YU ISSN 0015-3206
FZKAAA 19 (4) 375 (1987)

BASIC FEATURES OF HIGH-ENERGY NUCLEUS-NUCLEUS COLLISIONS

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Received 8 June 1987

Revised version received 22 October 1987

UDC 539.17

Original scientific paper

Starting from a straightforward analogy with the absorption of gamma-rays in matter a simple model of high-energy nucleus-nucleus collisions is developed. Closed expressions for basic observables — the average number of interacting nucleons in a projectile nucleus (the number of »wounded nucleons«) and the dispersion of this number — are derived. Comparisons with experimental results for d-Ta, α-Ta and C-Ta collisions at 4.2 GeV/c per nucleon confirm the adequacy of the model and show that collisions develop predominantly as a sequence of independent nucleon-nucleon collisions.

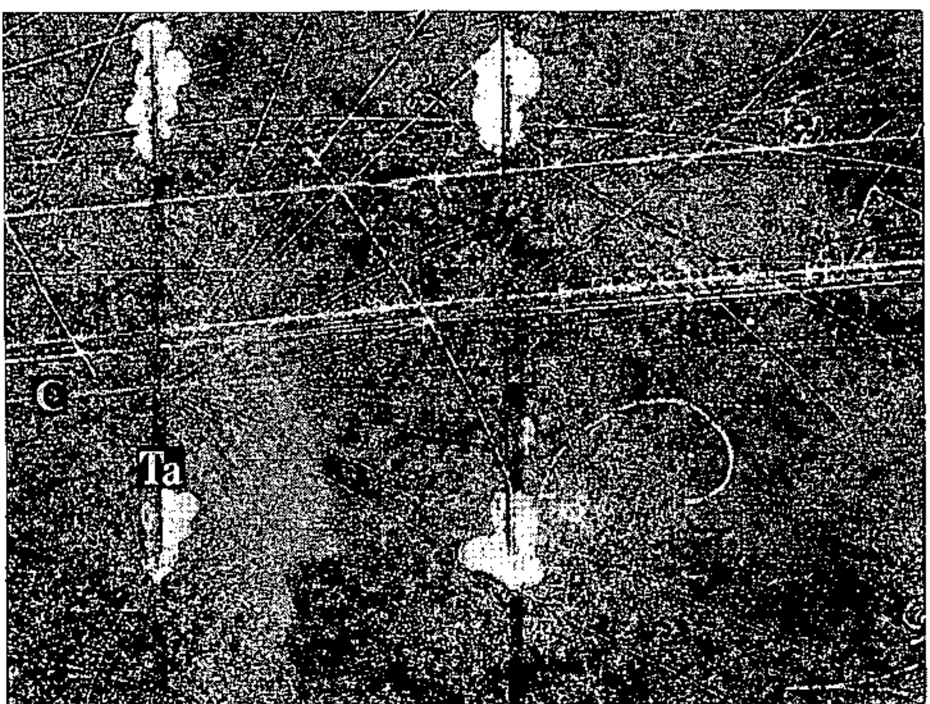
1. Introduction

High-energy nucleus-nucleus collisions are complex events with an enormously rich spectrum of possible outcomes. Initial nuclei are fragemented and multitude of secondary particles is produced. Ingenious methods have been deviced to cope with this complexity and at the present time many general features of such collisions are thought to be satisfactorily understood. Different aspects of this comparatively new field are summarized in a number of review articles[1-3].

However, most of the analyses are quite formal and leave certain unexpectedly simple properties of these seemingly very complicated interactions somewhat obscure. We hope that a simple model of such collisions, which we develop here, would be helpful in elucidating some of the basic physics displayed by these reactions.

