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COLLECTIVE MOTION OF ELECTRONS IN THE COULOMB FIELD OF NUCLEUS

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One-particle solutions describing collective motion of electrons in the Coulomb field of nucleus are investigated numerically. General character of motion and conditions at which the motion is periodic (electron trajectories are closed) are estimated. The class of closed trajectories which may represent the two-electron atomic shell, and particularly the shell of the helium atom, are determined.

1. Introduction

Twenty years of successful application of classical atomic collision theory to quantitative description of atomic collision experiments[1–9] has shown that in atomic systems (atoms, molecules) nuclei as well as electrons can be with a good accuracy considered as point particles carrying a point mass and a point charge, in other words — that the atom (molecule) can be considered as a collection of point particles the behaviour of which is in the first instance governed by the Coulomb interaction law and the Newtonian dynamics. Not long ago it was suggested that small, velocity dependent periodic perturbations, having the origin in gyromagnetic (spin) properties of the electron, can explain »quantization« of electron energy in the atom[10]. In view of all the above we consider the atom with localized electrons moving along definite orbits as a physical reality. Deciphering the details of this reality appears to be at the moment one of the most important problems of atomic physics.

The first attempts in this direction were done by Bohr and Sommerfeld at the very beginnings of atomic physics[11,12]. Unfortunately they made some mistakes which influenced strongly further development of the research. At first, Bohr by the unjustified elimination of the zero angular momentum Kepler orbit arrived at the wrong model of the hydrogen atom — see Fig. 1, and having the erronous

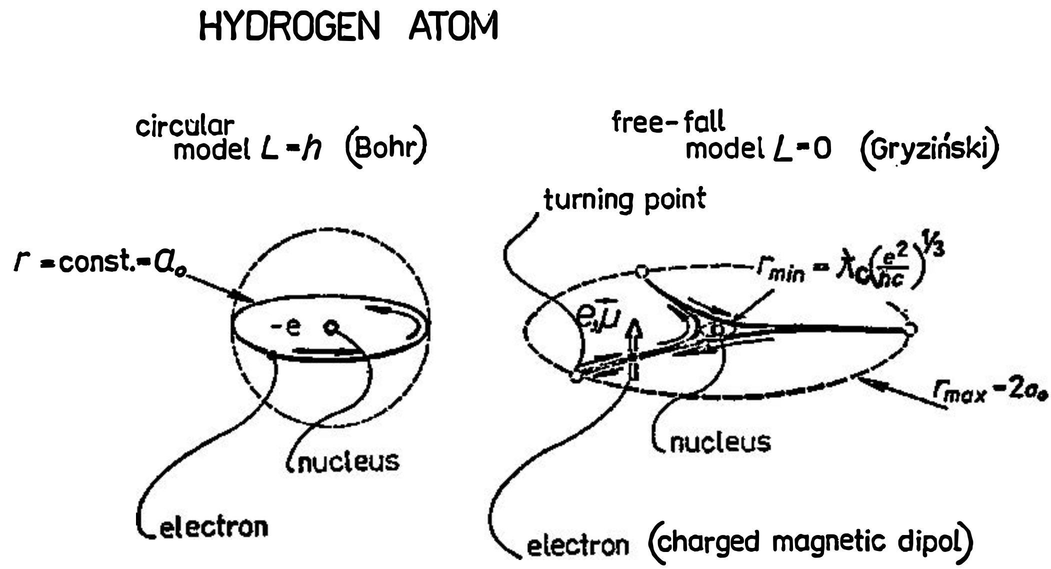

Fig. 1. Two alternative models of the hydrogen atoms. The circular model proposed by Bohr at the beginnings of atomic physics, and the free-fall atomic model proposed by the author. The latter was found to be consistent with spectroscopic observations and with atomic collision experiments.

model as a basis he was unable to make any progress in construction of many-electron atomic model. His circular model of the helium atom for instance — see Fig. 2, had, among others, evidently wrong magnetic and spectroscopic properties. On the other hand, Langmuir[13] trying to develope the model of many-electron atom had overlooked the whole class of collective orbits (those will be discussed latter) and proposed the model of the helium, as shown in Fig. 2, which could not explain the observed value of the binding energy.

Almost fifty years later the present author undertook the problem again and as result of a careful analysis of the huge experimental material in the field of atomic collisions and spectroscopy, proposed the atomic model, so called free-fall atomic model[14,15], see Figs. 1 and 2, which was deprived of the defficiences of the models mentioned above. Moreover, the free-fall atomic model was found to be consistent with a large number of atomic collision experiments[16-18], and was succesfully applied to quantitative interpretation of atomic energy level shifts and radiation line intensities[19], and enabled quantitative description of diamagnetic properties of matter[20].

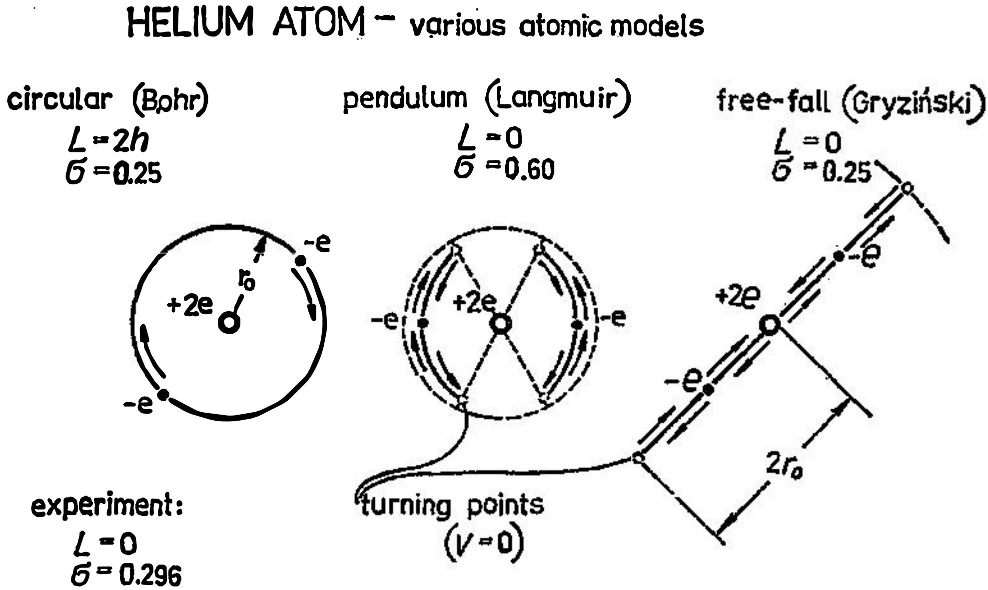

Fig. 2. Three different models of the helium atom.

