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LEPTONIC SCATTERING FROM HYDROGEN ATOM — A FIELD THEORETIC APPROACH

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A comparative study of the same generation lepton and anti-lepton scattering from hydrogen atom in a relativistic Lorentz gauge is done. The ratio of the two differential cross-sections ($\sigma_e^-(\Theta)$, $\sigma_e^+(\Theta)$) of electron and positron is found to be a function of scattering angle and energy of the projectile. However in cases of muons the ratio is unity.

1. Introduction

The elastic scattering of electron from hydrogen atom is the simplest collision problem and is studied by numerous model approximations[1,2] to justify the models as well as to explain the various scattering phenomena. But the situation is yet not conclusive. We apply the technique of Feynman diagrams, working in Lorentz gauge, to study the elastic scattering of leptons from bound targets. The formalism is valid for all energies, but we present results for low energies where experimental data is available. This method has presently successfully predicted the cross-sections for charge transfer phenomena[3,4] and photon scattering from bound target[5]. The philosophy behind the approach is to take the S-matrix expansion for the free interacting particles. The S-matrix is then operated upon by the initial and final physical states. The bound wave function[6] which is a solution of Dirac equation in Lorentz gauge, takes into account exchange of Coulomb photons of all orders[7]. The effect of binding on the amplitude comes through these bound state vectors. There is a scope to take the free projectile as a distorted wave in the Coulomb field of the bound target.

2. *Mathematical formalism*

The amplitude for the process is determined from the sum of the three Feyman diagrams with one virtual photon exchange in each case (Fig. 1, Fig. 2). At low energies proton form factor is not so important. With low target momentum the amplitude

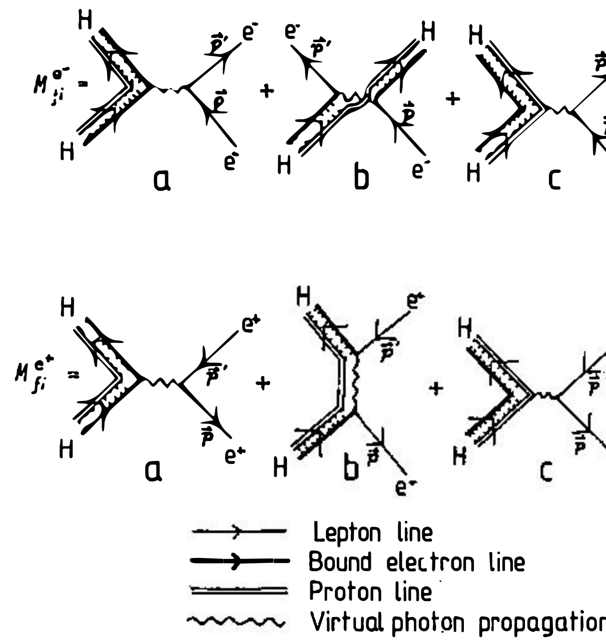

Fig. 1. Feynman diagram for electron-hydrogen elastic scattering.

(a) Electron-bound electron direct interaction.

(b) Electron-bound electron exchange interaction.

(c) Electron-proton interaction.

Fig. 2. The same as in Fig. 1 for positron-hydrogen scattering.

is mass independent. So contribution towards the amplitude from diagrams 1 (a) and 1 (c) ... or 2 (a) and 2 (c) are the same. Similar is the case for all leptons and anti-leptons. Interferences between the diagrams (a) and (b), respectively, with (c) for both the cases vanish identically because of vanishing trace between odd number of γ-matrices involved. In case of muons, scattering amplitude of μ^+ from hydrogen atoms are the sum of the two diagrams of the type (a) and (c) of Figs. 1 and 2, respectively. Eventually the cross sections are identical as the interference term does not contribute. However, in usual quantum mechanical calculations in Coulomb gauge, interference between electron-bound electron and electron-nucleus terms does contribute. The disappearance and appearance of interference terms in Lorentz gauge and Coulomb gauge, respectively, may be attributed to the gauge algebra. With higher order diagram it is worth seeing the contribution towards such interferences. In the case of positron the exchange term corresponding to diagram 2 (b) arises from annihilation. Contribution of the annihilation diagram is very low in the non-relativistic free-free cases. In the case of bound electron and free projectile interaction, the annihilation diagram 2 (b) is found to modify the cross-section through interference with direct term where a product of exchange integral I_E over bound wave function at the two vertices, and direct integral I_D over bound wave function at one vertex are involved. Eventually, the direct term, with Rutherford type of scattering cross-section damped by the direct integral I_D over bound state wave function in the space of transfered momentum, dominates the scene of scattering in the high energy region. In the region of few hundred electron volts, however, the elastic differential cross-sections of hydrogen target induced by electrons and positrons in the direction Θ_L are related as follows:

$$\sigma^- (\Theta_L) = \sigma^+ (\Theta_L) \, S (\Theta_L, p) \tag{1}$$

where

$$S (\Theta_L, p) = \left| \frac{M_{fi}^-}{M_{fi}^+} \right|^2 . \tag{2}$$

Amplitude for elastic scattering from electron (positron) $M_{fi}^{-(+)}$ is given by

$$M_{fi}^{-(+)} = S_a^{-(+)} - S_b^{-(+)} - S_c^{-(+)}. \tag{3}$$

$S_a^{-(+)}$, $S_b^{-(+)}$ and $S_c^{-(+)}$ correspond to the contribution of the diagrams a, b, c, of Fig. 1 (Fig 2), respectively.

$$S_a^{-(+)} = - \, ie^2 \frac{m^2}{\varepsilon_{1s} E_p} \, \delta (E_i - E_f) \, T_D^{-(+)} \, I_D^{-(+)}$$

$$S_b^{-(+)} = - \, ie^2 \frac{m^2}{\varepsilon_{1s} E_p} \, \delta (E_i - E_f) \, T_E^{-(+)} \, I_E^{-(+)}$$

$$S_c^{-(+)} = - \, ie^2 \frac{mM}{M E_p} \, \delta (E_i - E_f) \, T_D^{-(+)} \, I_D^{-(+)}$$

M = proton mass

m = lepton mass

Proton recoil is neglected.

ε_{1s} = Energy of the target electron = m_e + *binding energy* $\approx m_e$

$$E_p = \textit{projectile energy} = m + \frac{p^2}{2m} \approx m, \quad \text{for } p \ll m$$

$$T_D^- = \{\bar{u} (p') \, \gamma_\mu \, u (p)\} \{\bar{u} (0) \, \Gamma_\mu \, u (0)\}$$

$$T_D^+ = \{\bar{v}(-p')\,\gamma_\mu\, v(-p)\}\,\{\bar{u}(0)\,\Gamma_\mu\, u(0)\}$$
$$T_E^- = \{\bar{u}(0)\,R^+\,\gamma_\mu\, u(p)\}\,\{\bar{u}(p')\,\gamma_\mu\, Ru(0)\}$$
$$T_E^+ = \{\bar{u}(0)\,R^+\,\gamma_\mu\, v(-p)\}\,\{\bar{v}(-p')\,\gamma_\mu\, Ru(0)\}$$
$$T_D^- = \{\bar{U}(0)\,\gamma_\mu\, U(0)\}\,\{\bar{u}(p')\,\gamma_\mu\, u(p)\}$$
$$T_D^+ = \{\bar{U}(0)\,\gamma_\mu\, U(0)\}\,\{\bar{v}(-p')\,\gamma_\mu\, v(-p)\}.$$

u and v are the Dirac spinors for lepton and antilepton, respectively. $\varphi_{1s}(r)\,Ru(0)$ is the bound electron wave function and the bound particle spinor at zero momentum is in accordance with Ref. 6. U is the Dirac spinor for proton taken in static approximation.

$$\Gamma_\mu = R^+\,\gamma_\mu\, R$$

$$R = 1 + a\,\gamma_4 \frac{\gamma \cdot \vec{r}}{|r|}$$

$$I_D = \frac{1}{q^2} \int e^{-i(\vec{p}-\vec{p}')\cdot y}\,\varphi_{1s}(y)\,d^3y, \quad I_D = I_D^- = I_D^+$$

$$q^2 = (\vec{p} - \vec{p}')^2 = -4P^2 \sin^2\Theta/2, \; \Theta = \frac{\pi}{2} - \Theta_L$$

$\vec{p}$ and $\vec{p}'$ are the momenta of the projectile lepton before and after interaction.

$$I_E^{-(+)} = \int \frac{\varphi_{1s}(x)\,\varphi_{1s}(y)}{|x \mp y|} \exp\{-i\,\omega\,|x \mp y| - i\,(px \mp p'y)\}\,d^3x\,d^3y$$
$$\omega = p_0 - p_0' \quad \omega/c \ll 1, \; \exp(-i\,\omega\,|x \mp y|) \approx 1.$$

The different traces involved are listed below. These are computed taking only 1st term of R, the 2nd term gives only a correction of order a^2 to the cross-section.