2. The basic observables

Fig. 1 presents two typical collisions of a carbon-12 nucleus accelerated to 4.2 GeV/c per nucleon with a stationary tantalum-181 target nucleus. The pictures are from the Dubna 2m propane bubble chamber where collisions of deuterons, alpha particles and carbon-12 with tantalum-181 were recently experimentally investigated[4]. Fig. la presents a collision of the type frequently referred to as »peripheral«, while Fig. 1b shows the collision of the type which would be classified as »central«. The terms used imply that in the first case the nuclei collide at the larger impact parameter, i. e. with the incident nucleus passing further away from the center of the target nucleus, than in the second case. The two types are distinguished by the number of forward peaked nucleons or nuclear fragements (only charged nucleons or fragments are seen in the bubble chamber pictures). The total charge of such fragements can be determined by the ionization of tracks and the total number of nucleons, both charged and neutral, emitted in the direction of the incoming beam, can thereby be deduced by combinatorics[4]. It is also easily seen from the figures that from a collision with more forward tracks there are less



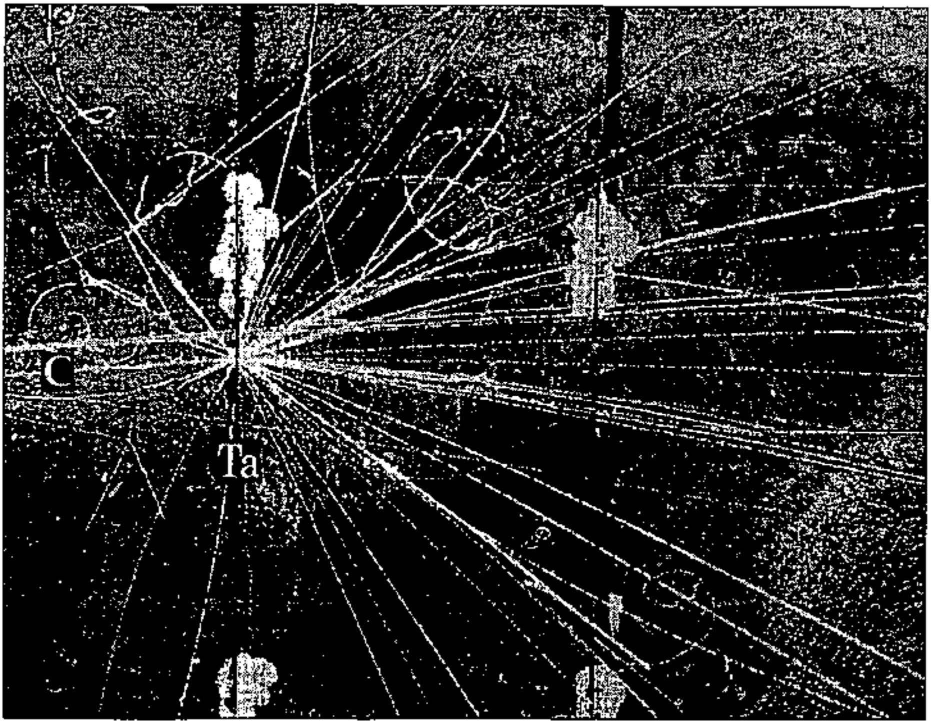


Fig. 1. Two typical collisions of carbon-12 nucleus accelerated to 4.2 GeV/c per nucleon with a stationary tantalum-181 nucleus. The pictures are from the Dubna 2m propane bubble chamber. Note that to the case with fewer forward peaked reaction products there corresponds a richer star (Fig. 1b), and vice versa (Fig. 1a). The difference arrises from different impact parameters, in the two cases.

starlike ones (a peripheral collision) and vice versa; when there are fewer forward peaked tracks a richer star is produced (a central collision). The curvature of tracks and subsequent kinematical analyses show that the momenta of the nucleons and nuclear fragments emitted in the direction of the projectile are comparable to the momenta of the projectile nucleons. This leads to the conclusion that they must be the nucleons or parts of the projectile nucleus that either did not interact with the target nucleus at all, or have at best experienced elastic interactions only. From the number of such nucleons we can finally infer how many nucleons of the projectile interacted inelastically with a target nucleus. These would be held responsible for the creation of a star which consists of the products from inelastic projectile nucleon-target interactions, the nucleons and their groups from the two nuclei and the produced secondary particles, mostly pions. We can now better justify the terms »pheripheral« and »central« since it is obvious that in the first case the probability of an inelastic nucleon-nucleon interaction is smaller then in the second, resulting in the characteristic topologies described above.