It was obvious, however, from the very beginning that the free-fall atomic model, describing relatively precisely the hydrogen atom — the latter was successfully used by Grujić et al.[21] for description of scattering of low energy electrons, in the case of multi-electron atoms may be considered as a rough description of the real situation only. There were two at last reasons for that.

Firstly — because of magnetic spin properties of electron being neglected (in the immediate vicinity of the nucleus the magnetic spin interaction of electrons plays an important role), secondly — because of the evident mechanical inconsistency (exactly radial free-fall trajectories cannot form the many electron shell configuration). There were, moreover, the other facts — like the quadrupole character of small angle scattering of slow electrons from noble gases[17] — which suggested that electron orbits in many electron atoms are not so simple and the more sophisticated two or three particle motion takes place.

It is the aim of the present paper, still neglecting magnetic properties of the electron, to investigate collective motion of electrons in configuration which might represent the atomic shell.

It is worthy to note that great effort towards the construction of the advanced many electron atomic model is nowadays observed — see for instance: Dimitrijević et al.[22], who investigated the synchronous and asynchronous free-fall motion of the two electrons, Harcourt[23] who proposed to introduce some modifications into the Bohr atomic model of helium, Leopold et al.[24,25] and Conveney and Child[26], who have investigated the classical two electron systems from the point of view of general quantization rules. Recently Dimitrijević and Grujić[27,28] applaying semiclassical quantization procedure have reexamined the Langmuir's model.

Unfortunately, neither of these calculations has led to conclusive results and the problem of the model of the helium atom is still open for discussion.

2. Mathematical formulation of the problem

Approaching the discussion on the structure of atomic systems one must keep in mind two firmly established facts: firstly, that properties of atoms and molecules do not change in time, and secondly, that properties of atoms (molecules) of the same kind are identical (the last statement represents simply the definition of atom). This means that for the given set of differential equations postulated for description of the atomic system only stationary solutions may represent the atom, and that the solution which pretends to represent in the deterministic approach the particular atom must be uniquely identified.

In the actual work we will assume, having in view that on the appreciably greater part of the electron orbit the Coulomb interaction dominates and electron velocities are appreciably smaller than the velocity of light, that the interaction between atomic particles (nuclei and electrons) is purely electrostatic and the motion is nonrelativistic. The behaviour of electrons in the atom, therefore, has to be consistent with the following set of equations:

$$m_i \frac{\mathrm{d}^2 \vec{r}_i}{\mathrm{d} t^2} + \nabla_i \left(\frac{Z e^2}{r_i} - \frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{e^2}{r_{ij}} \right) = 0 \quad i = 1, 2, \cdots, n, \tag{1}$$

where n is the number of electrons in the considered atom and Ze is the charge of nucleus. The above set of equations has practically uncountable number of solutions, but only among those describing relatively simple stationary motion we should search for the real ground-state electron orbits. But even then our atomic problem is too widely formulated to be effectively solved — it is almost identical with the problem of the solar system stability, which being intensively investigated more than two hundred years still remains unsolved. Fortunately, the preliminary confrontation of theoretical estimations with some experimental facts has shown that the situation in the atom is much simpler than in the solar system, i. e. the distribution of electrons around the nucleus is highly symmetrical, the motion of all atomic electrons is synchronous and the system as a whole does not possess the angular momentum. The requirement of symmetry is the factor, which makes theoretical analysis possible (recently on the grounds of very general considerations there were derived some relations describing stable periodic solutions for collectively moving Coulomb particles — Davies et al.[29]).

The assumption that there exists in the atom the perfect symmetry in distribution of electrons and the motion is perfectly synchronous is consistent with the well proved concept of the atomic shell according to which all electrons of the shell are exactly equivalent. From the point of view of electron orbits this means that all electrons in the shell move in an identical way and that in the case of the one isolated shell the many body problem can be reduced to the one particle problem with the following simple equation of motion:

$$m\frac{d^2\vec{r}}{dt^2} = \nabla\left(\frac{(Z-\sigma(\hat{r}))\,e^2}{r}\right), \tag{2}$$

where the screening factor $\sigma(\hat{r})$ represents electrostatic interaction between electrons. Keeping in mind the fact that the angular momentum of the atomic system being in the ground-state is zero, distribution of electrons in the shell and their velocities should all the time satisfy the following relation:

$$\sum_{i=1}^{n_s}[\vec{r}_i \times \vec{v}_i] = 0, \tag{3}$$

where n_s is the number of electrons in the shell.

Previously we have discussed a very particular case of the problem i. e. when the motion is perfectly radial ($\vec{r}_i \times \vec{v}_i = 0$). In this case $\sigma(\hat{r})$ is all the time a constant quantity

$$\sigma(\hat{r}) = const = \sigma^{ff},$$

and the free-fall Kepler orbit is the solution of the problem.

In the free-fall case the basic shell parameters, i. e. the radius of the shell (identified with the extremal distance of the electrons from the nucleus) and the shell frequency, are respectively:

$$r_s^{ff} = \frac{(Z-\sigma^{ff})\cdot e^2}{W}, \tag{4}$$

$$\omega_s^{ff} = \frac{1}{(Z-\sigma^{ff})\cdot e^2}\sqrt{\frac{(2W)^3}{m}}, \tag{5}$$

where W is the binding energy of the electron in the shell. This energy can be calculated from the total energy of the shell, which is equal to the sum of successive ionization potentials of the shell:

$$W_s = W\cdot n_s = \sum_{j=1}^{n_s}(U_i)_j. \tag{6}$$

Dynamics of the shell, that is the radius of the shell as a function of time, is in the *ff*-case simply given by:

$$t = \frac{1}{\omega_s^{ff}}\left[\arccos\left(1-\frac{2r}{r_s^{ff}}\right) - \sqrt{1-\left(1-\frac{2r}{r_s^{ff}}\right)^2}\,\right]. \tag{7}$$

Approaching the analysis of the more general case of the collective motion of electrons in the field of nucleus one must specify first the form of the screening factor $\sigma(\hat{r})$. The latter, for the particular distribution of electrons in the shell, can be easily derived from the potential energy formula.

