

OPTICAL-MODEL ANALYSIS OF $^{13}\text{C}(^{16}\text{O}, ^{16}\text{O})^{13}\text{C}$
IN THE RANGE $E_{\text{c.m.}} = 19\text{--}30$ MeV

SANTANU DATTA and NIKOLA CINDRO

Ruder Bošković Institute, 41001 Zagreb, Croatia, Yugoslavia

and

RICHARD M. FREEMAN, CHRISTIAN BECK, FLORENT HAAS and ABDELATIF
MORSAD

Centre de Recherches Nucléaires and Université Louis Pasteur, 67200 Strasbourg, France

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A six-parameter optical-model analysis of $^{13}\text{C}(^{16}\text{O}, ^{16}\text{O})^{13}\text{C}$ is performed. The obtained values of the real and imaginary potential depths increase with energy, consistent with the findings of other authors. The scattering-equivalent potentials intersect at a radial distance of 7.6 fm for this system.

1. Introduction

Elastic scattering from heavy-ion collisions in the mass range $A_P, A_T < 20$ has been studied quite extensively, especially with the isotopes of oxygen and carbon. Nevertheless, because of its basic theoretical and practical importance, a more complete understanding of this process is required.

The elastic scattering of ^{16}O on ^{13}C was studied by Gobbi et al.¹⁾ in the energy range 6—12.5 MeV in the centre-of-mass system. They made a parametrized phase-shift analysis of the excitation function and angular distribution.

In this paper we try to analyze the angular distribution of ^{13}C (^{16}O , ^{16}O) ^{13}C in the centre-of-mass energy range 19–30 MeV, using a six-parameter optical model. Our intention is to check for possible regularities of the parameters of the real and imaginary parts of the simple Wood-Saxon potential when applied to this system. To our knowledge, this is the first analysis of this kind of the $^{16}\text{O} + ^{13}\text{C}$ system.

2. Data and analysis

The measurement of the elastic scattering of ^{16}O on ^{13}C was performed by Freeman et al. at the CRN Strasbourg tandem accelerator²⁾. The data consist of cross sections measured in steps of $\Delta\theta_{\text{c.m.}} = 0.8^\circ$ in the angular range $42^\circ\text{--}92^\circ$ in the centre-of-mass system at incident energies $E_{\text{c.m.}} = 19.0, 21.5, 22.4, 24.0, 24.8, 25.6, 26.5, 28.0, 28.9, 29.7$ MeV, respectively.

TABLE 1.

$E_{\text{c.m.}}$	V_0	r_{0V}	a_V	W_0	r_{0W}	a_W
19.0	16.9	1.35	0.509	4.5	1.43	0.231
21.5	14.9	1.35	0.534	5.5	1.41	0.444
22.4	19.2	1.35	0.509	7.2	1.41	0.231
24.0	18.7	1.35	0.509	6.5	1.34	0.279
24.8	15.7	1.35	0.509	7.2	1.33	0.377
25.6	16.3	1.35	0.545	7.0	1.41	0.324
26.5	20.8	1.35	0.463	9.8	1.34	0.253
28.0	20.0	1.35	0.509	16.3	1.37	0.276
28.9	20.1	1.35	0.509	20.2	1.35	0.295
29.7	20.9	1.35	0.509	13.7	1.38	0.259

Best-fit parameters from the optical-model fit of ^{13}C (^{16}O , ^{16}O) ^{13}C .

The optical-model calculation was performed using the code PTOLEMY³⁾. The best-fit values of the optical-model parameters are given in Table 1. Fig. 1 shows optical-model fits and experimental values at incident energies $E_{\text{c.m.}} = 19.0, 21.5$ and 22.4 MeV. In Fig. 2 we show the same for $E_{\text{c.m.}} = 25.6, 28.0$ and 29.7 MeV. The obtained fits are satisfactory. The peaks of the experimental data and the calculated fit values differed only at large angles and at higher energies. In Fig. 3 the imaginary and real potential depths V_0 and W_0 from Table 1 and their ratio W_0/V_0 are plotted vs. the centre-of-mass energy. The overall trend of the potential depths shown in the figure is to increase with increasing energy. The obtained best-fit line ($V_0 = 0.45 E_{\text{c.m.}} + 6.77$) and the optical-model fit values of the real potential are consistent with what was obtained by other authors, as shown in Table 2 for similar systems. The imaginary potential depths increase gradually from 4.5 MeV to 7.0 MeV as $E_{\text{c.m.}}$ increases from 19.0 MeV to 25.6 MeV, following a best-fit line ($W_0 = 0.42 E_{\text{c.m.}} - 3.39$). After that W_0 shoots up to 20.2 MeV at $E_{\text{c.m.}} = 28.9$ MeV. It falls down again to 13.7 MeV at $E_{\text{c.m.}} = 29.7$