We therefore see that the number of projectile nucleons that have experienced at least one inelastic interaction is the first observable and significant quantity characteristic of a high-energy nuclear collision. This number is called the number of wounded nucleons. However, as we have already seen, this quantity can have different values for different impact parameters. Besides, due to the stochastic nature of the interactions, the number of wounded nucleons would be different even for the same impact parameter. This results in a particular probability distribution for the number of wounded nucleons which can be described by two parameters:

1. *The average number of wounded nucleons* which, for the projectile of A nucleons, we shall denote as w_A, and

2. *The dispersion of this number* defined as the square root of the difference between the square of the average number of wounded nucleons and the average of the square of this number, we shall denote this by D_w.

The way in which these quantities are recovered from raw experimental data is described in Ref. 4. It is these two quantites that we shall consider as the basic observables in high-energy nucleus-nucleus collisons and we shall attempt to reproduce these two experimental quantities in the simple model which follows. Successful reproduction of these quantities would verify the initial assumptions of the model thus setting forth clearly the general properties of such collisions.

3. *The model*

Majority of models in nuclear physics is built upon analogies with other microscopic and even macroscopic systems and processes. The familiar process that bears many similarities to the one just described is the passage of gamma-rays through matter. In the case of gamma-ray absorption we usually consider the parallel beam of radiation passing through a homogenious macroscopic slab of material, the scattering centers being the atoms of the absorber. In our case we can consider the incoming projectile nucleus as a parallel beam of A independent nucleons and the target nucleus with B nucleons as a microscopic and inhomogenious absorber, the scattering centers now being the nucleons of the target nucleus (Fig. 2). Knowing the cross sections for inelastic nucleon-nucleon interaction and the nuclear densities we can find the number of wounded nucleons which would be a quantity analogous to the intensity of gamma-rays lost in the course of passage through the absorber. To do so we shall be forced to make a number of assumptions about the properties of the interactions involved. The differences from the common gamma-ray case would be self-evident wherever they appear.

The De-Broglie wavelength of the projectile nucleon with momentum p (GeV/c) is $\lambda = 0.2/p$ (GeV/c) which, at $p = 4$ GeV/c, is equal to about 1/100 of the radius of the target nucleus (tantalum in our case). This enables us to define the impact parameter b (Fig. 2) for a given projectile nucleon with respect to the center of the target nucleus. Along the path through the target nucleus at a given b, the incoming nucleon passes through the region of different nuclear densities and we have to make use of the average surface density of absorbing centers instead of the straightforward surface density, common in the case of the homogenious absorber. For a known nuclear density distribution of the target nucleus, $\varrho(r) = \varrho(\sqrt{b^2 + x^2})$, the average surface density of apsorbing centers, $T(b)$, is obviously equal to (see Fig. 2):

$$T(b) = B \int_{-\infty}^{+\infty} \varrho(\sqrt{b^2 + x^2})\, \mathrm{d}x. \tag{1}$$

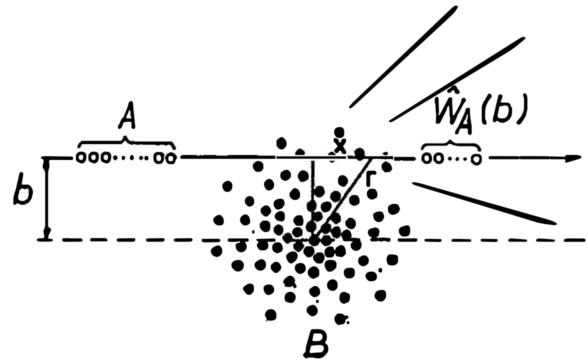


Fig. 2. Schematic diagram of a high-energy nucleus-nucleus collision with definitions of relevant quantities (see text).

In order to simplify the model we further assume that the radius of the projectile nucleus is much smaller then that of the target nucleus. This being reasonably so in all the cases we shall consider, we take that all the nucleons of the projectile have the same impact parameter b.

Another essential assumption is that each projectile nucleon, as it ploughs through the target nucleus, interacts only with individual nucleons of the target and not with the nucleus as a whole. This is again justified by the smallness of the nucleon wavelength at high energy and by the fact that at high energy nuclei behave as loosely bound systems of quasi-free nucleons. If we now assume that the cross section for an inelastic nucleon-nucleon interaction which ejects the nucleon from the »beam« equals σ then by analogy with the gamma-ray case, we obtain the average or the probable number of nucleons that shall pass through the target undisturbed at a given b:

$$\hat{w}_A(b) = A\,\mathrm{e}^{-\sigma T(b)}. \tag{2}$$

Although it is not a common practice to treat the absorption process as a binomial experiment it is evident that it is its typical representative. To clarify this let us quote the properties of the binomial experiment[5] as they would apply to our particular case:

a) The experiment consists of n repeated and independent trials; in our model A mutualy non-interacting nucleons independently and in succession try to pass through the target nucleus.

b) Each trial results in the outcome that might be classified as a success or a failure; in our model each of A nucleons may or may not get wounded while passing through the target nucleus.

c) The probability of a success, q, remains constant from trial to trial; in our model the target nucleus looks the same to each projectile nucleon.

