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$U_\pi^B(6) \times U_\nu^B(6) \times U_\pi^F(12) \times U_\nu^F(12)$ AND $U_\pi^B(6) \times U_\nu^B(6) \times U_\pi^F(12) \times U_\nu^F(10)$ DYNAMICAL SYMMETRIES IN THE 28—50 VALENCE SHELL

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A symmetry/supersymmetry scheme for odd-odd nuclei, with proton- and neutron-bosons, odd quasiproton in $j_\pi = 1/2, 3/2, 5/2$ and odd quasineutron in $j_\nu = 1/2, 3/2, 5/2$ or $j_\nu = 9/2$ orbitals is developed. Illustrative calculations for the positive- and negative-parity levels in ^{76}As are presented.

1. Introduction

The interacting boson-fermion model represents a description of the states in odd-even nuclei in terms of a system of interacting bosons and fermions. In this algebraic theory the symmetry considerations play a decisive role, as well as possible extensions to supersymmetry[2–8]. The last approach is based on the idea that a larger class of symmetry transformations may exist, involving boson-fermion operations which are called supersymmetry transformations. The traditional methods of Lie algebras and group theory are thus generalized to apply symmetry considerations to mixed systems of bosons and fermions. First, the theoretical applications have centered on the elementary particle and gravitational physics but recently the IBFM has been treated as a testing ground for testing supersymmetries. In a first stage, isomorphisms between the fermion and boson groups have been exploited, giving rise to the spinor groups, which can be embedded into larger superalgebra. It enables classification of both even-even and odd-even nuclei in single supermultiplet.

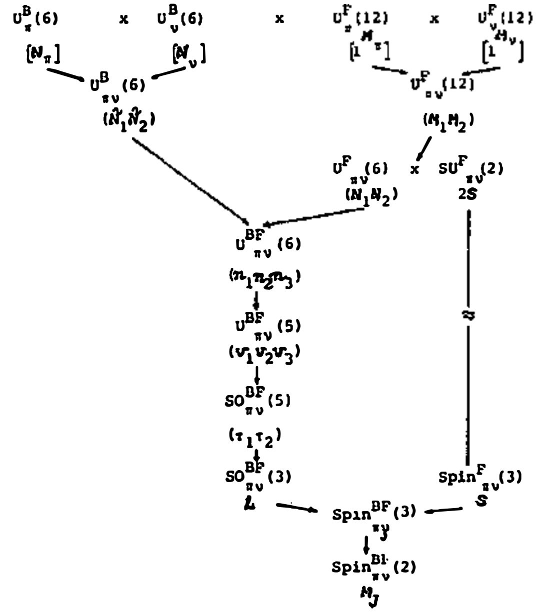
Recently, the boson-fermion approach was extended to odd-odd nuclei[12–24]. A new model for odd-odd nuclei was introduced, referred to as interacting boson-fermion-fermion model (IBFFM), and the related dynamical symmetries and supersymmetries have been developed, encompassing odd-odd in addition to even-even and odd-even nuclei.

In Ref. 24 the boson-fermion-fermion dynamical symmetry associated with U (5) boson core is introduced for odd odd nuclei with an odd proton and neutron in $j = 3/2$ configurations. An algorithm was constructed for the subsequent boson--fermion-fermion group chain and the corresponding energy formula was given and exemplified for ^{62}Cu in the case of the associated supersymmetry $U(6/4_{\pi} \oplus 4_{\nu})$.

Here we extend this symmetry approach to a multi-configuration case corresponding to 28—50 shell: for positive-parity levels protons and neutrons are considered in the orbitals $j_{\pi} = 1/2, 3/2, 5/2$, $j_{\nu} = 1/2, 3/2, 5/2$ and for negative-parity in $j_{\pi} = 1/2, 3/2, 5/2$, $j_{\nu} = 9/2$. This corresponds to the physical situation in nuclei with less than half-filled $Z = 28$—50 and more than half-filled $N = 28$—50 shell. In this region there are nuclei with U (5) boson core.

