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LETTER TO THE EDITOR

SPACE-TIME DEPENDENT DIELECTRIC RESPONSE FUNCTION OF ONE-COMPONENT CLASSICAL RARE PLASMA

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The variation with frequency and wave-vector at temperatures 300 K and 600 K, of the space-time temperature dependent dielectric response function appropriate for the classical rare plasma, is studied in the following article.

Many physical systems such as semiconductors at low temperature, ionised gases prevalent in ionosphere and other low density two-component plasma, where the motion of the heavy positive charge can be neglected may be approximated by one-component classical rare plasma. Such a system is quite different from the extensively studied one component degenerate plasma where the expression for space-time dependent dielectric function, $\varepsilon(\vec{q}, w)$, is valid for $T = 0$ K. However, it is becoming increasingly clear that in both the systems the frequency dependence of the dielectric function, plays a very important role in the physical studies where the time dependent behaviour of the system cannot either be ignored or approximated by some time averaged behaviour. For example, the problem of the determination of the static structure factor of the electrons about a finite mass charged impurity placed in one-component plasma, be it degenerate[1)] or rare[2)] involves the use of the full knowledge of the space and time dependent complex dielectric response function.

System in which the interparticle spacing, r, is much greater than the thermal de Broglie wavelength $\lambda_{th} = \hbar/\sqrt{2\,m\,k\,T}$ where m stands for mass of the particle, k is the Boltzmann constant, T the temperature of the system, $\hbar$ the Planck's constant is termed as classical plasma. The classical rare plasma therefore occurs at low density and high temperature, and thus in this case the one-component plasma can be described by Maxwell-Boltzmann distribution function. Using the transport theory formalism an appropriate expression for the dielectric function of the classical rare plasma was obtained by Tewari and Yadav[3] a few years ago, and is given as follows

$$\varepsilon(\vec{q}, w) = \varepsilon_1(\vec{q}, w) + \mathrm{i}\,\varepsilon_2(\vec{q}, w) \tag{1}$$

where

$$\varepsilon_1(\vec{q}, w) = 1 - \frac{4\sqrt{\pi}\, n e^2}{q^2\, k\, T}\, J\left(\sqrt{\frac{m\,\omega^2}{2\,q^2\,k\,T}}\right). \tag{2}$$

$J\left(\sqrt{\frac{m\,\omega^2}{2\,q^2\,k\,T}}\right)$ is a function defined by

$$J(y) = \mathrm{P}\int_{-\infty}^{\infty} \frac{x\, e^{-x^2}}{y - x}\, \mathrm{d}x.$$

P stands for Cauchy principal value and $y = \sqrt{mw^2/2\,q^2kT}$. $J(y)$ can be evaluated analytically as a function of y and we obtain the expression for $\varepsilon_1(\vec{q}, w)$ as follows

$$\varepsilon_1(\vec{q}, \omega) = 1 + \frac{4\pi\, n\, e^2}{q^2\, k\, T} - $$
$$- \frac{8\pi\, n\, e^2}{q^2\, k\, T} \cdot \frac{m\,\omega^2}{2\,q^2\,k\,T}\, \mathrm{e}^{-m\omega^2/2q^2kT}\; {}_1F_1\left(\frac{1}{2};\ \frac{3}{2};\ \frac{m\,\omega^2}{2\,q^2\,k\,T}\right) \tag{3}$$

${}_1F_1\left(\frac{1}{2}, \frac{3}{2}, \frac{m\,\omega^2}{2\,q^2\,k\,T}\right)$ being a hypergeometric function. This is the same expression as obtained by other authors[4].

