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DIAGRAMS AND DEPENDENCE ON THE GAUGE-FIXING VECTOR

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Using diagrammatic methods, we show new identities, additional to BRST identities, which forbid the unwanted counter-terms for non-covariant gauges.

1. Introduction

Non-covariant gauges have raised considerable interest because they are ghost free. Recently the focus has been on the light-cone gauge, which is most suitable for the development of string theories. However, an explanation has been needed concerning the question why the counter-terms which could spoil renormalization are forbidden.

We start by considering the planar gauge[1)] without introducing any regularization. The propagator in this gauge is given by

$$D_{\mu\nu}^{ab} = \delta_{ab}\,(k^2 + i\varepsilon)^{-1}\,[g_{\mu\nu} - (n \cdot k)^{-1}\,(k_\mu n_\nu + k_\nu n_\mu)]. \tag{1}$$

Then the self-energy to one-loop order reads

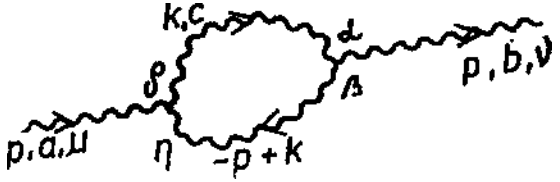


Fig. 1.

$$\Pi^{ab}_{\mu\nu} = \frac{1}{2} g^2 \, C(R)\, \delta_{ab} \int \frac{d^4 k}{(2\pi)^4} \left[g_{\alpha\beta}(2k - p)_\nu + g_{\alpha\nu}(-p - k)_\beta + g_{\nu\beta}(2p - k)_\alpha\right] \times$$

$$\times \frac{1}{k^2} \left[g_{\alpha\varrho} - \frac{k_\alpha n_\varrho + k_\varrho n_\alpha}{n \cdot k}\right] \times \left[g_{\eta\mu}(2p - k)_\varrho + g_{\mu\varrho}(-k - p)_\eta + g_{\varrho\eta}(2k - p)_\mu\right] \times$$

$$\times \frac{1}{(p - k)^2} \left[g_{\eta\beta} - \frac{(p - k)_\eta n_\beta + (p - k)_\beta n_\eta}{n(p - k)}\right]. \tag{2}$$

The simple observation that applying the operation $\partial/\partial n_\gamma$ to the propagator (1) leaves us with the terms proportional to either k_μ or k_ν, leads to new identities[2] in their simplest form. The factor k_μ acting on the vertex gives the usual Ward identity.

To show this in detail, for the self-energy we have

$$\frac{\partial}{\partial n_\gamma} \Pi^{ab}_{\mu\nu} = A \left[g_{\alpha\beta}(2k - p)_\nu + g_{\alpha\nu}(-p - k)_\beta + g_{\nu\beta}(2p - k)_\alpha\right] \times$$

$$\times \frac{1}{k^2 \, n \cdot k} \left[k_\alpha \left(\frac{k_\gamma n_\varrho}{n \cdot k} - \delta_{\varrho\gamma}\right) + k_\varrho \left(\frac{k_\gamma n_\alpha}{n \cdot k} - \delta_{\alpha\gamma}\right)\right] \times$$

$$\times \left[g_{\eta\mu}(2p - k)_\varrho + g_{\mu\varrho}(-k - p)_\eta + g_{\varrho\eta}(2k - p)_\mu\right] \times$$

$$\times \frac{1}{(p - k)^2} \left[g_{\eta\beta} - \frac{(p - k)_\eta n_\beta + (p - k)_\beta n_\eta}{n(p - k)}\right] +$$

$$+ A \left[g_{\alpha\beta}(2k - p)_\nu + g_{\alpha\nu}(-p - k)_\beta + g_{\nu\beta}(2p - k)_\alpha\right] \times$$

