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SELFINTERACTION AND ITS EFFECTS IN THE NEW DIRAC FIELD THEORY*

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Selfinteraction of an electron (positron) in the new Dirac field theory is analysed. It is shown that this interaction leads to consistent results, both in mathematical and physical sense. Only the Dirac field without external sources is considered. The results strenghten the new approach to the electron-positron theory and point out the error which has been done in the history of the quantum field theory in a new light.

1. Introduction

In the article concerning self-energy and stability of the classical electron[1] Rohrlich writes: *We conclude that, after correction of the definitions of energy and momentum of the Coulomb field, the classical electron theory exhibits exactly the same structure as quantum electrodynamics, both as to the mass problem and as to the stability of charge.* In this article we want to show how this problem of selfinteraction looks like in the new Dirac field theory[2,3] and what are the basic effects. In order to keep the problem as simple as possible we restrict ourself to the Dirac field without external sources. The presence of external sources (fields) are considered in subsequent papers.

The selfinteraction problem in the new theory differs from the conventional quantum theory in the following fundamental aspect: the fields which describe

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objects in the new theory are not quantized. The field variables are not, therefore, operators, physical quantities are not associated by operators either, and, consequently, there are not states, including ground state (vacuum), on which these operators act. The selfinteraction problem in the new theory is completely connected with the fields themselves as classical relativistic fields. Its solution has to be found within this framework.

For the information about the selfinteraction in the electrodynamics one may consult books or articles due to Pauli[4], Dirac[5], Rohrlich[1,6,7] and Schwinger[8].

In § 2 the problem is formulated. The solution of the problem is given in § 3. Conclusions are given in § 4.

2. *Formulation of the selfinteraction problem*

(1) The new Dirac field interacting with the electromagnetic field is described by the Lagrangian density[9,3]

$$\mathscr{L} = \mathscr{L}_D + \mathscr{L}_{em} \tag{2.1}$$

where

$$\mathscr{L}_D = K \{[(-\mathrm{i}\partial_\alpha - eA_\alpha)\, \overline{\Phi}\gamma^\alpha]\, [(\mathrm{i}\partial_\beta - eA_\beta)\, \gamma^\beta \Phi] - \varkappa^2 \overline{\Phi}\Phi\}, \tag{2.2}$$

and

$$\mathscr{L}_{em} = \frac{1}{8\pi}\left(-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + F^2 - G^2\right), \tag{2.3}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{2}\, \varepsilon_{\mu\nu\xi\zeta} (\partial^\xi b^\zeta - \partial^\zeta b^\xi), \tag{2.4}$$

$$G = \partial_\alpha A^\alpha, \qquad F = \partial_\beta b^\beta. \tag{2.5}$$

The Lagrange's variables of the Dirac field are Φ and $\Phi^\dagger$ and of the electromagnetic field A^α and b^α.

We use relativistic notation $x^\alpha = (x^0, x^1, x^2, x^3) = (ct, \vec{x})$, the metric

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1, \qquad g_{\alpha\beta} = 0, \qquad \alpha \neq \beta,$$

the standard representation of the γ-matrices,

$$\gamma^l = \begin{pmatrix} 0 & \sigma_l \\ \sigma_l & 0 \end{pmatrix}, \qquad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^5 = -\mathrm{i}\gamma^0 \gamma^1 \gamma^2 \gamma^3,$$

and units $c = \hbar = 1$. K and $\varkappa$ are constants.

The canonical equations of the Dirac field are[9]

$$
\begin{aligned}
&[(\mathrm{i}\partial_\alpha - e A_\alpha)\gamma^\alpha - \varkappa]\Psi_I = 0,\\
&[(\mathrm{i}\partial_\alpha - e A_\alpha)\gamma^\alpha + \varkappa]\Psi_{II} = 0,\\
&(-\mathrm{i}\partial_\alpha - e A_\alpha)\overline{\Psi}_I\gamma^\alpha - \varkappa\overline{\Psi}_I = 0,\\
&(-\mathrm{i}\partial_\alpha - e A_\alpha)\overline{\Psi}_{II}\gamma^\alpha + \varkappa\overline{\Psi}_{II} = 0,
\end{aligned}
\tag{2.6}
$$

where

$$
\begin{aligned}
&\Psi_I = \frac{1}{\sqrt{2}}\left(\varkappa\Phi + \frac{\mathrm{i}}{K}\Pi_\Phi{}^\dagger\right),\\
&\Psi_{II} = \frac{1}{\sqrt{2}}\left(\varkappa\Phi - \frac{\mathrm{i}}{K}\Pi_\Phi{}^\dagger\right),\\
&\Pi_\Phi{}^\dagger = K[-\mathrm{i}(\mathrm{i}\partial_\beta - e A_\beta)\gamma^\beta\Phi],\\
&\Pi_\Phi = K[-(\mathrm{i}\partial_\beta + e A_\beta)\overline{\Phi}\gamma^\beta\mathrm{i}\gamma^0]
\end{aligned}
\tag{2.7}
$$

and Π_Φ, $\Pi_\Phi{}^\dagger$ are conjugate momenta to Φ, $\Phi^\dagger$.