And so, in the case of k electrons situated on the surface of the sphere centered on the nucleus the potential energy can be written in the following way:

$$V_k = -k\left(\frac{Ze^2}{r}\right) + \frac{e^2}{r}\cdot\frac{1}{2}\sum_{\substack{i,j\\ i\neq j}}^{k}\frac{1}{2\sin(a_{ij}/2)}, \tag{8}$$

where a_{ij} denotes the angle between radius vectors of i-th and j-th electrons. Electrons can be situated symmetrically on the surface of the sphere in various ways depending upon the number of electrons. In the case of odd number of electrons there is the only possibility — electrons must be placed in the same plane at the corners of a regular polygon. In the case of even number of electrons ($n_e = 2k$) the simplest configuration is that formed by two identical polygons situated symmetrically with respect to the nucleus. In the both cases the system has k-fold axis of symmetry. In the present paper we will confine our analysis to these two particular configurations which may represent the incomplete atomic shell (with odd number of electrons) and the closed atomic shell (with even number of electrons).

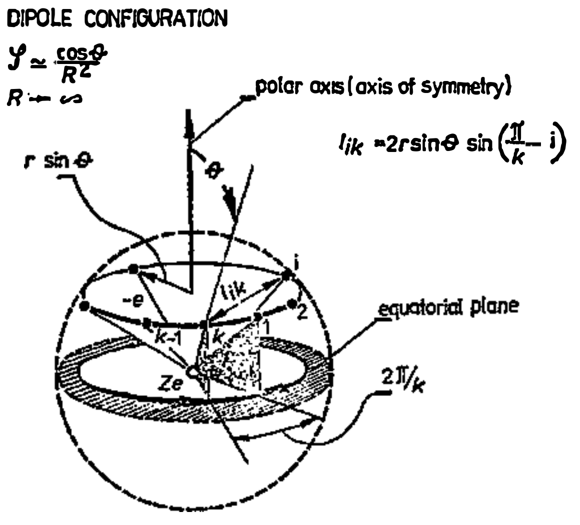

Fig. 3. The figure shows the simplest symmetric distribution of k-electrons around the nucleus: the so-called dipole configuration.

If electrons are situated on the surface of the sphere in such a particular way that they form a regular polygon, see Fig. 3, then:

$$\sin(a_{ij}/2) = \sin\theta\cdot\sin\left(\frac{\pi\cdot(i-j)}{k}\right) \tag{9}$$

and the potential energy formula assumes the form:

$$V_k(r, \theta) = -k \cdot \left(\frac{e^2}{r}\right) \cdot (Z - \sigma_k(\theta)), \tag{10}$$

where:

$$\sigma_k(\theta) = \frac{1}{4 \cdot \sin\theta} \sum_{i=1}^{k} \frac{1}{\sin\left(\pi \frac{i}{k}\right)}. \tag{11}$$

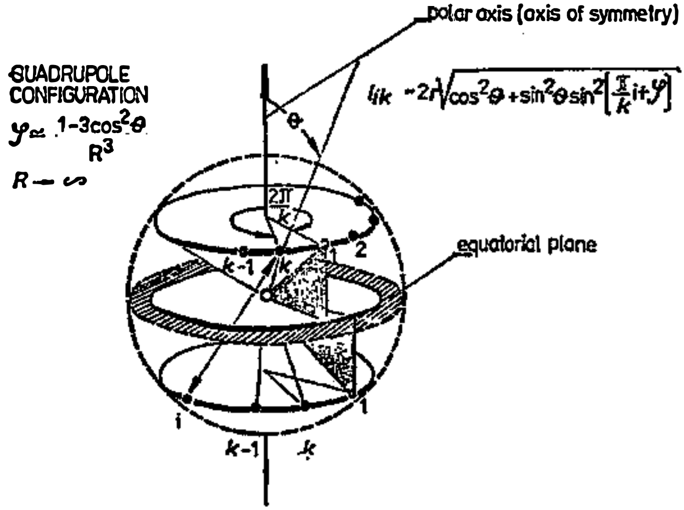

Fig. 4. The symmetrical distribution of $2k$ electrons grouped into two identical subsystems (subshells): the so called quadrupole configuration. In this case the two subshells can oscillate around the axis of symmetry of the system and orbital motion is, in general, three-dimensional.

In the case of shell electrons forming two identical polygons situated symmetrically with the respect to the nucleus, see Fig. 4, one has:

$$V_{2k} = 2 \cdot V_k + V_{kk}, \tag{12}$$

where V_k represents the potential energy of the isolated k-electron polygon (subshell) and V_{kk} represents the interaction energy between the polygons (between the subshells). The interaction energy of the two subshells shifted in the azimuth by the angle 2φ, as it can be easily deduced from Fig. 4, is given by:

$$V_{kk}(r, \theta, \varphi) = k\left(\frac{e^2}{r}\right) + \frac{1}{4} \sum_{i=1}^{k} \left(\cos^2\theta + \sin^2\theta \cdot \sin^2\left(\frac{\pi}{k} i + \varphi\right)\right)^{-1/2}. \tag{13}$$

The potential energy of the system with two subshells, therefore, is:

$$V_{2k}(r) = -2 \cdot k \left(\frac{e^2}{r}\right) \cdot [Z - \sigma_{2k}(r)], \tag{14}$$

where

$$\sigma_{2k}(\hat{r}) = \sigma_k(0) + \frac{1}{8} \sum_{i=1}^{k} \left(\cos^2 \theta + \sin^2 \theta \cdot \sin^2 \left(\pi \frac{i}{k} + \varphi\right)\right)^{-1/2}. \tag{15}$$

With specification of the form of the screening factor $\sigma(\hat{r})$ general character of the collective motion becomes determined.

The shape of the particular orbit, however, depends upon initial conditions, that is initial values of $\vec{r}$ and $\vec{v}$. Specifying initial conditions for the particular problem one must keep in mind that localization and velocity of electrons are not quite independent quantities but are related through the integral of energy, that is they must satisfy the following relation:

$$\frac{mv^2}{2} - \frac{e^2}{r}[Z - \sigma(\hat{r})] = E. \tag{16}$$

Equation of motion (2), Eqs. (11) and (15) defining the screening factor $\sigma(\hat{r})$, and the integral of motion (16) form the complete set of equations describing electron motion in the isolated atomic shell.

Unfortunately, although the integral of motion is known and equations of motion are relatively simple, numerical analysis is the only way of solving the problem.

3. *General considerations*

Prior to undertaking the numerical analysis of the particular case it is worth to discuss briefly the general aspects of the problem. At first, however, by introducing the proper units of length and time it is convenient to write the equation of motion and the energy in the dimensionless form.

