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LETTER TO THE EDITOR

PROPERTIES OF THE COMPTON EFFECT

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In the simplest possible way, in the semiclassical approximation of Schrödinger, the background of the kinematical and optical properties of the Compton effect is discussed.

The Compton effect is a well understood elementary process. Thus, this letter in response to some recent publications can contribute only to conceptual clarification.

Elbaz considering optical[1] and kinematical[2] properties of the Compton effect noted the link with relativistic optical properties of matter and explained the effect as a pure specular reflection in the proper frame of the scattering element[3]. The results as such are sound, however, their dynamical background should not be concealed.

In general, the Compton effect can be treated at three levels: QED, semiclassical approximation and classical conjecture[4]. The second approach in which the electron is described within quantum mechanics and the radiation within classical electrodynamics gives results equivalent to the ones of QED. In fact, the scattering cross-section was first obtained in the semiclassical approximation describing the electron with the Dirac equation[5]. A description of the electron with the Schrödinger equation gave an approximate cross-section[6]. The first proposal to tackle the Compton effect in the semi-classical approximation is due to Schrödinger[7].

In the semiclassical approximation the Planck constant is entering in a natural way through the wave function of the electron. Photons are not introduced explicitely, but owing to energy and momentum conservation an amount of energy and momentum of the radiation field, a tantum[8], is interacting with the electron. Other processes, e. g. spontaneous emission, which cannot be consistently described in the semiclassical approximation, give evidence that QED is the superior theory.

The classical conjecture considering the electron a point charge and describing the radiation within classical electrodynamics[9] does not represent a consistent approach. The Planck constant has to be introduced in a separate step identifying a coefficient, containing it, in the derived expression for the frequency of the scattered radiation with measured data[8,9]. Besides, one has to introduce the ad hoc assumption that the electron does not radiate untill it is accelerated to the final velocity. To dwell on the kinematical and optical properties of the Compton effect only, may obscure the distinction of the three approaches.

In the simple Schrödinger's procedure in the laboratory frame S the electron recoiling in the direction of the x-axis is described with a relativistic Schrödinger wave function composed of two parts, pertaining to the initial and final state, respectively. In this frame the probability density is not stationary. It is, however, stationary in reference frame S' in which the electron in the initial and final states has equal and opposite momentum $P' = mc\,(2\,(\nu_0 - \nu)/\nu_c)^{1/2}$ along the x'-axis[4]. Thereby ν_0 and ν are the frequency of the incident and scattered X-rays, respectively, in the laboratory frame S and $\nu_c = mc^2/h$ is the Compton frequency of the electron. In the reference frame S' the stationary probability density $\cos^2(P'x'/2\hbar)$ is periodic with the period $a' = h/P'$. On this periodic structure the X-rays are scattered as, e. g., light on standing ultrasound waves.

We consider a simplified stationary scattering problem in the reference frame S' in which the frequency of the incident X-rays amounts to ν_0'. Plane waves with this frequency are incident with an angle of incidence Φ' on N parallel planes of maximum probability density and the scattered waves are observed very far away. This problem is related to a Fraunhofer type diffraction problem on an extended $\cos^2$-grating with N lines. If E_0' is the amplitude of the electric field in the incident wave the electric field in the scattered wave is

$$E_s' = q\,(q\,E_0'/m)\,(4\pi\,\varepsilon_0\,c^2\,r')^{-1}\,e^{2\pi i\nu_0' r'/c} \int_{-Na'/2}^{Na'/2} e^{ig'x'} \cos^2(\pi x'/a')\,dx'/(Na'/2) =$$

$$= E_0'\,(r_{c1}/r')\,e^{2\pi i\nu_0' r'/c}\,\sin(Ng'a'/2)/(Ng'a'/2)\,(1 - g'^2\,a'^2/4\pi^2).$$

The incident waves are taken to be linearly polarised and scattered waves with the same polarisation are considered. $r_{c1} = q^2/4\pi\,\varepsilon_0\,m\,c^2$ is the classical radius of the electron and $g' = (2\pi\,\nu_0'/c)\,(\sin\Phi' - \sin\Phi_s')$ determines the phase difference between the incident and the scattered wave. We have taken N odd to ensure symmetry, but the same result is got with even N if the probability density is shifted by $a'/2$ along the x'-axis.

As N is increasing the function $f(g') = \sin(Ng'\,a'/2)/(Ng'\,a'/2)$ having the maximum 1 at $g' = 0$ is becoming narrower and narrower. In the limit $N \to \infty$ it is zero everywhere except at $g' = 0$ where it has the value 1. Thus, the only direction in which scattered waves occur is given by $\sin \Phi' - \sin \Phi'_s = 0$, i. e. the scattering angle Φ'_s equals the angle of incidence Φ'. This explains the specular reflection in the reference frame S' as a result of interference.

The intensity of the scattered waves in the stated direction

$$I'_s = \tfrac{1}{2} c \varepsilon_0 E'^*_s E'_s = I'_0 (r_{c1}/r')^2 \tag{1}$$

with the intensity of the incident waves $I'_0 = \frac{1}{2} c \varepsilon_0 E'^2_0$ does not depend on the angle of incidence Φ'. The scattering is isotropic in the reference frame S'. It is, however, not isotropic in the laboratory frame S. Using the transformations for the electric and magnetic field the Lorentz transformations

$$I'_0 = I_0 (\nu'_0/\nu_0)^2 \qquad I'_s = I_s (\nu'_0/\nu)^2 \tag{2}$$

for the intensities are obtained. A related situation is encountered in transforming the cosmic background radiation from the proper frame in which it is isotropic to the rest frame of the earth[10].

If in Eq. (1) r' is replaced by r, Eq. (2) leads to

$$I_s = (\nu/\nu'_0)^2 I'_s = (\nu/\nu'_0)^2 I'_0 (r_{c1}/r)^2 = (\nu/\nu_0)^2 (r_{c1}/r)^2 I_0. \tag{3}$$

This should be compared with the equation of Klein and Nishina in the case that the linear polarisation of the scattered waves is the same as of the incident ones[5]:

$$I_s = (\nu/\nu_0)^2 (r_{c1}/r)^2 I_0 (1 + \nu^2/\nu_0^2)/2.$$

Thereby the ratio of the frequencies of the incident and the scattered X-rays is given by the well-known Compton equation

$$\nu_0/\nu = 1 + (\nu_0/\nu_c)(1 - \cos \Theta)$$

if Θ is the scattering angle, i. e. the angle between the scattered and the incident X-rays, in the laboratory frame S.

Eq. (3) was derived in a naive way. One could object to the equation for E'_s and would more carefully put

$$1/r'^2 = (1/r^2) [(\nu_0 - \nu)^2 (\nu_0 - \nu + 2\nu_c)/2\nu_c (\nu_0 + \nu)^2 + 1]$$

so that the last bracket would multiply the result in Eq. (3). A more detailed calculation, including the case of general polarisation, may give even better agreement. Nevertheless, our intention was not to derive the exact result, which in the semi-classical approximation was obtained long ago, but to try to divert the attention in the simplest possible way to the dynamical background of the stated kinematical and optical properties of the Compton effect[1-3].

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LASTNOSTI COMPTONOVEGA POJAVA

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Na najpreprostejši mogoč način, v Schrödingerjevemu polklasičnemu približku, je obdelano ozadje kinematičnih in optičnih lastnosti Comptonovega pojava.