The Lagrange's equations of the electromagnetic field are

$$\partial_\alpha\partial^\alpha A^\mu = 4\pi j^\mu, \tag{2.8}$$

$$\partial_\alpha\partial^\alpha b^\mu = 0, \tag{2.9}$$

where

$$j^\mu = e(\overline{\Psi}_I\gamma^\mu\Psi_I - \overline{\Psi}_{II}\gamma^\mu\Psi_{II}) \equiv e\overline{\Psi}\tau_+\gamma^\mu\Psi, \tag{2.10}$$

$$\Psi = \begin{pmatrix}\Psi_I\\ \Psi_{II}\end{pmatrix}, \qquad \tau_+ = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{2.11}$$

The canonical equations of the electromagnetic field are

$$
\begin{aligned}
&\partial_\alpha(F^{\alpha\beta} + G g^{\alpha\beta}) = 4\pi j^\beta,\\
&\partial_\alpha(\tilde{F}^{\alpha\beta} + F g^{\alpha\beta}) = 0,\\
&F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha - \frac{1}{2}\varepsilon^{\alpha\beta\xi\zeta}(\partial_\xi b_\zeta - \partial_\zeta b_\xi),\\
&\tilde{F}^{\alpha\beta} = \partial^\alpha b^\beta - \partial^\beta b^\alpha + \frac{1}{2}\varepsilon^{\alpha\beta\xi\zeta}(\partial_\xi A_\zeta - \partial_\zeta A_\xi),\\
&G = \partial_\alpha A^\alpha, \qquad F = \partial_\beta b^\beta.
\end{aligned}
\tag{2.12}
$$

Selecting the solution $b^\mu = 0$, we get the system

$$[(\mathrm{i}\partial_\alpha - e A_\alpha)\gamma^\alpha - \varkappa]\Psi_I = 0,$$
$$[(\mathrm{i}\partial_\alpha - e A)_\alpha \gamma^\alpha + \varkappa]\Psi_{II} = 0, \text{ and h. c.,} \tag{2.13}$$
$$\partial_\alpha \partial^\alpha A^\mu = 4\pi j^\mu.$$

These are the basic equations in the problem we are considering in this article.

The general solution of the last of Eqs. (2.13) is given by

$$A^\mu = A^\mu_h + A^\mu_s \tag{2.14}$$

where A^μ_h is the general solution of the homogeneous equation

$$\partial_\alpha \partial^\alpha A^\mu_h = 0$$

and A^μ_s is a particular solution. We are not interested here in A^μ_h (!). Thus we take $A^\mu_h = 0$. The particular solution is determined by the current density (2.10). Since this current is determined by the Dirac field, the corresponding electromagnetic field belongs to the Dirac field particles and in the Dirac field equation (2.13) A^μ_s describes interaction of the Dirac field with its own electromagnetic field, i. e. selfinteraction. What are the consequencies of this selfinteraction is the problem of selfinteraction in the new Dirac field theory.

The particular solution A^μ_s we take in the form

$$A^\mu_s = \int \frac{j^\mu(\vec{x}', x^0 - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|}\, \mathrm{d}^3 x'.$$

(2) In the next we need scalar and energy-momentum constants of motion of the system.

By making use of (2.13) it follows

$$\partial_\alpha j^\alpha = 0. \tag{2.15}$$

From here we get the scalar constant of motion

$$Q = const_Q \int j^0\, \mathrm{d}^3 x \tag{2.16}$$

or

$$Q = const_Q\, e \int \Psi^\dagger \tau_+ \Psi\, \mathrm{d}^3 x. \tag{2.17}$$

The energy-momentum constant of motion comes from

$$\partial_\beta T_\alpha{}^\beta = 0 \tag{2.18}$$

where

$$T_\alpha^\beta = \partial_\alpha \eta_\delta^\dagger \frac{\partial \mathscr{L}}{\partial (\partial_\beta \eta_\delta^\dagger)} + \frac{\partial \mathscr{L}}{\partial (\partial_\beta \eta_\delta)} \partial_\alpha \eta_\delta - \delta_\alpha^\beta \mathscr{L}. \tag{2.19}$$