$$\times \frac{1}{k^2} \left[g_{\alpha\varrho} - \frac{k_\alpha n_\varrho + k_\varrho n_\alpha}{n \cdot k}\right] \times \left[g_{\eta\mu}(2p - k)_\varrho + g_{\mu\varrho}(-k - p)_\eta + g_{\varrho\eta}(2k - p)_\mu\right] \times$$

$$\times \frac{1}{(p-k)^2 \, n(p-k)} \left\{ (p-k)_\eta \left[\frac{(p-k)_\gamma n_\beta}{n(p-k)} - \delta_{\gamma\beta} \right] + \right.$$

$$\left. + (p-k)_\beta \left[\frac{(p-k)_\gamma n_\eta}{n(p-k)} - \delta_{\gamma\eta} \right] \right\}, \tag{3}$$

$$A = \frac{1}{2} g^2 \, C(R) \, \delta_{ab} \int \frac{d^4 k}{(2\pi)^4}. \tag{4}$$

Then, the simple Ward identity

$$k_\alpha \left[g_{\alpha\beta} (2k-p)_\nu + g_{\alpha\nu} (-p-k)_\beta + g_{\nu\beta} (2p-k)_\alpha \right] =$$

$$= g_{\nu\beta} \, p^2 - p_\nu \, p_\beta - g_{\nu\beta} (p-k)^2 + (p-k)_\nu (p-k)_\beta \tag{5}$$

indicates that the tensor structure will have a factor of the form $(g_{\nu\beta} \, p^2 - p_\nu p_\beta)$, if we can show the cancellation of the second factor within the diagrams of the same order in the coupling constant.

Actually, after evaluating these integrals[3], we obtain

$$\frac{\partial}{\partial n_\gamma} \Pi_{\mu\nu} = 2 g^2 \, C(R) \, \delta_{ab} \left\{ \frac{n_\mu}{n^2} \left(\delta_{\beta\gamma} - \frac{n_\beta \, n_\gamma}{n^2} \right) (g_{\nu\beta} \, p^2 - p_\nu \, p_\beta) + \right.$$

$$+ \frac{n_\varrho}{n^2} \left(\delta_{\nu\gamma} - \frac{n_\nu \, n_\gamma}{n^2} \right) (g_{\mu\varrho} \, p^2 - p_\mu \, p_\varrho) + \frac{n_\nu}{n^2} \left(\delta_{\beta\gamma} - \frac{n_\beta \, n_\gamma}{n^2} \right) (g_{\mu\beta} \, p^2 - p_\mu \, p_\beta) +$$

$$\left. + \frac{n_\varrho}{n^2} \left(\delta_{\mu\gamma} - \frac{n_\mu \, n_\gamma}{n^2} \right) (g_{\nu\varrho} \, p^2 - p_\nu \, p_\varrho) \right\}. \tag{6}$$

Then a counter-term such as $n_\mu n_\nu F^{\mu\lambda} F^\nu_\lambda$ does not satisfy this identity, as it contains the term $\delta_{\mu\nu} \, (n \cdot p)^2$ and there is no such term in (6) (i. e. the term $\delta_{\mu\nu} \, (n \cdot p) \, p_\gamma$). This counter-term, if it existed, although allowed by gauge invariance, would be dangerous, as it does not vanish on the mass shell and depends on the gauge parameter n.

In the same way, by insering the light-cone gauge propagator of Mandelstam or Leibbrandt[4], we can show why there are no counter-terms such as

$$n_\mu n^*_\nu F_{\mu\lambda} F_{\nu\lambda} \text{ and } (n_\mu n^*_\nu F_{\mu\nu})^2.$$

2. *The ghost-gluon transition*

To derive diagrammatic identities for the planar gauge, let us introduce a new vertex

$$\left(\frac{n_\xi\, n_\alpha}{n^2} - \delta_{\xi\alpha}\right). \tag{7}$$

It symbolizes the ghost-gluon transition (for the planar gauge we can choose $n^2 = 1$, $n_\xi\, \delta n_\xi = 0$) and then

$$\delta\, n_\xi\, \eta \left(\frac{n_\xi\, n_\alpha}{n^2} - \delta_{\xi\alpha}\right) A_\alpha = -\eta\, \delta\, n \cdot A. \tag{8}$$