MeV. The reason for the peaking of W_0 is not obvious, so the best-fit line for the W_0 's was drawn only up to $E_{c.m.} = 25.6$ MeV (solid part) and then extrapolated up to $E_{c.m.} = 29.7$ MeV (dashed part) for comparison. The best-fit line for the W_0/V_0 vs. $E_{c.m.}$ was plotted in a similar way. The obtained values of W_0/V_0 up to $E_{c.m.} = 25.6$ MeV are also consistent with those of other authors⁷⁾.

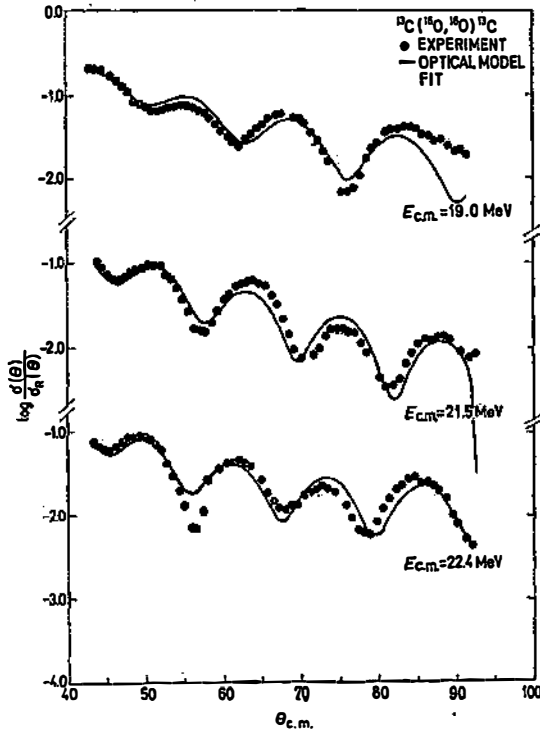


Fig. 1. Ratio of elastic to Rutherford scattering cross section vs. the centre-of-mass angle at incident energies $E_{c.m.} = 19.0, 21.5$ and 22.4 MeV, respectively.

TABLE 2.

System	$E_{c.m.}$	V_0	r_{0V}	a_V	W_0	r_{0W}	a_W	Ref.
$^{16}\text{O} + ^{16}\text{O}$	12~40	17	1.35	0.49	$0.4 + 0.1 E_{c.m.}$	1.35	0.49	4
$^{12}\text{C} + ^{12}\text{C}$		14	1.35	0.35	$0.4 + 0.1 E_{c.m.}$	1.40	0.35	5
$^{14}\text{N} + ^{14}\text{N}$		15	1.35	0.49	$0.4 + 0.125 E_{c.m.}$	1.35	0.49	5
$^{16}\text{O} + ^{14,15}\text{N}$	10~28	$7.5 + 0.5 E_{c.m.}$	1.35	0.49	$0.4 + 0.15 E_{c.m.}$	1.35	0.49	6
$^{16}\text{O} + ^{16}\text{O}$	12~40	$12 + 0.25 E_{c.m.}$	1.35	0.49	$0.4 + 0.1 E_{c.m.}$	1.35	0.49	6
$^{16}\text{O} + ^{18}\text{O}$	12~32	$12 + 0.25 E_{c.m.}$	1.35	0.49	$0.4 + 0.1 E_{c.m.}$	1.35	0.49	6
$^{16}\text{O} + ^{13}\text{C}$	19~30	$6.77 + 0.45 E_{c.m.}$	1.35	0.51*	$-3.39 + 0.42 E_{c.m.}$	1.38*	0.296*	Present work

* average values

Optical-model potential parameters in the neighbouring systems at similar energies.