For the Lagrangian (2.1) it is

$$T_\alpha^\beta = T_{\alpha D}^\beta + T_{\alpha em}^\beta \tag{2.20}$$

where

$$T_{\alpha D}^\eta = \partial_\alpha \Phi^\dagger \gamma^0 \gamma^\beta \Pi_\Phi{}^\dagger + \Pi_\Phi \gamma^0 \gamma^\beta \partial_\alpha \Phi - \delta_\alpha^\beta \left(\frac{1}{K} \Pi_\Phi \gamma^0 \Pi_\Phi{}^\dagger - K \varkappa^2 \overline{\Phi} \Phi \right) = \tag{2.21}$$

$$= \frac{K}{2i\varkappa} [(\partial_\alpha \overline{\Psi}_I \gamma^\beta \Psi_I - \overline{\Psi}_I \gamma^\beta \partial_\alpha \Psi_I) - (\partial_\alpha \overline{\Psi}_{II} \gamma^\beta \Psi_{II} - \overline{\Psi}_{II} \gamma^\beta \partial_\alpha \Psi_{II}) +$$

$$+ \partial_\alpha (\overline{\Psi}_{II} \gamma^\beta \Psi_I - \overline{\Psi}_I \gamma^\beta \Psi_{II})] + K \delta_\alpha^\beta (\overline{\Psi}_I \Psi_{II} + \overline{\Psi}_{II} \Psi_I),$$

$$T_{\alpha em}^\beta = \frac{1}{4\pi} \Big[-\partial_\alpha A_\eta F^{\beta\eta} + \partial_\alpha b_\eta \widetilde{F}^{\beta\eta} - G \partial_\alpha A^\beta + F \partial_\alpha b^\beta -$$

$$- \frac{1}{2} \delta_\alpha^\beta \left(- \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + F^2 - G^2 \right) \Big]. \tag{2.22}$$

A redefinition of $T_{em}^{\alpha\beta}$ by an addition of the tensor

$$t^{\alpha\beta} = \frac{1}{4\pi} \partial_\eta (A^\alpha F^{\beta\eta} - b^\alpha \widetilde{F}^{\beta\eta}), \tag{2.23}$$

without changing the corresponding energy-momentum constant of motion (tensor in the bracket is antisymmetric in the last indices), is useful.

By making use of (2.12), the new tensor

$$T_{emN}^{\alpha\beta} = T_{em}^{\alpha\beta} + t^{\alpha\beta} \tag{2.24}$$

becomes

$$T_{emN}^{\alpha\beta} = \frac{1}{4\pi} \Big[\partial^\eta A^\alpha - \partial^\alpha A^\eta) F_\eta^\beta - (\partial^\eta b^\alpha - \partial^\alpha b^\eta) \widetilde{F}_\eta^\beta - G \partial^\alpha A^\beta + \tag{2.25}$$

$$+ F \partial^\alpha b^\beta + A^\alpha \partial^\beta G - b^\alpha \partial^\beta F + \frac{1}{2} g^{\alpha\beta} \left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - F^2 + G^2 \right) \Big] - A^\alpha j^\beta.$$

Introducing the notation

$$F_A^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha,$$
$$F_b^{\alpha\beta} = \partial^\alpha b^\beta - \partial^\beta b^\alpha, \tag{2.26}$$

is goes over into

$$T^{00}_{emN} = \frac{1}{8\pi} [(\vec{E}_A^2 + \vec{B}_A^2 + G^2) - (\vec{E}_b^2 + \vec{B}_b^2 + F^2) + 2 (A^0 \partial^0 G - b^0 \partial^0 F - G \partial^0 A^0 + F \partial^0 b^0)] - A^0 j^0,$$
$$T^{i0}_{emN} = \frac{1}{4\pi} [(\vec{E}_A \times \vec{B}_A + \vec{A} \partial^0 G + G \nabla A^0) - (\vec{E}_b \times \vec{B}_b + \vec{b} \partial^0 F + F \nabla b^0)]^i - A^i j^0. \tag{2.27}$$

For $b^\alpha = 0$ and $G = 0$ (see Ref. 3)

$$T^{00}_{emN} = \frac{1}{8\pi} (\vec{E}_A^2 + \vec{B}_A^2) - A^0 j^0,$$
$$T^{i0}_{emN} = \frac{1}{4\pi} (\vec{E}_A \times \vec{B}_A)^i - A^i j^0. \tag{2.28}$$