The imaginary part of the dielectric response function, $\varepsilon_2(\vec{q}, \omega)$[3] is given by

$$\varepsilon_2(\vec{q}, \omega) = \frac{8\pi^3\, e^2\, m^2\, n}{(2\pi\, m\, k\, T)^{3/2}}\, \frac{\omega}{q^3}\, \mathrm{e}^{-m\omega^2/2q^2kT} \tag{4}$$

By applying the low-frequency expansion for $\varepsilon_1(\vec{q}, \omega)$, the total dielectric response function can be rewritten as:

$$\varepsilon(\vec{q}, \omega) = 1 + \frac{4\pi\, n\, e^2}{q^2\, k\, T} - \frac{8\pi\, n\, e^2}{q^2\, k\, T} \cdot \frac{m\,\omega^2}{2\,q^2\,k\,T}\, \mathrm{e}^{-m\omega^2/2q^2kT} \Big[1 +$$

$$+\frac{1}{3}\left(\frac{m\,\omega^2}{2\,q^2\,k\,T}\right)+\frac{1}{10}\left(\frac{m\,\omega^2}{2\,q^2\,k\,T}\right)^2+\cdots\Bigg]+$$

$$+\frac{8\,\pi^3\,e^2\,m^2\,n}{(2\,\pi\,m\,k\,T)^{3/2}}\cdot\frac{\omega}{q^3}\cdot e^{-m\omega^2/2q^2kT}\qquad \omega\ll\sqrt{\frac{2\,k\,T}{m}}. \tag{5}$$

In order to have a feel of the frequency and temperature dependence of the dielectric function we have numerically evaluated $\varepsilon_1(\vec{q},\omega)$ and $\varepsilon_2(\vec{q},\omega)$, for a physical system having the density n of electrons equal to 10^{15} cm^{-3}, which corresponds to mean interparticle distance 0.6×10^{-5} cm. We consider two different temperatures of the system namely 300 K and 600 K. The two values of the wave vector $q=\left(\frac{2\pi}{\lambda},\right.$ where λ is the wavelength$\left.\right)$, are taken to be 0.5×10^6 cm^{-1} and 1.0×10^6 cm^{-1}.

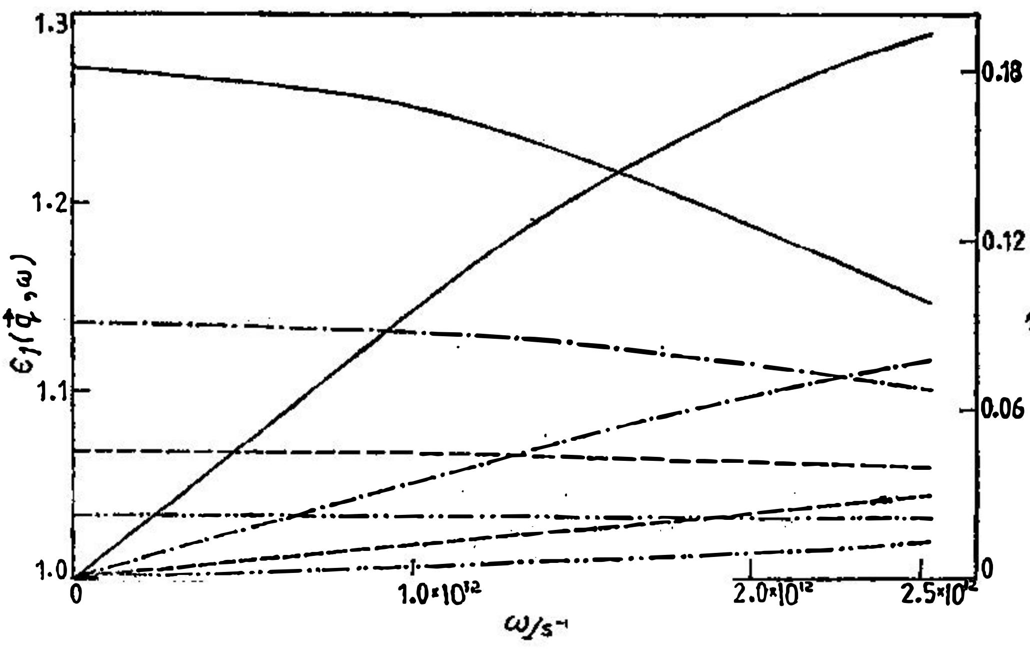

Figure 1. Real and imaginary part of dielectric response function, $\varepsilon_1(\vec{q},\omega)$ and $\varepsilon_2(\vec{q},\omega)$, respectively, for different values of the wave vector q and different temperatures T.