It seems reasonable to have the motion in the free-fall configuration as a standard and to use the following units:

a) the distance between the nucleus and the turning point of the free-fall orbit as the unit of length

$$l_1 = \frac{(Z - \sigma^{ff}) \cdot e^2}{W}, \tag{17}$$

b) reciprocal of the orbital frequency as the unit of time

$$t_1 = \frac{1}{\omega_s^{ff}} = \frac{l_1}{\sqrt{\frac{2W}{m}}}. \tag{18}$$

Applying these units and introducing the following notation:

$$\varrho = \frac{r}{l_1}, \tag{19}$$

$$\tau = \frac{t}{t_1}, \tag{20}$$

$$u = \frac{v}{(l_1 / t_1)}, \tag{21}$$

equations of motion and integral of energy, written in the spherical system of coordinates with the polar axis directed along the axis of symmetry of the electron distribution, will assume the form:

$$(Z - \sigma^{ss}) \left(\frac{\mathrm{d}\vec{u}}{\mathrm{d}\tau}\right) \cdot \hat{u}_\varrho = - \frac{z(\hat{\varrho})}{\varrho^2}, \tag{22}$$

$$(\quad - \sigma^{ss}) \left(\frac{\mathrm{d}\vec{u}}{\mathrm{d}\tau}\right) \cdot \hat{u}_\theta = \frac{1}{\varrho^2} \frac{\partial}{\partial \theta} [z(\hat{\varrho})], \tag{23}$$

$$(Z - \sigma^{ss}) \left(\frac{\mathrm{d}\vec{u}}{\mathrm{d}\tau}\right) \cdot \hat{u}_\varphi = \frac{1}{\sin\theta\, \varrho^2} \frac{\partial}{\partial \varphi} [z(\hat{\varrho})], \tag{24}$$

$$u^2 - \frac{2 \cdot z(\hat{\varrho})}{\varrho} = \mathrm{sgn}(E), \tag{25}$$

where the reduced effective charge $z(\hat{\varrho})$ is defined in the following way:

$$z(\hat{\varrho}) = \frac{Z - \sigma(\hat{\varrho})}{Z - \sigma^{ss}}. \tag{26}$$

Undertaking the general discussion of the problem it is worthy to note that for the region of the bound motion of electrons (then sign $(E) = -1$) is determined by the zero velocity surface equation:

$$\frac{2\, z(\hat{\varrho})}{\varrho} = 1, \tag{27}$$

which one obtains from Eq. (25) by putting $u = 0$. For the two particular cases, i. e. of the one-polygon configuration, which we will call from now on the *dipole confi-*

guration, and of the two-polygon configuration, which we will call from now on the *quadrupole configuration*, the regions of the bound motion of electrons are shown in Fig. 5.

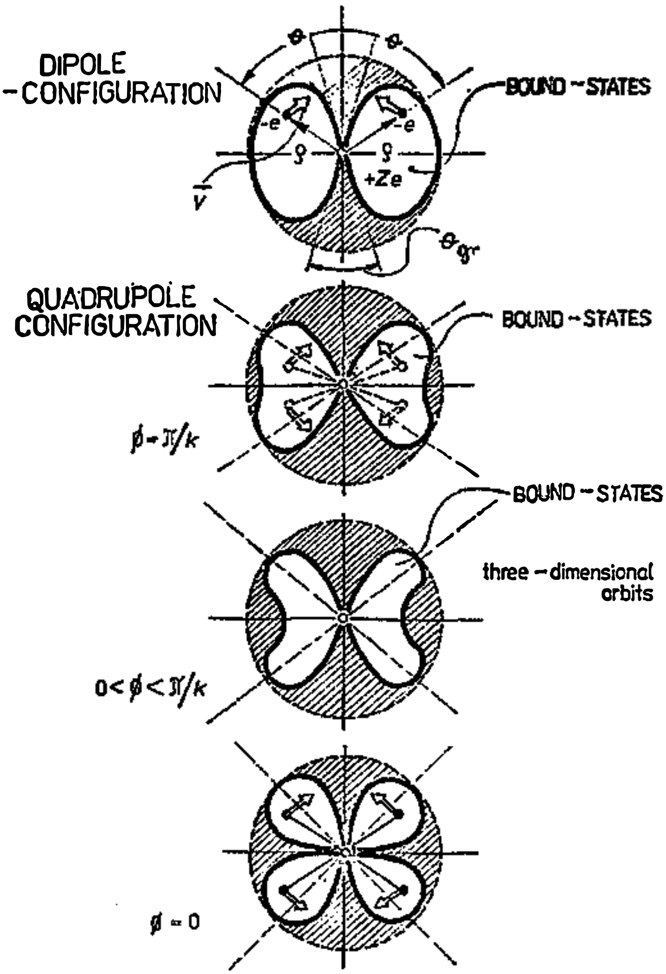

Fig. 5. The regions accessible for collectively moving electrons, in absence of rotation of the system: on the top - in the case of dipole-configuration, and below - in the case of quadrupole configuration (at three azimuthal shifts of the subshells).

Analysis of the potential energy as a function of the angle θ, see Figs. 4 and 6, shows that the potential energy in the dipole configuration has a minimum at θ equal to $\pi/2$, while the potential energy in the quadrupole configuration has the two minima located symmetrically on both sides of the equatorial plane.

It is clearly seen from Fig. 6 that for identical number of electrons the potential energy minimum for the quadrupole configuration is lower than for the dipole configuration. This means that the dipole configuration with the even number of electrons ($n_e = 2k$) is energetically unstable and that for $k > 1$ there should be a tendency to splitting the configuration into two k-electron subsystems. The potential ener-