The energy-momentum constant of motion of the total system is then

$$P^\alpha = const_P \int T^{\alpha 0} \, \mathrm{d}^3 x. \tag{2.29}$$

For $\Psi' = 0$ it becomes the energy-momentum vector of the electromagnetic field and from here it follows that $const_P = 1$. On the other hand for $A^\alpha = 0$ it gives the average energy-momentum values of the Dirac field particles and from here it follows that $K = \varkappa$. Thus, we have

$$P^0 = \int [\Psi'^\dagger \tau_+ (\mathrm{i}\partial^0) \Psi' - A^0 j^0 + \frac{1}{8\pi} (\vec{E}_A^2 + \vec{B}_A^2)] \, \mathrm{d}^3 x,$$
$$P^i = \int [\Psi'^\dagger \tau_+ (\mathrm{i}\partial^i) \Psi' - A^i j^0 + \frac{1}{4\pi} (\vec{E}_A \times \vec{B}_A)^i] \, \mathrm{d}^3 x, \tag{2.30}$$

or

$$P^0 = \int [\Psi^\dagger \tau_+ (i\partial^0 - e A^0) \Psi + \frac{1}{8\pi} (\vec{E}^2 + \vec{B}^2)] d^3x,$$
$$P^i = \int [\Psi^\dagger \tau_+ (i\partial^i - e A^i) \Psi + \frac{1}{4\pi} (\vec{E} \times \vec{B})^i] d^3x, \tag{2.31}$$

where we have made use of

$$\int A^0 j^0 d^3x = \int e A^0 \Psi^\dagger \tau_+ \Psi d^3x = \int \Psi^\dagger \tau_+ e A^0 \Psi d^3x$$

and dropped the index A.

It is useful to introduce a Hamiltonian operator of the Dirac field by rewriting the Dirac field equations (2.13) in the form

$$i\partial_0 \Psi_I = (-i\partial_j \alpha^j + e A_\alpha \gamma^0 \gamma^\alpha + \gamma^0 \varkappa) \Psi_I,$$
$$i\partial_0 \Psi_{II} = (-i\partial_j \alpha^j + e A_\alpha \gamma^0 \gamma^\alpha - \gamma^0 \varkappa) \Psi_{II}, \tag{2.32}$$

and from here

$$i\partial_0 \Psi = H \Psi \tag{2.33}$$

where

$$H = \begin{pmatrix} -i\partial_j \alpha^j + e A_\alpha \gamma^0 \gamma^\alpha + \gamma^0 \varkappa & 0 \\ 0 & -i\partial_j \alpha^j + e A_\alpha \gamma^0 \gamma^\alpha - \gamma^0 \varkappa \end{pmatrix}. \tag{2.34}$$

By making use of (2.33) the energy (2.31) can be written also in the form

$$P^0 = \int [\Psi^\dagger \tau_+ (H - e A^0) \Psi + \frac{1}{8\pi} (\vec{E}^2 + \vec{B}^2)] d^3x. \tag{2.35}$$

It is instructive to make here a comparison with the classical point-particle theory. Using $P^j = -i\partial_j$ the Hamiltonian operator reads

$$H = (P^j - e A^j) \alpha^j + e A^0 + \varkappa \gamma^0 \tau_+$$

and

$$P^0 = \int \Psi^\dagger \tau_+ (H - e A^0) \Psi d^3x + \int [\frac{1}{8\pi} (\vec{E}^2 + \vec{B}^2)] d^3x = P^0_{(1)} + P^0_{(2)},$$

$$P^i = \int \Psi^\dagger \tau_+ (P^i - e A^i) \Psi d^3x + \frac{1}{4\pi} \int (\vec{E} \times \vec{B})^i d^3x = P^i_{(1)} + P^i_{(2)}.$$

$P_{(1)}$ corresponds to the energy-momentum of the point-particle and $P_{(2)}$ to the energy-momentum of the electromagnetic field. This leads to

$$P^0_{cl} = \sqrt{\varkappa^2 + \vec{p}^2} + \frac{1}{8\pi} \int (\vec{E}^2 + \vec{B}^2)\, d^3x,$$

$$P^i_{cl} = p^i + \frac{1}{4\pi} \int (\vec{E} \times \vec{B})^i\, d^3x,$$

where

$$p^i = P^i - e A^i.$$

In the nonrelativistic limit it is

$$P^0_{cl\ nonrel} = \varkappa + \frac{\vec{p}^2}{2\varkappa} + \frac{1}{8\pi} \int (\vec{E}^2 + \vec{B}^2)\, d^3x,$$

$$P^i_{cl\ nonrel} = \varkappa v^i + \frac{1}{4\pi} \int (\vec{E} \times \vec{B})^i\, d^3x,$$

and from here, the classical mass renormalization follows. Thus, we have full correspondence of Eqs. (2.30) with the classical picture. But the essential difference is in the description of the point-particle by the Dirac field functions. What are the consequencies of this difference is considered in the following section.