2. *Boson-fermion-fermion dynamical symmetry* $U^{B}_{\pi}(6) \times U^{B}_{\nu}(6) \times U^{F}_{\pi}(12) \times U^{F}_{\nu}(12)$ *and supersymmetry* U (12/24)

Let us first consider the case with one proton quasiparticle in $j_{\pi} = 1/2, 3/2, 5/2$ orbitals and one neutron quasiparticle in $j_{\nu} = 1/2, 3/2, 5/2$ orbitals, coupled to the $U^{B}(5)$-type boson core which consists of proton- and neutron-bosons. The corresponding group chain is:



(1)

In the group decomposition (1) the relevant group labels are also presented. The symbols N_{π}, N_{ν}, M_{π} and M_{ν} are the number of proton bosons, neutron bosons, protons and neutrons, respectively. This and remaining group labels are sufficient to designate corresponding irreps.

In coupling $U^B_\pi(6)$ (U (6) group for proton-bosons) and $U^B_\nu(6)$ (U (6) group for neutron-bosons) we restrict our considerations to the symmetric irreducible representations (irreps) (for example, in the case of ^{76}As it is (50) irreps), since the other irreps correspond to the mixed symmetry states, which are higher-lying. On the other hand, by coupling fermion irreps $[1^{M_\pi=1}]$ and $[1^{M_\nu=1}]$ into $U^F_{\pi\nu}(12)$ there appear symmetric and antisymmetric irreps (20) and (11), which are further decomposed into the irreps of $U^F_{\pi\nu}(6) \times SU^F_{\pi\nu}(2)$. Further decomposition is carried out according to the standard group-theoretical methods[28-30].

The Hamiltonian corresponding to the group chain (1) reads:

$$H = A_1 \cdot C_2\,[U^F_{\pi\nu}(12)] + A_2 \cdot C_2\,[U^F_{\pi\nu}(6)] + A_3 \cdot C_2\,[U^{BF}_{\pi\nu}(6)] + A_4 \cdot C_1\,[U^{BF}_{\pi\nu}(5)] + A_5 \cdot C_2\,[U^{BF}_{\pi\nu}(5)] + A_6 \cdot C_2\,[SO^{BF}_{\pi\nu}(5)] + A_7 \cdot C_2\,[Spin^F_{\pi\nu}(3)] + A_8 \cdot C_2\,[SO^{BF}_{\pi\nu}(3)] + A_9 \cdot C_2\,[Spin^{BF}_{\pi\nu}(3)], \qquad (2)$$

where C_1 and C_2 denote the first-and second-order Casimir operators and A_i are the parameters. Here, we have neglected terms which contribute only to the binding energies.

The basis states are characterized by labels defined in (1)

$$|N_\pi N_\nu M_\pi M_\nu (\tilde{N}_1 \tilde{N}_2)(M_1 M_2)(N_1 N_2)(n_1 n_2 n_3)(v_1 v_2 v_3)(\tau_1 \tau_2)\, a\, LSJM_J\rangle. \qquad (3)$$

Additional label a is introduced in order to fully specify decomposition of irreps of $O^{BF}(5)$ into the irreps of $O^{BF}(3)$.

For the low-lying states the number of parameters in (2) can be reduced: if the parameters associated with Casimir operator of a certain group are sufficiently large and negative (positive) we can restrict the consideration to the irreps with highest (lowest) values of the Casimir operator. In this way we can, for instance, fix the contributions from $C_2\,[U^F_{\pi\nu}(6)]$ and $C_2\,[U^{BF}_{\pi\nu}(6)]$. For ^{76}As we just made it because we had observed that quantum numbers of these irreps have negligible effect at reconstruction energy spectrum. In the group $U^F_{\pi\nu}(6)$ there appear irreps $(N_1 N_2) = (20)$ and (11), in which $C_2\,[U^F_{\pi\nu}(6)]$ has the values $N_1(N_1+5) + N_2(N_2+3) = 14$ and 10, respectively. (These irreps appear in the decomposition

$$(20)_{U^F_{\pi\nu}(12)} \supset \{(20)\times(2) + (11)\times(0)\}_{U^F_{\pi\nu}(6)\times SU^F_{\pi\nu}(2)}$$

and

$$(11)_{U^F_{\pi\nu}(12)} \supset \{(11)\times(2) + (20)\times(0)\}_{U^F_{\pi\nu}(6)\times SU^F_{\pi\nu}(2)}.)$$

In the group $U^{BF}_{\pi\nu}(6)$ there are irresp $(n_1 n_2 n_3) = (700)$, (610), (520) and (511) with the corresponding eigenvalues of C_2 are $n_1(n_1+5) + n_3(n_2+3) + n_3(n_3+1) = 84, 70, 60$ and 56, respectively. Therefore, we can restrict the consideration for the low-lying levels to the irreps (20) of $U^F_{\pi\nu}(6)$ and (70) of $U^{BF}_{\pi\nu}(6)$, if the parameters A_2 and A_3 are sufficiently large and negative. In this way, the corresponding terms in the eigenvalue contribute only to the binding energy and can be omitted from the energy formula.