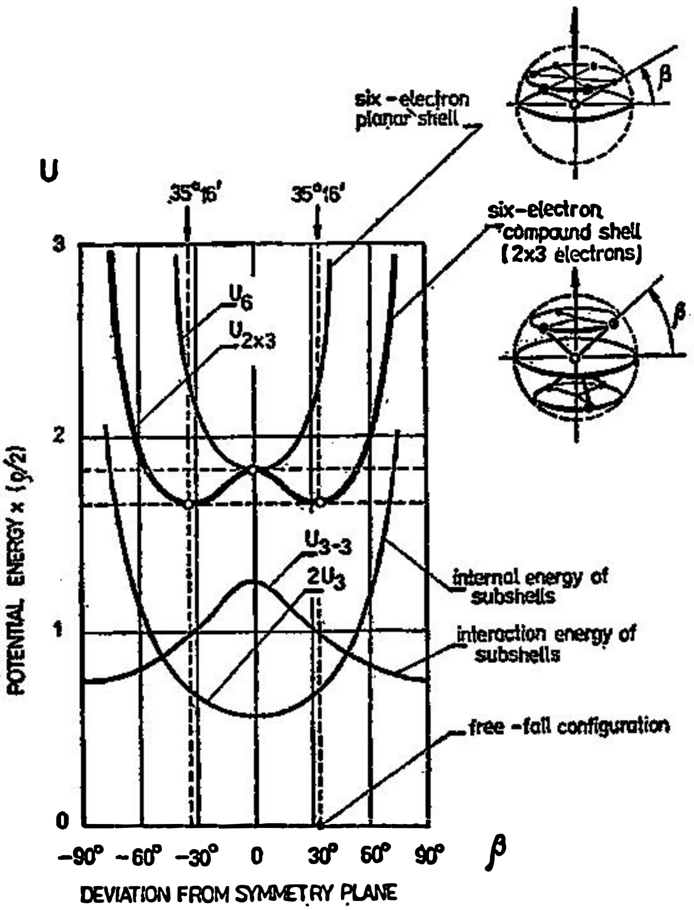

Fig. 6. Potential energy of $2k$ electrons in the case of dipole and quadrupole configurations. From the figure the conclusion can be drawn that in many electron systems a tendency to formation of subsystems (subshells) should exist.

gy of the quadrupole configuration, considered as a function of the azimuthal angle φ, shows a minimum when the subshells are twisted in the azimuth by the angle $\varphi = \pi/k$. One can, therefore, suppose that such an orientation should be typical for many-electron atomic systems. The orientations which correspond to the potential energy extrema are those which determine the free-fall configuration. They are determined by:

$$\frac{\partial \sigma(\hat{\varrho})}{\partial \theta} = 0, \tag{28}$$

and

$$\frac{\partial \sigma(\hat{\varrho})}{\partial \varphi} = 0. \tag{29}$$

If only θ and φ are different from θ_{min} and φ_{min}, which define the potential energy minima, transversal oscillations are imposed on the radial motion. Period of the radial motion and of the transversal oscillations are not necessarily commensurable and in general trajectories are open. The motion is periodic (orbits are closed) in very particular situation only. Unfortunately, conditions at which orbits are closed are a priori unknown, and the latter can only be found by numerical solving of the equations of motion.

4. *Periodic solutions-closed orbits*

Since we are interested in orbits which may represent the ground-state atom we will confine our investigations to the rotationless case. This limitation arises from the well established fact that spins of electrons play the important role in the formation of atomic shells and the spin-orbit coupling and spin-spin interaction may attain appreciable values in the close vicinity of nucleus only Gryziński[14,15] and this region at the non zero centrifugal force would be unaccessible for electrons. In general there is an infinite number of closed orbits. We can find them solving numerically equations of motion. For the dipole case, then the screening factor $\sigma(\hat{\varrho})$ is given by Eq. (11), the two simplest closed orbits, that is the free-fall orbit and the »pendulum« orbit, are shown in Fig. 2. The set of more complicated orbits, which correspond to a greater number of transversal oscillation, are shown in Figs. 7—11.

Among the various closed orbits we can distinguish the two following classes of orbits: the class of orbits which have the zero-velocity point, as shown in Figs. 7 and 8 and the class of continual orbits, as shown in Fig. 10. Orbits with the zero-velocity starting point, if the latter is located further from the nucleus than the zero-velocity point of the »pendulum« orbit — see Fig. 9, we will call quasi-free-fall orbits. If the zero velocity point of the closed orbit is located closer to the nucleus than the zero velocity starting point of the »pendulum« orbit such an orbit we will call collisional orbit.

Initial conditions specifying the particular type of the closed orbit are for the given value of the screening parameter uniquely determined. In the case of the zero-velocity starting point orbit it is convenient to define the initial conditions specifying the particular orbit by the value of the polar angle of the zero-velocity point — see Fig. 11. In general, however, it is more convenient to specify the orbit by the value of energy of transversal oscillations. This energy is directly related to the value of the angular momentum p_θ of a single electron at the symmetry of point of the orbit. Assuming that the starting point of the electron is located at the point of symmetry of the orbit, where $v_r = 0$, we will have:

$$p_0 = \sqrt{2\, m\, (Z - \sigma^{ff})\, e^2 \cdot r_s^{ff}} \cdot \sqrt{y_0\, (1 - y_0)}. \tag{30}$$

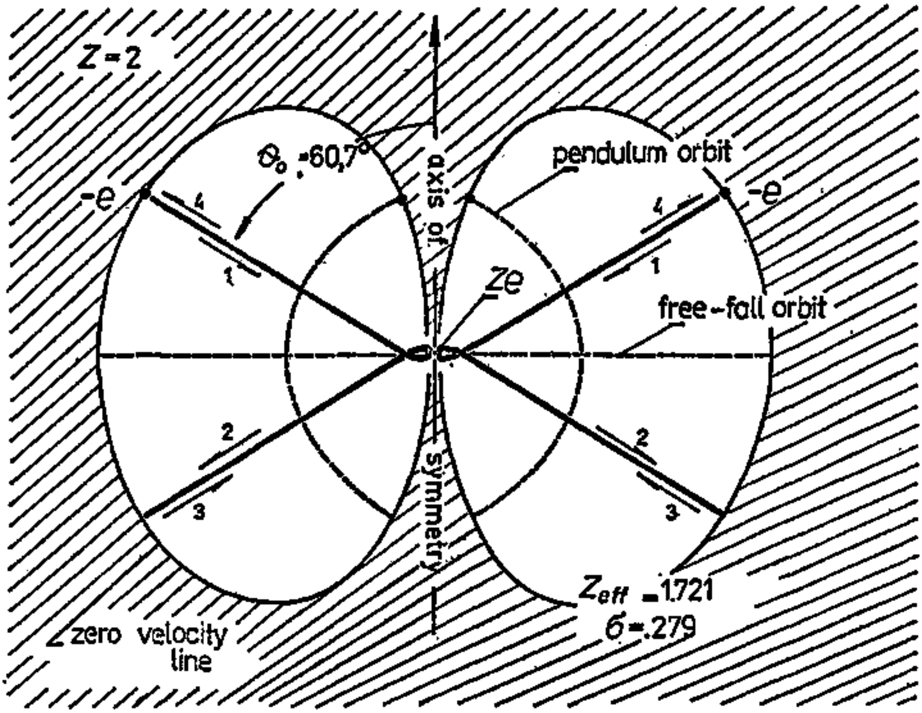

Fig. 7. The simplest quasi free-fall orbit.

