIJE SVJETLOSTI NAKON PROLAZA KROZ SLOJ TEKUĆINE KOJA JE IZMEĐU DVA PROZIRNA ZIDA

JOSIP MOZER i LJILJANA JANIĆIJEVIĆ

Fizički institut, Prirodno-matematički fakultet, p. p. 162, 91 000 Skopje

UDK 535.51

Originalni znanstveni rad

Određuje se stanje polarizacije linearno polarizirane monohromatske upadne svjetlosti nakon prolaza kroz planparalelni sloj tekućine koja se nalazi između dvaju planparalelnih zidova od prozirnog izotropnog materijala. Prolazna je svjetlost eliptički polarizirana. Opće rješenje se specijalizira za slučaj prazne posude i za slučaj same tekućine.