Thus, the energy formula corresponding to the group chain (1) reads:

$$E = A_1 [M_1 (M_1 + 11) + M_2 (M_2 + 9)] + A_4 (v_1 + v_2 + v_3) + \\ + A_5 [v_1 (v_1 + 4) + v_2 (v_2 + 2) + v_3^2] + A_6 [\tau_1 (\tau_1 + 3) + \\ + \tau_2 (\tau_2 + 1)] + A_7 S (S + 1) + A_8 L (L + 1) + A_9 J (J + 1). \qquad (4)$$

Employing this energy formula we have fitted the positive-parity levels of ${}^{76}_{33}As_{43}$, which has five protons and seven neutron holes in 28—50 proton and neutron shells, respectively.

The quality of fits is quantified by the values of Φ and σ

$$\Phi = \{\sum_i \mid E_i^{exp} - E_i^{the} \mid / \sum_i E_i^{exp}\} (\%) \qquad (5)$$

$$\sigma = [\sum_i (E_i^{exp} - E_i^{the})^2 / (n - k)]^{1/2} (\text{keV}) \qquad (6)$$

where n is the number of levels included in the fit, k is the number of parameters, and $n - k$ is the number of degrees of freedom.

In Fig. 1 we present the theoretical spectrum corresponding to the parametrization $A_1 = 9.9$, $A_4 = -298$, $A_5 = 55.3$, $A_6 = 11.5$, $A_7 = -144$, $A_8 = 5.8$, $A_9 = 41.2$ (all in keV), in comparison to experiment[32]. The corresponding quality of fit is given by $\Phi = 12\%$, $\sigma = 97$ keV. Each level is labelled by the quantum numbers $J (v_1 v_2 v_3) (\tau_1 \tau_2) LS$. We note that by fitting to the experimental positive parity levels we obtain an additional 0^+ level at 0.285 MeV. On the other

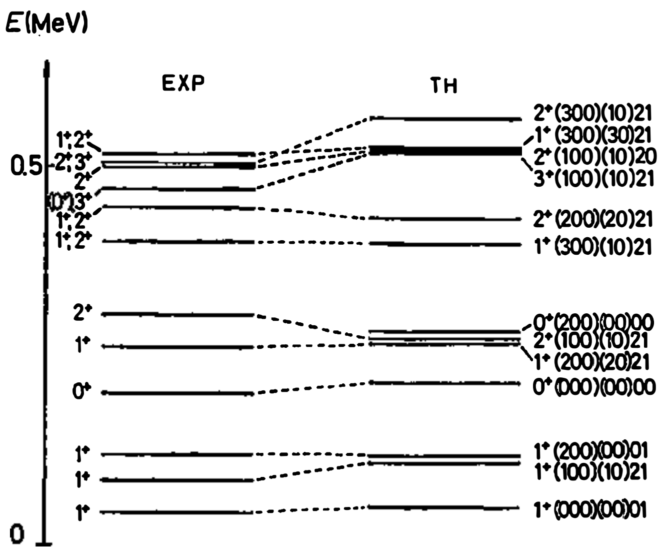


Fig. 1. Theoretical positive-parity spectrum of the $U_\pi^B(6) \times U_\nu^B(6) \times U_\pi^F(12) \times U_\nu^F(12)$ scheme in comparison to the available positive-parity experimental levels of ${}^{76}As$.

hand, at 0.2926 MeV there is an experimental level with unknown spin and parity. This state could be an experimental counterpart of the theoretical level at 0.285 MeV.