The parameter p_θ defining the orbit is a measure of θ-oscillations. With $y_0 \to 0$ or $y_0 \to 1$ the amplitude of θ-oscillations tends to zero and in the limit the orbit is a free-fall orbit, and with $y_0 \to 1/2$ the amplitude of θ-oscillations tends to maximum and the orbit takes the form of the pendulum orbit.

5. »May be« atomic orbits

Which of the closed collective orbits may represent the real situation in the atom is an open question. There are in principle two ways of answering the question: the one which can be called theoretical and the other which can be called experimental.

The purely theoretical way seems to be at the moment very unrealistic. The reason is that the set of postulates describing the properties of the electron is too far incomplete — we do not know for instance how far the radiation process depends upon gyromagnetic properties of the electron[10]. Moreover, the existence of various paradoxes, like the Aharonov-Bohm effect[30] indicates that something in the electrodynamics of the electron may be wrong.

All what we can do at the moment is to take into considerations spin-magnetic interactions of electrons and to use some of the experimentally verified »quantum« rules, although their causal explanation not necessarily must be clear.

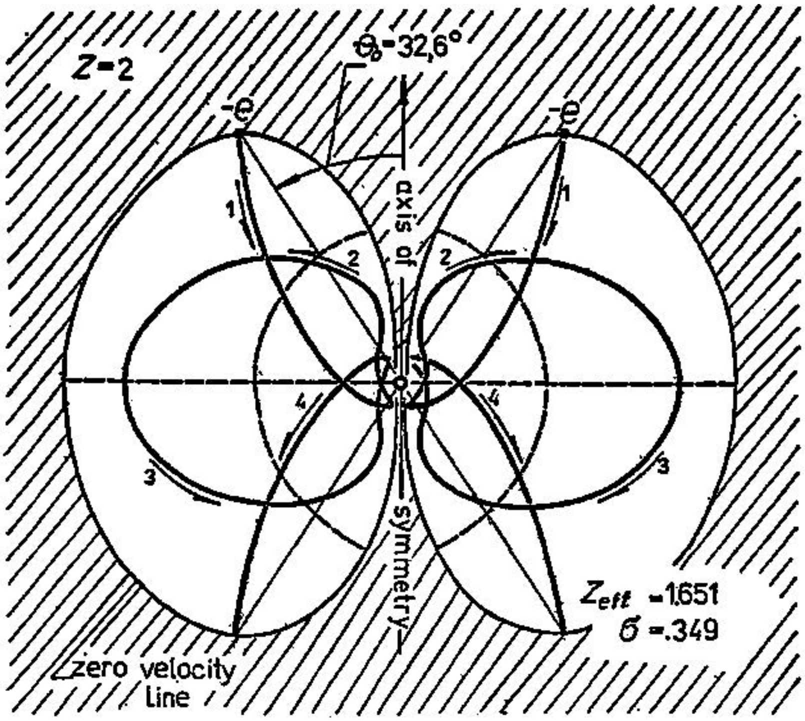

Fig. 8. The quasi-free-fall orbit of the more complicated form.

Assumption that the electron has a magnetic moment implies, according to the postulates of classical field theory, modification of equations of motion by adding two terms at least. The first, which describes magnetic interactions between the shell electrons, and the second, which describes magnetic interaction of the moving electron with the charged nucleus. All this, however, is insufficient to solve the whole problem theoretically.

The other way, which may be called experimental, is based on a direct confrontation of the properties of the particular atomic model with the observed properties of atoms. One can, for instance, calculate for the given orbit the momentum distribution, similarly as it has been done for the *ff*-orbit[31], and compare the latter with the momentum distribution derived from high energy electron Compton scattering[32,33]. The other way of checking the model is analysis of inner shell ionization by protons correlated with deflection of the latter in the field of nucleus[34–37].

In view of mathematical difficulties, incompletness of theoretical description of the electron and limited experimental informations about atoms, the combination of the both ways — i. e. experimental and theoretical seems to be the most pragmatic procedure towards deciphering the electronic structure of the atom.

And so one can by applying semiclassical quantisation rules as given by *Old Quantum Theory* to determine energy spectrum of the system for various collective orbits — as it has been done by Dimitrijević and Grujić[27,28] for the pendulum

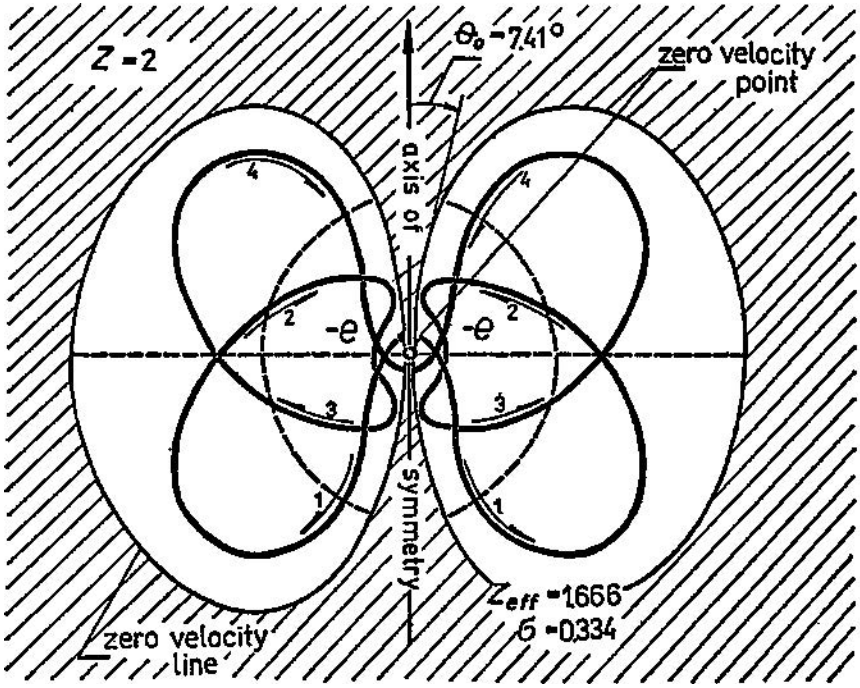

Fig. 9. The simplest collisional orbit (see text).

orbit. Unfortunately there are some ambiguities in practical use of these rules (results depend upon the choice of system of coordinates used for calculations) and moreover applicability of these rules to many electron systems has been never proved.