$$T_D^{-*}T_D^- = 4\left(1 + \frac{p^2}{m^2}\cos^2\Theta/2\right)$$

$$T_D^{+*}\,T_D^+ = T_D^{-*}\,T_D^- = T_{D'}^{-*}\,T_{D'}^-$$

$$T_D^{-*}\,T_D^- + T_E^{-*}\,T_D^- = 4\left(1 + \frac{p^2}{m^2}\sin^2\Theta/2\right).$$

$$T_D^{+*}\,T_E^+ + T_E^{+*}\,T_D^* = -4\left[3 + \frac{p^2}{m^2}(1 + \sin^2\Theta/2)\right]$$

$$T_E^{-*}\,T_E^- = T_D^{-*}\,T_E^-; \quad T_E^{+*}\,T_E^+ = 4\left[3 + \frac{p^2}{m^2}(3 - \cos\Theta)\right]$$

where

$$\Theta = \Theta_L - \pi/2.$$

With this we find

$$S_a^{-*} S_a^- = S_c^{-*} S_c^- = S_a^{+*} S_a^+ = S_c^{+*} S_c^+.$$

Finally for $p \ll m$

$$S(\Theta_L, p) = \frac{1 + (1/2)\, I^- + (1/2)\,(I^-)^2}{1 - (3/2)\, I^+ + (3/2)\, I^{+2}}$$

where

$$I^{-(+)} = I_E^{-(+)}/I_D =$$

$$= 4\,(1 + p^2 \sin^2 \Theta/2)\,[X\{2X^4 + X^2\,(1 + p^4) - 2p^2\}\,/\,(1 + p^2)^2 + 2\,(X^2 + p^2) \sin^{-1}\{X/(1 + X^2)^{1/2}\}]\,/\,\{X\,(1 + p^2)\}^3$$

where

$$x = p \sin \Theta/2 \text{ for electron}$$
$$= p \cos \Theta/2 \text{ for positron.}$$

3. *Results and discussion*

The ratio $S(\Theta_L, p)$ is always greater than one. This shows the scattering of electron off hydrogen target is favoured more than that of the positron. But hydrogen atom does not show this difference in behaviour for the second generation leptons (μ^+, μ^-) for which the ratio is always one. This may be attributed to the fact that the target and projectile leptons are of different generation. Other existing results for e^- and e^+ also yield the ratio $S(\Theta_L, p)$ to be greater than one. Our result is compared with result of Ref. 8 and shown in Table 1. In principle present calculation compares well with the existing results. However there is a scope to improve our result by including distortion of the projectile wave function.

TABLE 1.

Energy	100 eV					200 eV		300 eV		400 eV	
Angle (deg.)	FT	UEBS	EBS	MO	W	FT	UEBS	FT	UBES	FT	UEBS
5	1.67	2.67	3.18	3.05	1.24	2.19	1.87	2.45	1.57	2.56	1.33
10	2.51	2.28	2.41	2.69	1.406	2.89	1.6	2.83	1.31	2.69	1.21
20	1.9	3.2	2.11	2.67	1.66	2.50	1.36	2.74	1.23	1.94	1.17
30	2.25	1.69	2.05	1.88	1.77	1.6	1.33	1.51	1.24	1.41	1.18
40	1.59	1.64	1.6	1.75	1.82	1.31	1.36	1.22	1.25	1.18	1.19
50	1.29	1.67	1.59	1.71	1.82	1.14	1.39	1.20	1.25	1.08	1.18
60	1.15	1.72	1.74	1.71	1.81	1.06	1.39	1.04	1.25	1.03	1.18
70	1.09	1.76	1.87	1.71	1.79	1.03	1.39	1.02	1.26	1.01	1.17
80	1.06	1.70	1.52	1.70	1.76	1.02	1.39	1.0	1.24	1.0	1.16

FT = Present field theoretic method; EBS = Eikonal Born series[9] UEBS = Uniterised EBS[10]; MO = Molecular orbital[11]; W = Wallace amplitude[11].

Ratio of cross sections $S(\Theta_L, p)$ for scattering of electron and positron from hydrogen atom.

It is inspiring to note that this compact yet simple formalism gives good description of the leptonic elastic scattering processes from bound targets. Extension of quantum field theory in Lorentz gauge to atomic physics has its own intrinsic appeal. It reveals effect of charge conjugation on lepton-atom elastic scattering in a most elegant form.

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RASPRŠENJE LEPTONA NA VODIKOVOM ATOMU — PRISTUP TEORIJE POLJA

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Načinjena je usporedna analiza raspršenja iste generacije leptona na vodikovom atomu uz Lorentzov uvjet. Nađeno je da omjer diferencijalnih udarnih presjeka ($\sigma_e^-(\Theta)$, $\sigma_e^+(\Theta)$) elektrona i pozitrona ovisi o kutu i energiji projektila. U slučaju muona taj omjer jednak je jedinici.