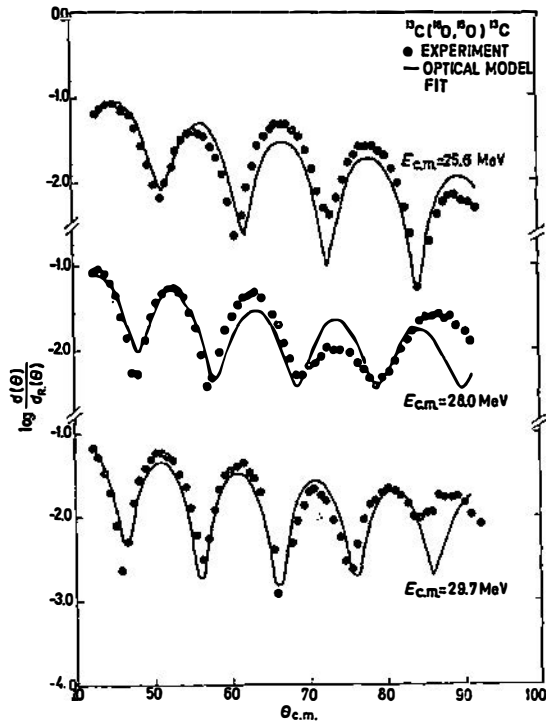


Fig. 2. Ratio of elastic to Rutherford scattering cross section vs. the centre-of-mass angle at incident energies $E_{c.m.} = 25.6, 28.0$ and 29.7 MeV, respectively.

TABLE 3.

$E_{c.m.}$	V_0	r_{0V}	a_V	W_0	r_{0W}	a_W
28.0	45.5	1.35	0.374	8.3	1.41	0.086
29.7	15.0	1.35	0.686	3.5	1.49	0.217

Alternative scattering-equivalent potential parameters for $^{13}\text{C} (^{16}\text{O}, ^{16}\text{O}) ^{13}\text{C}$.

3. Discussion

The values of the parameters given in Table 1 are in no way unique. Equally good fits can be obtained with other combinations of parameters. Two such sets for two energies, respectively, are given in Table 3. However, for $E_{c.m.} = 28.0$ MeV, the value required for the imaginary diffuseness is extremely small and appears to be unacceptable. Similarly for $E_{c.m.} = 29.7$ MeV, the values of V_0 and W_0 are low. Such low values are off the general trend that the potential depths increase with incident energies⁶⁾.

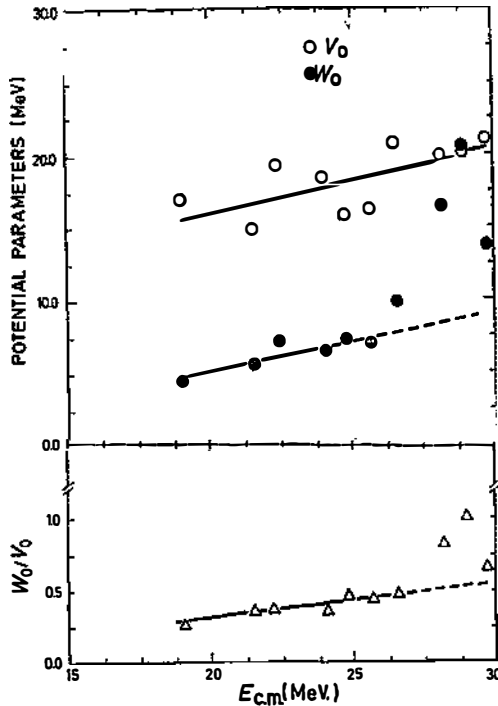


Fig. 3. Variation of the best-fit values of imaginary and real potential depths and their ratios vs. the centre-of-mass energy.

It is well known⁸⁾ that scattering-equivalent optical-model real potentials of Woods-Saxon type intersect at radial distances near their tails, which is a consequence of the fact that the heavy-ion interaction takes place at the nuclear surface. As shown in Fig. 4, in our case this is around $r = 7.6 \pm 0.2$ fm.