From here we see that in addition to the assumptions we already introduced we have to make another one, required by c), which seems also plausible.

We can thus assume that the number of nucleons that did not interact inelastically is binomially distributed and that the average number of such nucleons equals the number of the attempts, A, times the probability of a success in a single trial, $p(b)$ (Eq. (2)). From here we deduce that the probability for a single nucleon to pass, at a given b, through the target nucleus without being wounded is $p(b) = \exp(-\sigma T(b))$. The probability we are actually interested in, that at the same b the nucleon will get wounded, is then equal to:

$$q(b) = 1 - p(b) = 1 - e^{-\sigma T(b)}. \tag{3}$$

Applying the binomial distribution, the probability that, at given b, j projectile nucleons out of A shall get wounded is:

$$q_j(b) = \binom{A}{j} q^j p^{A-j} = \binom{A}{j} [1 - e^{-\sigma T(b)}]^j [e^{-\sigma T(b)}]^{A-j} \tag{4}$$

where the binomial coefficient is deffined as:

$$\binom{A}{j} = \frac{A!}{j!(A-j)!}.$$

However, since the target is microscopic, these quantities are not observables, and to obtain observable probabilities we have to integrate over all values of b:

$$Q_j = \int q_j(b)\, d^2 b. \tag{5}$$

Since the probabilities $q_j(b)$ are normalized to one it is obvious that the probabilities Q_j are normalized as:

$$\sum_{j=0}^{A} Q_j = \int d^2 b. \tag{6}$$

Again, since the target is microscopic, the probability that no nucleon is wounded,

$$Q_0 = \int q_0(b)\, d^2 b = \int e^{-A\sigma T(b)}\, d^2 b \tag{7}$$

is not an observable quantity since it can not be distinguished from the case when the projectile misses the target nucleus altogether. The correctly normalized probability that j nucleons out of A shall get wounded is thus finally

$$Q_j = \frac{1}{N} \int \binom{A}{j} [1 - e^{-\sigma T(b)}]^j [e^{-\sigma T(b)}]^{A-j}\, d^2 b \tag{8}$$

with normalization constant N equal to:

$$N = \int d^2 b - Q_0 = \int [1 - e^{-\sigma A T(b)}]\, d^2 b. \tag{9}$$

Now we are able to find the average number of wounded nucleons and its dispersion as the parameters of this probability distribution. Following the definitions, after some algebra we find that the mean of the distribution is equal to:

$$w = \langle j \rangle = \sum_{j=0}^{A} j\, Q = A \frac{\int [1 - e^{-\sigma T(b)}]\, d^2 b}{\int [1 - e^{-A\sigma T(b)}]\, d^2 b} \tag{10}$$

and that dispersion of this number, or standard deviation of the mean equals:

$$D_w^2 = \sum_{j=0}^{A} (j - \langle j \rangle)^2 Q_j = w_A - w_A^2 + 2(A-1) w_A - A(A-1) \cdot \cdot \frac{\int [1 - e^{-2\sigma T(b)}]\, d^2 b}{\int [1 - e^{-A\sigma T(b)}]\, d^2 b}. \tag{11}$$

The last term in this expression is proportional to the ratio of the number of wounded nucleons for projectiles with A and 2 nucleons respectively (see Eq. (10)) and Eq. (11) finally becomes:

$$D_w^2 = w_A - w_A^2 + 2(A-1) w_A - A(A-1) \frac{w_A}{w_2}. \tag{12}$$

All the results of the model are summarized in Eqs. (10) and (12). These are practically contained in the expressions obtained by A. Bialas et al. within the conceptually and formally somewhat more complicated multiple scattering model. Their results are, howerer, also more general and take into account the finite size of the projectile as well. For the interesting details the reader is refered to the original paper[6]. Direct comparison with other theoretical work[7] is not possible since explicit expressions for corresponding quantities are not given. In the next section we compare the values from our expressions with the experimental ones for the set of results for d-Ta, α-Ta and C-Ta collisions from Ref. 4.