In view of the above and in view of the recently formulated hypothesis, which relates stability of electron orbit with perturbations caused by translational precession of electron spin axis, it seems justified to use quantisation rule in the form

$$\oint m\vec{v} \cdot \mathrm{d}\vec{l} = n\,h, \tag{31}$$

where $\mathrm{d}\vec{l}$ is an element of the electron path along the given orbit. In this case the binding energy of the system is given by:

$$W = U_i^H (Z - \bar{\sigma})^2, \tag{32}$$

where U_i^H is the ionization potential of the hydrogen atom and $\bar{\sigma}$ is the mean value of the screening parameter, which for particular orbit can be easily calculated:

$$\bar{\sigma} = \frac{1}{2\pi}\int_0^{2\pi} \frac{\sigma\,[\hat{\varrho}\,(\tau)]}{\varrho\,(\tau)}\,\mathrm{d}\tau. \tag{33}$$

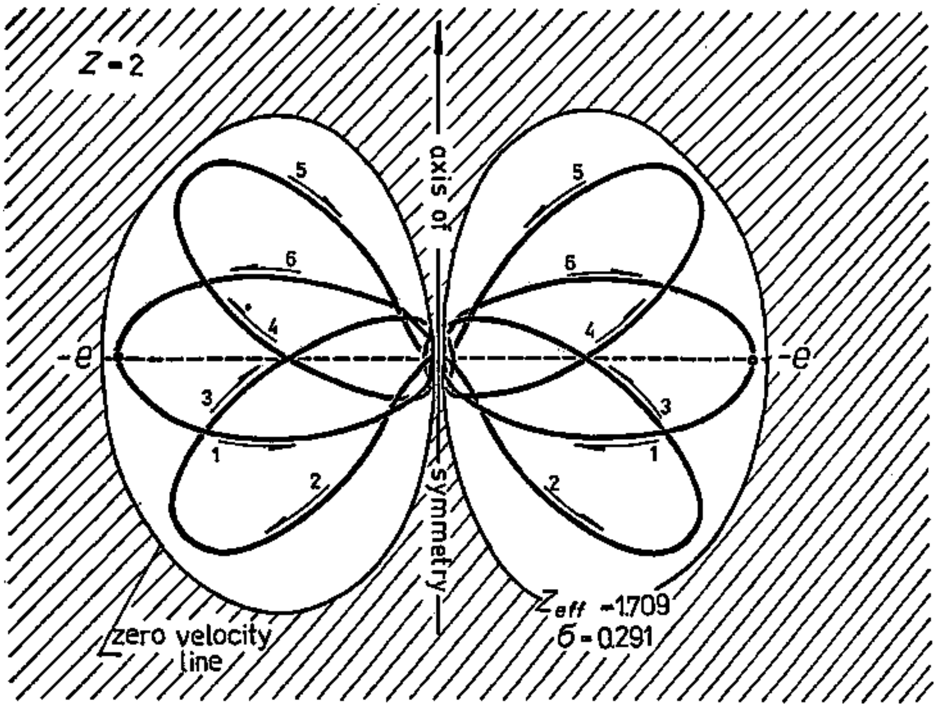

Fig. 10. The simplest continual orbit (the velocity of the electron is always different from zero).

Comparing the theoretical binding energy W, calculated from Eq. (32), with the binding energy determined experimentally, one can estimate the real situation in the atom.

In the particular case of helium atom we have:

$$W^{exp} = \frac{1}{2} (U_I + U_{II}) = 39.4 \text{ eV}, \qquad (34)$$

and, therefore, the experimental value of δ is

$$\delta^{exp} = Z - \sqrt{\frac{W^{exp}}{U_I^H}} \approx 0.296. \qquad (35)$$

Comparing the experimental value of δ with the values calculated from Eq. (33), see Fig. 12, one finds that there is a class of orbits with the value of the screening parameter very close to that experimentally measured. Since trajectories of the most likely orbits of δ very close to 0.296 approach very closely to the nucleus — where magnetic interaction of electrons cannot be neglected — the final choice of the orbit cannot be done. Nevertheless, some class of orbits can be eliminated from fur-

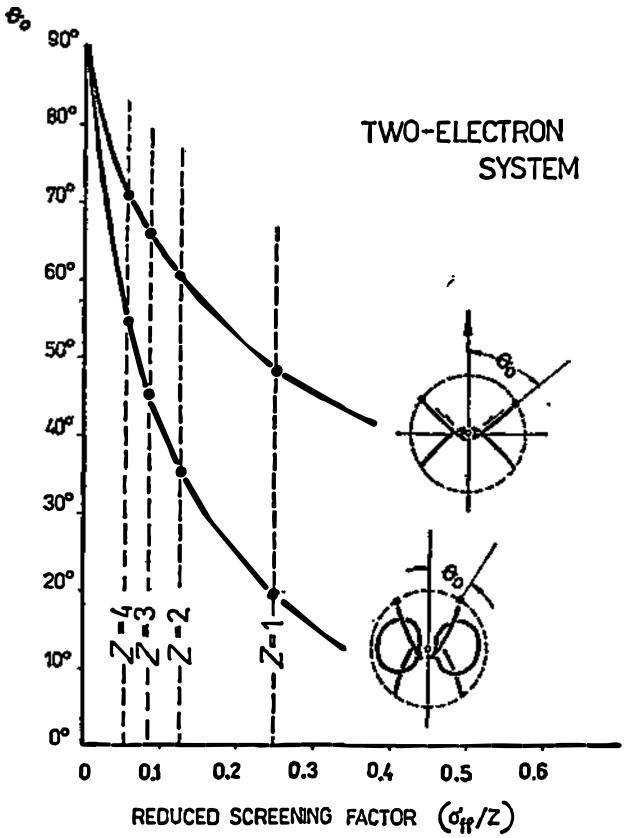

Fig. 11. Location of the zero-velocity starting points of electrons as a function of the value of the reduced screening parameters σ_{ff}/Z for the two quasi-free-fall orbits.

ther considerations. At first, following the general requirement of simplicity we should eliminate from considerations very complicated orbits. Among relatively simple orbits those with appreciable transversal energy, roughly with $p_0 > 0.3$ should be eliminated too, since then the screening factor is much too large and the magnetic interaction of electron is much too small to change the previous appreciably.

What exactly the form of the electron orbit of the helium atom is we shall try, taking into account magnetic properties of the electron, to show in the near future.