Then the structures $[(k_\gamma\, n_\varrho / n \cdot k) - \delta_{\varrho\gamma}]$ which occurred in (3) can be represented as the ghost-gluon transition through the new vertex

$$\frac{1}{n \cdot k}\left(\delta_{\gamma\alpha} - \frac{n_\gamma\, n_\alpha}{n^2}\right)\frac{1}{k^2}\left(\delta_{\alpha\varrho} - \frac{k_\alpha\, n_\varrho + k_\varrho\, n_\alpha}{n \cdot k}\right) = \frac{1}{n \cdot k\, k^2}\left(\delta_{\gamma\varrho} - \frac{k_\gamma\, n_\varrho}{n \cdot k}\right). \tag{9}$$

3. *The vertex graph*

Let us take three-gluon vertex and apply the $\partial / \partial n_\xi$ operation. It will act on each propagator, but let us follow what happens when it acts on the propagator with momentum r:

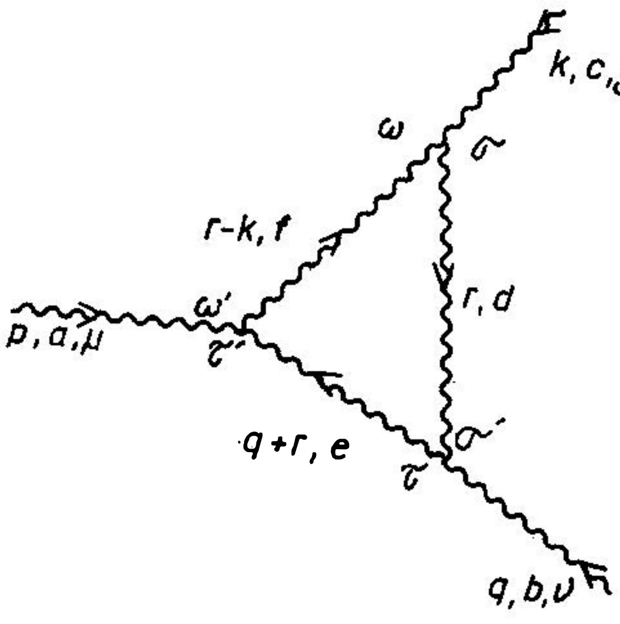
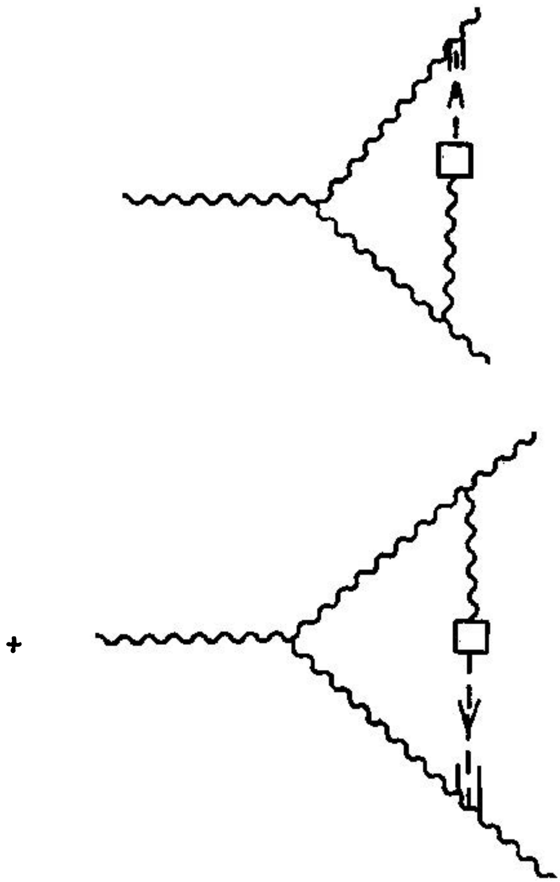