4. *Results*

To evaluate Eqs. (10) and (12) in a given case two quantities are needed: the values of inelastic nucleon-nucleon cross section σ and the function $T(b)$. The values of the cross section at a few GeV is known to be close to $30 \times 10^{-27}\,\mathrm{cm}^2$ from many nucleon-nucleon scattering experiments. Repeated calculations with different values for σ show that our quantities are not very sensitive to this parameter so we adopted $30 \times 10^{-27}\,\mathrm{cm}^2$ for all calculations. In order to find the function $T(b)$ one has to assume a certain nuclear density distribution $\varrho(r)$ for the target nucleus. Since tantalum is a heavy nucleus we used the common Saxon-Woods distribution:

$$\varrho_{SW}(r) = \frac{\varrho_0}{1 + e^{-\frac{R-r}{c}}} \tag{13}$$

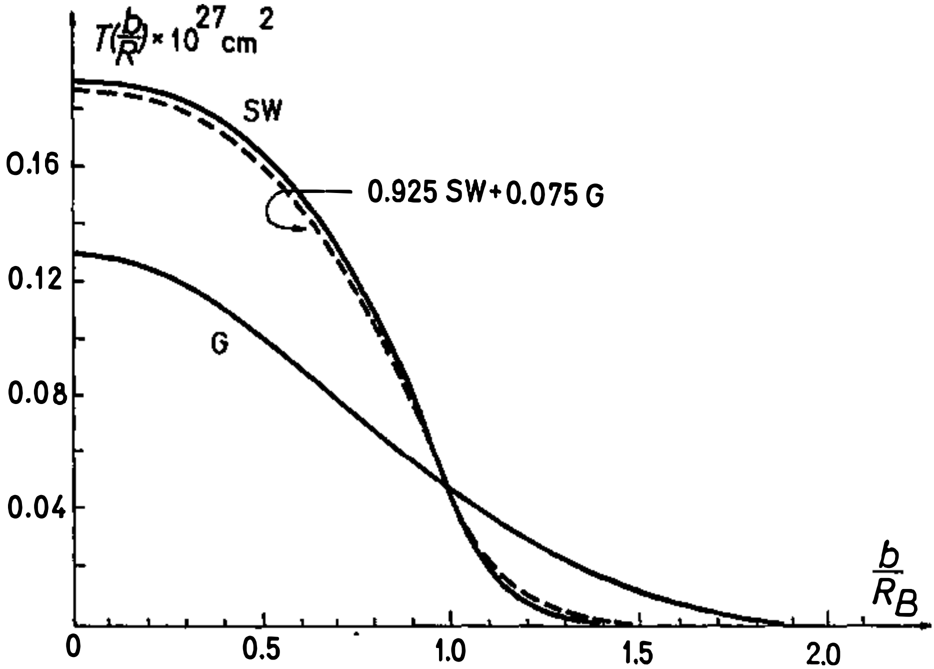


Fig. 3. Surface density of target nucleons, $T(b)$, as a function of the impact parameter b for the tantalum-181 target nucleus. Functions for different admixtures of Saxon-Woods (SW) and Gaussian (G) nuclear density distributions are presented.

with $R = 1.18\,B^{1/3}$ fm and $c = 0.54$ fm. The density ϱ_0 is obtained by normalization to $B = 181$. The resulting function $T(b)$ is shown in Fig. 3. The values of w_A are then found by numerical integration using Eq. (10) and from there the dispersions are calculated (Eq. (12)). The results for d-Ta, α-Ta and C-Ta collisions are summarized in Table 1 together with the experimental results from Ref. 4 (except for the last coloumn on which we shall comment later on). It is seen that the model values for w_A's are systematically higher then the experimental

TABLE 1.