Supersymmetry appears by embedding the chain (1) into the direct-product group

$$U\ (6/12) \times U\ (6/12) \subset U\ (12/24).$$

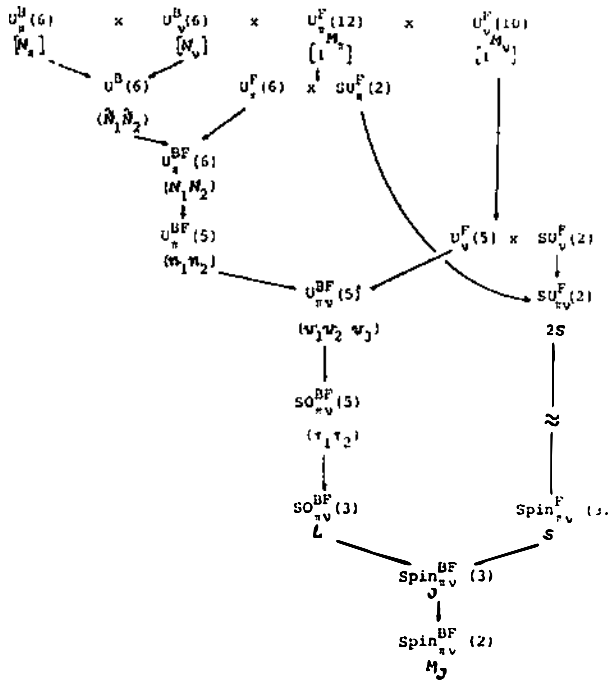
In this way the supermultiplets encompass quartets of nuclei and their properties are interrelated. Thus, the energy formula (3) can be used without reference to supersymmetry, i. e. as a dynamical boson-fermion symmetry for an odd-odd nucleus itself, or in a supersymmetric context, predicting the properties of the odd-odd nucleus from those of the other three nuclei in the quartet.

In our case, ^{76}As together with ^{75}As, ^{76}Se, ^{77}Se make quartel of nuclei which belong to supermultiplet $|7\}$ of supergroup U (12/24).

3. *Boson-fermion-fermion dynamical symmetry* $U^B_\pi(6) \times U^B_\nu(6) \times U^F_\pi(12) \times U^F_\nu(10)$

Let us now consider the case with one proton quasiparticle in $j_\pi = 1/2, 3/2, 5/2$ orbitals and one neutron quasiparticle in $j_\nu = 9/2$ orbital, coupled to the $U^B(5)$-type boson core which consists of proton- and neutron-bosons.

The corresponding group chain is:



(7)

The corresponding Hamiltonian reads

$$H = B_1 \cdot C_2 [U_{\pi}^{BF}(6)] + B_2 \cdot C_1 [U_{\pi}^{BF}(5)] + B_3 \cdot C_2 [U_{\pi}^{BF}(5)] + \\ + B_4 \cdot C_1 [U_{\pi\nu}^{BF}(5)] + B_5 \cdot C_2 [U_{\pi\nu}^{BF}(5)] + B_6 \cdot C_2 [SO_{\pi\nu}^{BF}(5)] + \\ + B_7 \cdot C_2 [Spin_{\pi\nu}^{F}(3)] + B_8 \cdot C_2 [SO_{\pi\nu}^{BF}(3)] + \\ + B_9 \cdot C_2 [Spin_{\pi\nu}^{BF}(3)]. \quad (8)$$

The basis states are in this case given by

$$|N_{\pi}N_{\nu}M_{\pi}M_{\nu}(\tilde{N}_1\tilde{N}_2)(N_1N_2)(n_1n_2)(v_1v_2v_3)(\tau_1\tau_2)\,a\,L\,S\,J\,M_J >. \quad (9)$$

Employing similar arguments as used in Sect. 2, because of the same reason, we restrict the considerations to the irreps $S = 0$ of $Spin_{\pi\nu}^{F}(3)$ which implies $L = J$, and the energy formula for description of the low-lying states is given by

$$E = B_1 [N_1 (N_1 + 5) + N_2 (N_2 + 3)] + B_2 (n_1 + n_2) + \\ + B_3 [n_1 (n_1 + 4) + n_2 (n_2 + 2)] + B_4 (v_1 + v_2 + v_3) + \\ + B_5 [v_1 (v_1 + 4) + v_2 (v_2 + 2) + v^2] + B_6 [\tau_1 (\tau_1 + 3) + \\ + \tau_2 (\tau_2 + 1)] + B'_7 J (J + 1) \quad (10)$$

where $B'_7 = B_8 + B_9$.