Fig. 2.

$$\frac{\partial}{\partial n_\xi} V^{abc}_{\mu\nu\varrho} =$$



+ (cyclic permutations of propagators)

Fig. 3.

where — → — ≡ · is the ghost propagator with the factor r_σ $(r_{\sigma'})$, respectively.

The factor r_σ with the vertex $V_{\sigma\varrho\omega}$ gives a Ward identity like the one in (5). Thus, the first diagram in Fig. 3 is

$$\frac{\partial}{\partial n_\xi} V^{abc}_{\mu\nu\varrho}(1) = -\mathrm{i}g^3 f_{afe} \left[g_{\mu\omega'} (-q - r - 2p)_{\tau'} + g_{\omega'\tau'} (2q + 2r + p)_\mu + \right.$$

$$+ g_{\tau'\mu} (p - q - r)_{\omega'}] \times f_{bed} \left[g_{\tau\nu} (-2q - r)_{\sigma'} + g_{\sigma'\tau} (2r + q)_\nu + g_{\nu\sigma'} (q - r)_\tau \right] \times$$

$$\times f_{fcd} \left[g_{\omega\varrho} k^2 - k_\omega k_\varrho - g_{\omega\varrho} (r - k)^2 + (r - k)_\omega (r - k)_\varrho \right] \times$$

$$\times \frac{1}{r^2\, n\cdot r}\left(\frac{r_\xi\, n_{\sigma'}}{n\cdot r}-\delta_{\xi\sigma'}\right)\times\frac{1}{(q+r)^2}\left[g_{\tau\tau'}-\frac{(q+r)_\tau\, n_{\tau'}+(q+r)_{\tau'}\, n_\tau}{n\,(q+r)}\right]\times$$

$$\times\frac{1}{(r-k)^2}\left[g_{\omega\omega'}-\frac{(r-k)_\omega\, n_{\omega'}+(r-k)_{\omega'}\, n_\omega}{n\,(r-k)}\right]. \qquad (10)$$

On the mass shell where $k^2 = 0$, $e^\varrho k_\varrho = 0$ (e^ϱ — the polarization vector), the tensor $(g_{\omega\varrho}\, k^2 - k_\omega\, k_\varrho)$ vanishes.

The other tensor $g_{\omega\varrho}\,(r-k)^2 - (r-k)_\omega\,(r-k)_\varrho$ reduces the $(r-k)^{-2}$ denominator, so we are left with the first diagrammatic identity (Fig. 4), where wc meet a special ghost-gluon vertex

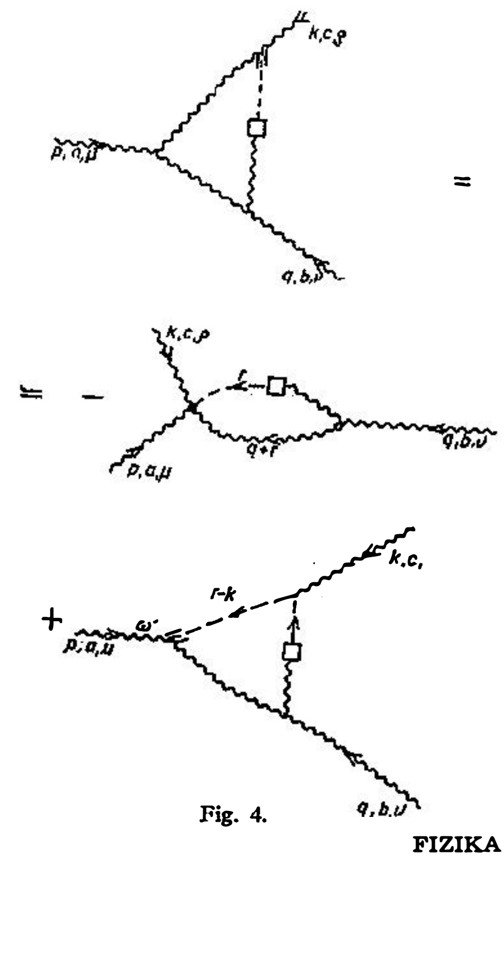


Fig. 4.