$\varepsilon_1(\vec{q},w)$ and $\varepsilon_2(\vec{q},w)$ for $q=0.5\times10^6$ cm^{-1} and $T=300$ K,
$\varepsilon_1(\vec{q},w)$ and $\varepsilon_2(\vec{q},w)$ for $q=0.5\times10^6$ cm^{-1} and $T=600$ K,
$\varepsilon_1(\vec{q},w)$ and $\varepsilon_2(\vec{q},w)$ for $q=1.0\times10^6$ cm^{-1} and $T=300$ K,
$\varepsilon_1(\vec{q},w)$ and $\varepsilon_2(\vec{q},w)$ for $q=1.0\times10^6$ cm^{-1} and $T=600$ K.

The calculated values of $\varepsilon_1 (\vec{q}, \omega)$ and $\varepsilon_2 (\vec{q}, \omega)$, are shown in Fig. 1. For the sake of convenience, ω has been expressed in terms of plasma frequency $\omega_p = \sqrt{\frac{4 \pi n e^2}{m}}$ of the system, in units of s^{-1}.

As is evident from the figure, for a fixed temperature and a given wave-vector the frequency dependence of both $\varepsilon_1 (\vec{q}, \omega)$ and $\varepsilon_2 (\vec{q}, \omega)$ is not at all simple. As the frequency increases while $\varepsilon_1 (\vec{q}, \omega)$ keeps decreasing from its Debye-Hückel value at $\omega = 0$, $\varepsilon_2 (\vec{q}, \omega)$ increases from its value equal to zero at $\omega = 0$ rather sharply. For a fixed temperature as q is allowed to vary, the detailed behaviour of both $\varepsilon_1 (\vec{q}, \omega)$ and $\varepsilon_2 (\vec{q}, \omega)$ become somewhat different. The temperature dependence of both $\varepsilon_1 (\vec{q}, \omega)$ and $\varepsilon_2 (\vec{q}, \omega)$, is also not simple. Though as temperature increases from its value at $T = 300$ K to 600 K the absolute values of $\varepsilon_1 (\vec{q}, \omega)$ and $\varepsilon_2 (\vec{q}, \omega)$ decreases for a given frequency, but, the amount of decrease in the entire frequency region is in general quite involved. For instance, for $q = 0.5 \times 10^6$ cm^{-1}, as the temperature increases from 300 K to 600 K, the difference in the real part of $\varepsilon (\vec{q}, \omega)$ starts decreasing from its value at $\omega = 0$ and becomes smaller (in not a simple way) with increase in ω and reduces to less than half for $\omega = 2.5 \times 10^{12}$ s^{-1}. For $\varepsilon_2 (\vec{q}, \omega)$, the situation is just the reverse: the difference between the absolute value of $\varepsilon_2 (\vec{q}, \omega)$ for the increase in temperature from 300 K to 600 K, decreases from its maximum value at $\omega = 2.5 \times 10^{12}$ s^{-1}, again in not a simple way, to the zero difference at $\omega = 0$. Thus we find the suggested frequency dependent complex dielectric response function in contrast to the real dielectric function of Debye-Hückel theory valid for $\omega = 0$, has in general the contribution of not only the imaginary part of $\varepsilon (\vec{q}, \omega)$ which will play a very important role in physical quantity involving the use of the total dielectric function, but also has temperature dependence quite different from that given by Debye-Hückel theory. The two match as is expected for $\omega = 0$.

We therefore conclude that Eq. (5) gives space-time temperature dependent dielectric function appropriate for classical rare plasma in a usable form and this tform can directly be verified experimentally for an electron system formed from ihermionic emission or in a semiconductor in which the hole concentration is neglgible.

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PROSTORNO-VREMENSKA OVISNOST DIELEKTRIČNE ODZIVNE FUNKCIJE JEDNOKOMPONENTNE KLASIČNE RAZRIJEĐENE PLAZME

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Razmatraju se promjene realnog i imaginarnog dijela dielektrične odzivne funkcije s frekvencijom za različite vrijednosti valnog vektora i temperature.