A	w_A^{exp}	w_A^{th}	D_w^{2exp}	D_w^{2th}	$D_w^{2th}(w_A^{exp})$
1	1	1	0	0	0
2	1.60 (4)	1.75	0.24 (2)	0.19	0.24 (8)
4	2.86 (10)	3.15	1.64 (10)	1.34	1.12 (50)
12	6.6 (3)	8.14	16.8 (10)	18.7	17.5 (25)

Experimental[4] and model values for the average number of wounded nucleons and dispersion of this number for p-Ta, d-Ta, α-Ta and C-Ta collisions at 4.2 GeV/c per nucleon.

ones by some 10 to 25% while the dispersions, due to the complex dependence do not show a definite trend.

It is, however, well known[8] and is easily seen if the integrations in Eq (10) are performed stepwise, that the quantities that depend upon $T(b)$ are most sensitive to the shape of $\varrho(r)$ in the region $r \cong R_B$, i. e. to the details of the nuclear surface. To examine this in our case we admixed different amounts of a Gaussian density distribution of the type:

$$\varrho_G = \varrho_0 \, e^{-\frac{r^2}{R^2}}$$

to Saxon-Woods distribution. The results for w_A's for an admixture of 7.5% are presented in Fig. 4. It can be seen that the agreement is much better than in the pure Saxon-Woods case. It is instructive to note in Fig. 3 how slightly this function $T(b)$ differs from the one corresponding to the pure Saxon-Woods density distribution. It is also most interesting to note in Fig. 4 that our quantities are most sensitive to the small departures from Saxon-Woods distribution. From these manipulations of our model expressions one gets the feeling that a satisfactory agreement of all the data could eventually be obtained by suitable minor changes in the shape of the nuclear density distribution which, of course, do not have to have much in common with our *ad hoc* trials with usual analytical functions. It is, however, also clear that such an attempt would not be justified since a simple model like ours cannot pretend to give a perfect fit due to the number of approximations which are certainly not completely justified in some cases. For instance, one of the doubious approximations in the case of carbon projectile might be the neglect its finite dimensions.

The fact that expression (12) gives the dispersions as an explicit function of the average number of wounded nucleons only, opens up the possibility of a parameter-free test of our model. The dispersions calculated by the use of Eq. (12) with the experimental values for w_A's would test the initial assumptions of the model only, irrespective of the values for σ and $T(b)$. These values are listed in the last column of Table 1. The quoted errors in calculated dispersions are obtained by the common procedure of the propagation of errors from experimental errors for w_A's. The magnitude of these errors shows that the dispersions are most sensitive to the values of w_A's. The model gives the dispersions that within the (large) error limits agree with the experimental ones. Our simple model thus seems to account for the majority of effects that influence the two observables we have discussed.

5. *Summary and discussion*

We have derived and tested the expressions for the average number of wounded nucleons of the projectile nucleus, Eq. (10), and for the dispersion of this number, Eq. (12), in high-energy nucleus-nucleus collisions. We have closely followed the analogy with the case of gama-ray absorption while to do so we had to make a number of more or less explicit assumptions about the interactions involved. These are:

1. The wavelength of the projectile nucleons is small as compared to the radius of target nucleus.
2. The radius of the projectile nucleus is much smaller than that of the target nucleus.
3. The projectile nucleons interact independently with individual nucleons of target nucleus only.