The last diagram in Fig. 4 can be further reduced to:

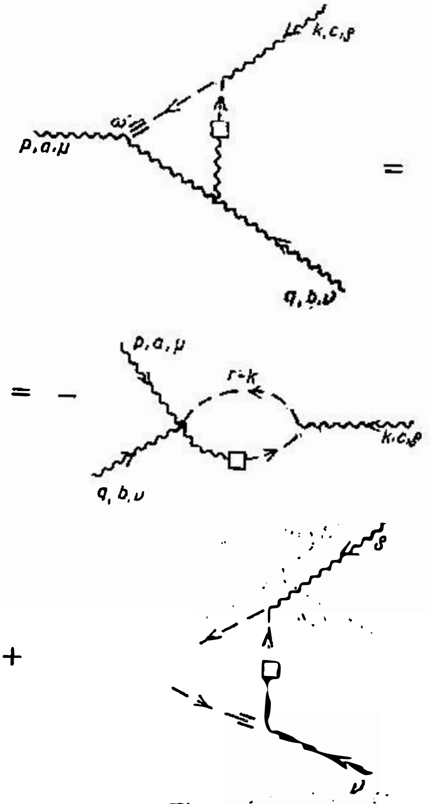


Fig. 5.

The last graph eventually becomes

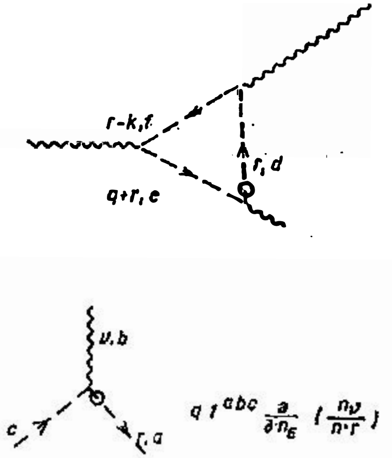


Fig. 6.

In this way we have

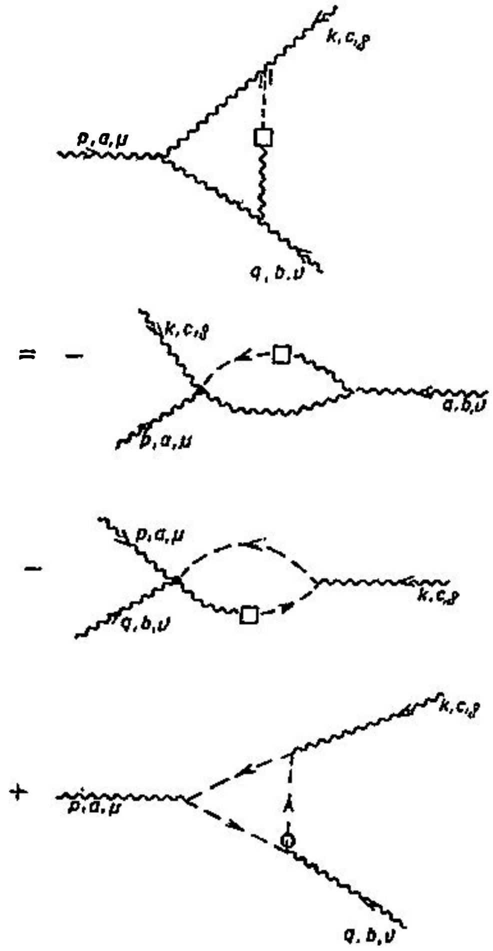


Fig. 7.