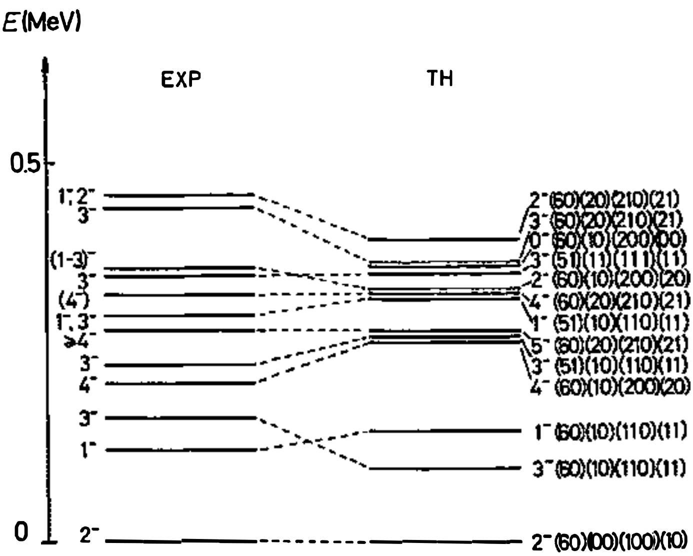


Fig. 2. Theoretical negative-parity spectrum of the $U_{\pi}^{B}(6) \times U_{\nu}^{B}(6) \times U_{\pi}^{F}(12) \times U_{\nu}^{F}(10)$ scheme in comparison to the available negative-parity experimental levels of ^{76}As.

In Fig. 2 we present the theoretical spectrum corresponding to the parametrization $B_1 = -14.6$, $B_2 = -610$, $B_3 = -30.4$, $B_4 = 732$, $B_5 = 52$, $B_6 = 0$, $B'_7 = -5.$ (all in keV), in comparison to experiment[32]. The quality of fit is given by $\Phi = 7.6\%$, $\sigma = 45$ keV. Each level is labelled by the quantum numbers $J \cdot \cdot (N_1 N_2)(n_1 n_2)(v_1 v_2 v_3)(\tau_1 \tau_2)$ from the chain (7).

In analogy to Sect. 2 we obtain the supersymmetry by embedding the chain (7) into the direct-product group $U_\nu(6/10) \times U_\pi(6/12)$.

4. *Conclusion*

We have developed the boson-fermion-fermion symmetry schemes associated with odd-odd nuclei which have less-than half-filled $Z = 28$—50 and more — than half-filled $N = 28$—50 shell, while the boson core is of $U^B(5)$ type. The corresponding energy formulae have been derived and exemplified for positive- and negative-parity levels in ^{76}As. As seen, the experimental levels are rather well reproduced. In this connection we note that number of parameters is rather large, which is an inherent feature of the interacting boson model. However, one should keep in mind that the specific form of the energy formula, in conjunction with allowed combinations of quantum numbers for the levels sizeably restrict the flexibility in the fitting procedure. We have tested the fact that the energy spectra in cases when the boson core is not of $U^B(5)$ type (as, for example, the spectrum of ^{198}Au) the present energy formulas do not enable good fits to experiment. Thus, inherent restrictions of the present energy formulas have the consequence that only certain types of spectra can be well fitted.

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$U^B_\pi(6) \times U^B_\nu(6) \times U^F_\pi(12) \times U^F_\nu(12)$ i $U^B_\pi(6) \times U^B_\nu(6) \times U^F_\pi(12) \times$ $\times$ $U^F_\nu(10)$ DINAMIČKE SIMETRIJE U VALENTNOJ LJUSCI 28—50

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Originalan znanstveni rad

Razvijena je simetrijska/supersimetrijska shema za neparno-neparne jezgre s protonskim i neutronskim bozonima, i s neparnim kvaziprotonima u orbitalama $j_\pi =$ $= 1/2, 3/2, 5/2$ i neparnim kvazineutronima u orbitalama $j_\nu = 1/2, 3/2, 5/2$ ili $j_\nu = 9/2$. Kao ilustracija, napravljen je proračun nivoa pozitivnog i negativnog pariteta jezgre ^{76}As.