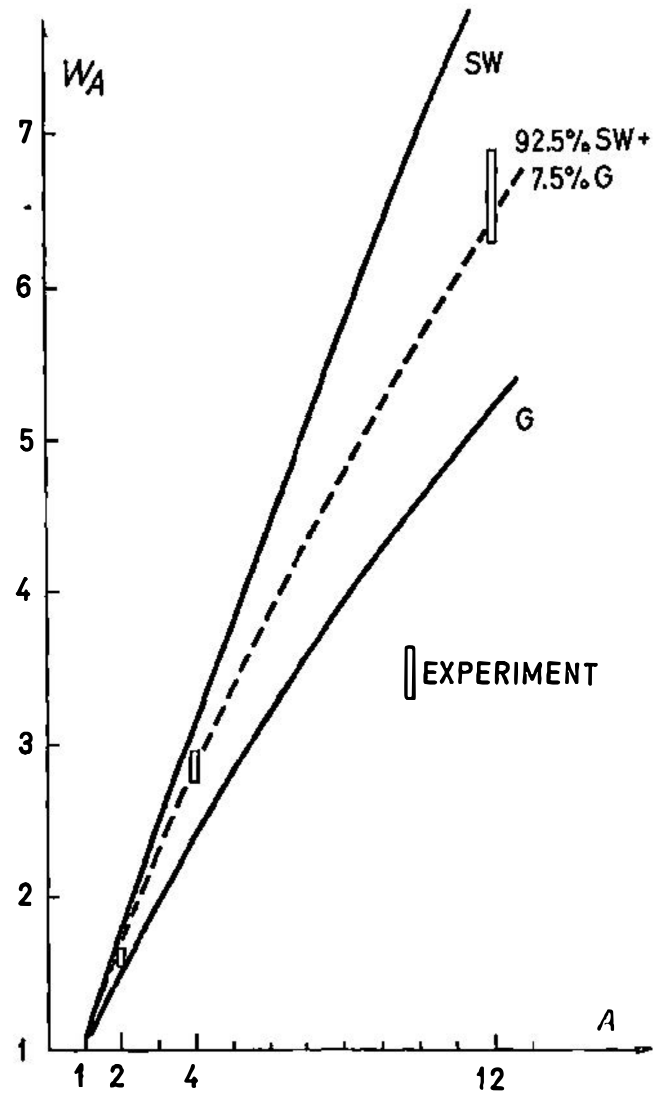


Fig. 4. Average number of wounded nucleons, w_A, for different projectile nuclei, A, and for different nuclear density distributions for the tantalum-181 target nucleus.

4. The interactions that knock the projectile nucleons out of their initial directions are described by the cross section for inelastic scattering of free nucleons.

5. The surface density of target nucleons is found as if their projections do not overlap.

6. Significant changes in the target nucleus, as induced by passage of the projectile, develop only after the last of the projectile nucleons would have traversed it.

We have here deliberately neglected a number of suspected higher-order effects which could play some role in these interactions. Without dealing with them in detail, they are:

a. The effects of the finite size of the projectile nucleus, as opposed to assumption 2. above.

b. The possible collective interactions between the colliding nuclei or their parts, known as the collective or coherent effects, whose existence opposes assumption 3. above.

c. The shadowing (or eclipsing) effects which violate assumption 5.

d. The trailing effects, which take into account the possibility that the successive projectile nucleons see the target perturbed by the passage of previous ones, which opposes assumption 6.

Fair agreement of experimental and model values for the two observables, as evidenced by the results listed in Table 1 and those presented in Fig. 4, show that the assumptions about the essential properties of high-energy nucleus-nucleus collisions are basically correct and that the neglected higher-order effects probably have only minor influence on the two quantities we have discussed. It follows that at high energies, as far as our two observables are concerned, the nuclei appear merely as space correlators of otherwise free nucleons and that they interact through individual nucleon-nucleon collisions only. Restating, we might say that high-energy nucleus-nucleus collisions belong to the realm of elementary particle physics rather than to that of nuclear physics. The question of at what lower energies typically nuclear properties of colliding nuclei again become relevant is best left to be decided by future experiments at intermediate energies. It can, however, be roughly predicted that this will happen when our first and third assumptions become invalid.

From the last column of Table 1 we see that the errors in the dispersions propagate quite fast from the errors in the number of wounded nucleons. The resulting large errors in calculated dispersions show that more experimental results with smaller errors are called for if the existence of higher-order effects is to be reliably judged by the two observables we have considered. Also, further refinements of the simple model developed here do not seem to be seriously required until experimental accuracy is significantly increased. An alternative conclusion is that the discussed observables are probably not best suited for the pursuit of higher-order effects in such collisions.

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OSNOVNE OSOBINE SUDARA ATOMSKIH JEZGARA NA VISOKIM ENERGIJAMA

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UDK 539.17

Originalni naučni rad

Oslanjajući se na direktnu analogiju sa apsorpcijom gama zračenja u materiji razvijen je jednostavan model sudara atomskih jezgara na visokim energijama. Broj ranjenih nukleona u jezgru projektilu je osnovna opservabla u ovakvom procesu i ovde su dobijeni izrazi za parametre distribucije ove veličine; njenu srednju vrednost i disperziju. Poređenje sa eksperimentalnim rezultatima za d-Ta, α-Ta i C-Ta sudare na 4.2 GeV/c po nukleonu potvrđuju osnovne postavke modela i pokazuju da se ovakvi sudari uglavnom odvijaju kao niz nezavisnih nukleon-nukleon sudara.