However, to the order g^3 there are the vertex graph coming from the four-gluon vertex and the reducible graph with the self-energy insertion:

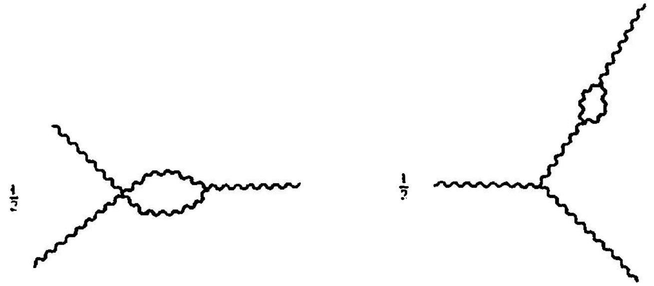


Fig. 8.

Applying the $\partial/\partial n_\xi$ operation to these graphs gives two graphs which cancel the first and the second graph on the right-hand side of Fig. 7.

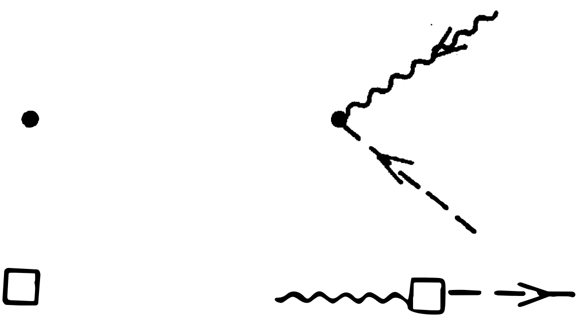
4. *Identities for the generating functional*

The diagrammatic identities (including the on mass-shell terms) can be written as identities for the generating functional:

$$\frac{\partial^3 \delta \Gamma^{(1)}}{\partial A(1)\,\partial A(2)\,\partial A(3)} = \frac{\partial \Gamma^{(1)}}{\partial A(1)\,\partial A(2)\,\partial \lambda\,\partial u^a_\mu(x)} \times \frac{\partial \Gamma^{(0)}}{\partial A^a_\mu(x)\,\partial A(3)} + $$

$$+ \frac{\partial \Gamma^{(1)}}{\partial A(1)\,\partial \lambda\,\partial u_\mu(x)} \, \frac{\partial \Gamma^{(0)}}{\partial A^a_\mu(x)\,\partial A(2)\,\partial A(3)} + \text{permutations of the points } (1,2,3), \tag{11}$$

where $\delta\Gamma$ is the variation of the generating functional with respect to the gauge parameter n, $u_\mu(x)$ is the source coupled to the ghost and the gluon and λ is the strength of the ghost-gluon transition:



$\Gamma^{(1)}$ and $\Gamma^{(0)}$ are one-loop and zero-loop generating functionals, respectively.

From Eq. (11) we can deduce the basic equation

$$\delta\Gamma^{(1)} = \int \mathrm{d}x \frac{\partial^2 \Gamma^{(1)}}{\partial \lambda\,\partial u^a(x)} \times \frac{\partial \Gamma^{(0)}}{\partial A^a(x)}. \tag{12}$$

Diagrammatic identities can obviously be derived for any number of external legs.

Formally, such identities can be phrased through BRST transformations[2].

However, the diagrammatic method shows why the light-cone gauge has to be treated as a special case and how to tackle this problem.

We shall discuss that in a forthcoming paper[5].

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DIJAGRAMI I OVISNOST O VEKTORU U BAŽDARNOM UVJETU

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Pomoću dijagramatskih metoda pokazani su novi identiteti, dodatni BRST identitetima, koji zabranjuju nepoželjne kontra-članove u nekovarijantnim baždarnim uvjetima.