6. *Conclusions*

The carried out analysis of the collective motion of electrons in the field of nucleus shows that it is possible in principle to construct the many-electron atomic model, which would describe correctly the observed properties of atoms. The

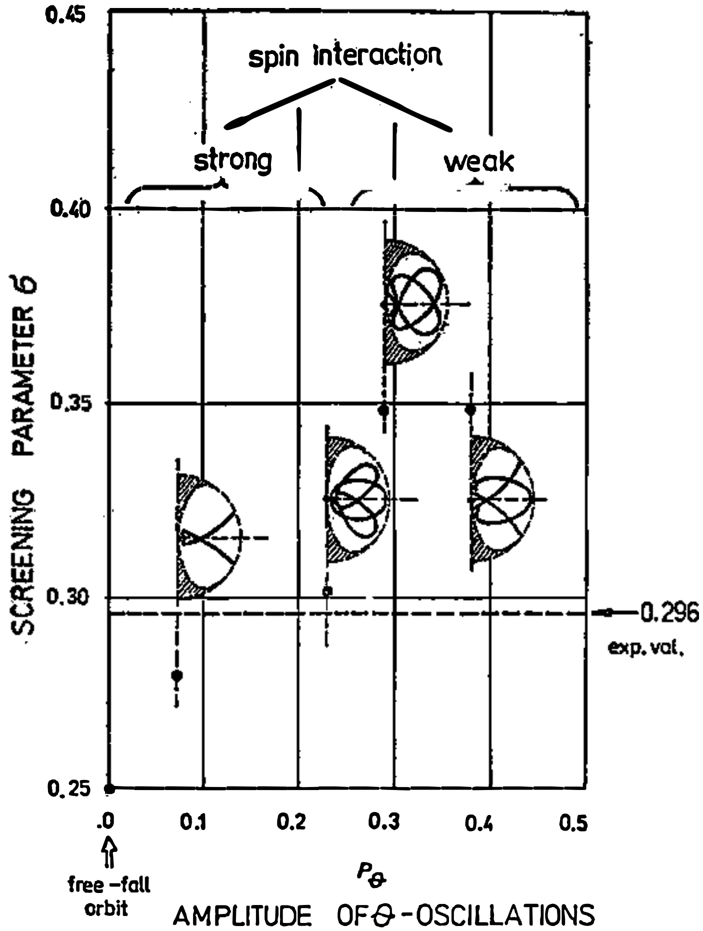

Fig. 12. Values of the screening parameter δ in the helium-like system ($Z = 2$ and $n_c = 2$) for various closed orbits as a function of transversal oscillations (p_θ— oscillations).

zero angular momentum collective orbits, with appreciably high energy of transverse oscillations, seem to be mechanically suficiently stable to form the basis for construction of the many-shell configuration consisting of the orbits of the similar form. In the case of valence electrons the motion should be not much different from the free-fall case and the trajectory should be similar to the hydrogen atom orbit.

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References

1) M. Gryziński, Phys. Rev. **115** (1959) 374;
2) R. C. Stabler, Phys. Rev. **133A** (1964) 1268;
3) M. Gryziński, Phys. Rev. **138A** (1965) 356;
4) R. Abrines and J. C. Percival, Proc. Phys. Soc. **88** (1966) 861;
5) J. L. Delcroix, *Physique des Plasmas* (Dunod, Paris) 1966;
6) A. Burgess and I. C. Percival, Adv. Atom. Molec Phys. **4** (1968) 109;
7) L. Vriens, *Case Studies in Atomic Collisions Physics*, eds Mc E. W. Daniel and Mc M. R. C. Dowell (North-Holland, Amsterdam 1970);
8) D. R. Bates, Physics Reports **35** (1978) 307;
9) D. Eihenauer, N. Grün and W. Scheid, J. Phys. **314** (1981) 3329;
10) M. Gryziński, Phys. Lett. **76A** (1980) 28;
11) K. Bohr, Phil. Mag. **26** (1913 : 1, 476 and 857;
12) A. Sommerfeld, Ann. d. Phys. **51** (1916) 1;
13) J. Langmuir, Phys. Rev. **16** (1921) 339;
14) M. Gryziński, Phys. Lett. **41A** (1972) 69;
15) M. Gryziński, Phys. Lett. **44A** (1973) 131;
16) M. Gryziński, J. Kunc and M. Zgorzelski, J. Phys. **36** (1973) 2292;
17) M. Gryziński, Jour. of Chem. Phys. **62** (1975) 2620 and 2629;
18) M. Gryziński, J. Physique Colloq. **40** (1979) C7-171;
19) M. Gryziński, Phys. Lett. **56A** (1976) 180;
20) M. Gryziński, *Diamagnetism of matter and the structure of atom*, accepted for publication in J. Magnetism and Magn. Materials;
21) P. Grujić, A. Tomić and S. Vučić, J. Chem. Phys. **79** (1983) 1776;
22) M. S. Dimitrijević, P. Grujić and S. Vučić, Fizika **13** (1981) 321;
23) R. D. Harcourt, J. Phys. B **16** (1983) 2647;
24) J. C. Leopold, I. C. Percival and A. S. Tworkowski, J. Phys. B **13** (1980) 1025;
25) J. G. Leopold, I. C. Percival, J. Phys. B **13** (1980) 1037;
26) P. V. Conveney and M. S. Child, J. Phys. B **17** (1984) 319;
27) M. S. Dimitrijević and P. Grujić, Z. Naturforsch. **39A** (1984) 930;
28) M. S. Dimitrijević and P. Grujić, Fizika **17** (1985) 1;
29) S. Davies, A. Truman and D. Williams, Phys. Lett. **99A** (1983) 15;
30) Y. Aharonov and D. Bohm, Phys. Rev. **115** (1959) 485;
31) M. Gryziński, J. Physique **43** (1982) L-425;
32) A. D. Barlas, W. H. E. Rueckner and E. F. Wellenstein, J. Phys. B **11** (1978) 3381;
33) A. L. Bennani, A. Duguet and H. F. Wellenstein, J. Phys. B **12** (1979) 461;
34) P. A. Amundsen, J. Phys. B **10** (1977) 2177 and J. Phys. B **11** (1978) 3197;
35) G. Lapicki, R. Laubert and W. Brandt, PRL **22** (1980) 1889;
36) E. Horsdal-Pedersen, F. Folkman and N. H. Pedersen, J. Phys. B **15** (1982) 739;
37) M. Gryziński and J. Kunc, J. Phys. B **13** (1986) 2273.

KOLEKTIVNO KRETANJE ELEKTRONA U KULONSKOM POLJU JEZGRE

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Ispitivana su numerički jednočestična rješenja koja opisuju kolektivno kretanje elektrona u kulonskom polju jezgre. Procenjen je opći karakter kretanja, kao i uslovi za periodično kretanje (zatvorene elektronske putanje). Određena je klasa zatvorenih trajektorija koje mogu da predstavljaju dvoelektronsku atomsku ljusku, sa posebnim osvrtom na ljusku atoma helija.