One can attempt to estimate the value of V_0 and a_{0V} from the liquid-drop model^{8,9)} using the expression

$$V_0 = 17 (A_P^{2/3} + A_T^{2/3} - (A_P + A_T)^{2/3}),$$

$$a_{0V} = \frac{0.356 (R_P + R_T) (A_P^{2/3} + A_T^{2/3} + (A_P + A_T)^{2/3})}{R_P R_T},$$

where the half-density radii are given by¹⁰⁾

$$R_j = 1.128 A_j^{1/3} (1 - 0.786 A_j^{-2/3}),$$

$j = P, T$ (P and T are the projectile and the target, respectively).

The above expression, although energy independent, gives satisfactory values of V_0 and a_{0V} for various systems⁹⁾. For the $^{13}\text{C}(^{16}\text{O}, ^{16}\text{O})^{13}\text{C}$ system, the

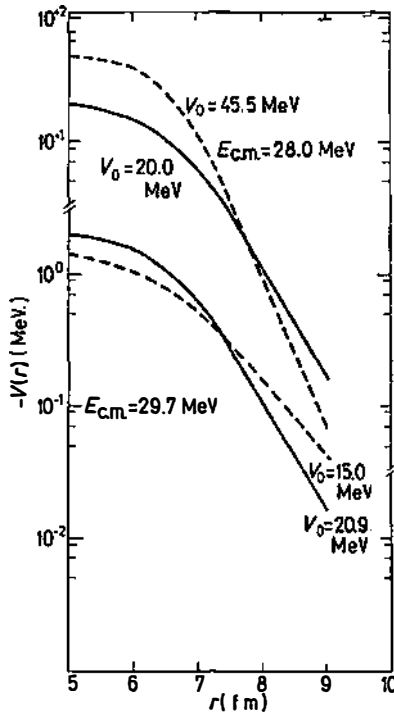


Fig. 4. Comparison between the tail region of the real parts of various scattering-equivalent Wood-Saxon potentials.

values of V_o and a_{oV} are 41.4 MeV and 0.798 fm, respectively. A scattering-equivalent value of $V_o = 45.6$ MeV requires $a_{oV} = 0.374$ fm (Table 3) and an unrealistic value of a_{oW} . So it is unacceptable and shows that the applicability of the above formulae for $A_p, A_T < 20$ is not obvious.

4. Conclusion

Summarizing, in the present study we have analyzed angular distributions of the $^{13}\text{C}(^{16}\text{O}, ^{16}\text{O})^{13}\text{C}$ reaction within the angular range $\Theta_{c.m.} = 42^\circ - 92^\circ$ and in the energy range $E_{c.m.} = 19 - 30$ MeV using the six-parameter optical-model code PTOLEMY. It leaves us with a set of optical-model parameters which give a satisfactory fit to the experimental data of the above scattering system. The values of potential depths V_o and W_o of this set of parameters increase with energy. The imaginary potential depths W_o shows a peculiar behaviour after $E_{c.m.} = 25.6$ MeV without any obvious reason. The best values of V_o and those of W_o (up to $E_{c.m.} = 25.6$ MeV) are consistent with observations by other authors⁴⁻⁶. The radial distance at which the scattering-equivalent potentials intersect has been found to be around 7.6 fm.

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ANALIZA RASPRŠENJA ^{13}C (^{16}O , ^{16}O) ^{13}C U PODRUČJU
 $E_{\text{c.m.}} = 19\text{--}30\text{ MeV}$ POMOĆU OPTIČKOG MODELA

SANTANU DATTA, NIKOLA CINDRO

Institut »Ruder Bošković«, 41000 Zagreb

RICHARD M. FREEMAN, CRISTIAN BECK, FLORENT HAAS i ABDELATIF MORSAD

Centre de Recherches Nucléaires and Université Louis Pasteur, 67200 Strasbourg, France

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Originalni znanstveni rad

Provedena je analiza raspršenja ^{13}C (^{16}O , ^{16}O) ^{13}C pomoću optičkog modela sa šest parametara. Dobivene vrijednosti dubina realnog i imaginarnog potencijala rastu s energijom u skladu s rezultatima drugih autora. Ekvivalentni potencijali za promatrani sustav se presijecaju na radijalnoj udaljenosti 7,6 fm.