An other observation is interesting and instructive. The energy P^0 can be written also in the form

$$P^0 = \int \Psi^\dagger \tau_+ (H_0) \Psi\, d^3x + \int A_\alpha j^\alpha\, d^3x + \frac{1}{8\pi} \int (\vec{E}^2 + \vec{B}^2)\, d^3x -$$

$$- \frac{1}{4\pi} \int A^0\, div\, \vec{E}\, d^3x.$$

If we now take $\Psi = \begin{pmatrix} \Psi_I \\ \Psi_{II} \end{pmatrix} \to \Psi_I \equiv \Psi$, the last two terms as the energy of the free electromagnetic field and apply quantum rules for the fields according to the conventional quantum field theory, we obtain the Hamiltonian (operator) of the conventional quantum electrodynamics. Thus, we have a basis for comparison of our theory with the conventional quantum field theory.

3. *Selfinteraction effects*

(1) The stationary states of the Dirac field without the selfinteraction in the new theory are given by[3)]

$$i\partial_0 \Psi = H_0 \Psi,$$

$$\Psi(\vec{x}, x^0) = e^{-i\omega x^0}\,\Psi_\omega(\vec{x}), \qquad H_0 \Psi_\omega(\vec{x}) = \omega\,\Psi_\omega(\vec{x}),$$

$$\omega = \pm k_0, \qquad k_0 = \sqrt{\varkappa^2 + k^2}, \tag{3.1}$$

$$\Psi_{+k_0\vec{k}}(\vec{x}, x^0) = \frac{1}{L^{3/2}}\begin{pmatrix} u_{\vec{k}} \\ 0 \end{pmatrix} e^{-ik_\alpha x^\alpha}, \qquad \Psi_{-k_0\vec{k}}(\vec{x}, x^0) = \frac{1}{L^{3/2}}\begin{pmatrix} 0 \\ u_{\vec{k}} \end{pmatrix} e^{-ik_\alpha x^\alpha},$$

where

$$u_{\vec{k}} = \sqrt{\frac{\varkappa + k_0}{2k_0}}\left\{\begin{bmatrix} 1 \\ 0 \\ \dfrac{k_z}{\varkappa + k_0} \\ \dfrac{k_x + ik_y}{\varkappa + k_0} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ \dfrac{k_x - ik_y}{\varkappa + k_0} \\ \dfrac{-k_z}{\varkappa + k_0} \end{bmatrix}\right\} \text{and } \int \Psi^\dagger_{\pm k_0\vec{k}} - \Psi_{\pm k_0\vec{k}}\, d^3x = 1. \tag{3.2}$$

The corresponding energy-momenta are

$$P^\alpha = k^\alpha. \tag{3.3}$$

To these states we may come from the state of the Dirac field particle at rest. Let us denote the corresponding reference system by S'. In this reference system the current probability is equal to zero,

$$j'^i = \Psi'^\dagger \tau_+ \alpha^i \Psi' = 0, \qquad i = 1, 2, 3, \tag{3.4}$$

or explicitely

$$\Psi_I'^\dagger \alpha^i \Psi_I' - \Psi_{II}'^\dagger \alpha^i \Psi_{II}' = 0.$$

Writing

$$\Psi_I' = \begin{pmatrix} \chi_I' \\ \eta_I' \end{pmatrix}, \qquad \Psi_{II}' = \begin{pmatrix} \chi_{II}' \\ \eta_{II}' \end{pmatrix}. \tag{3.5}$$

it is

$$\chi_I'^\dagger \sigma^i \eta_I' + \eta_I'^\dagger \sigma^i \chi_I' - (\chi_{II}'^\dagger \sigma^i \eta_{II}' + \eta_{II}'^\dagger \sigma^i \chi_{II}') = 0 \tag{3.6}$$

The following solutions are of interest:

$$\text{a)} \qquad \chi_I' \neq 0, \quad \eta_I' = 0, \quad \chi_{II}' = \eta_{II}' = 0,$$

$$\text{b)} \qquad \chi_I' = \eta_I' = 0, \quad \chi_{II}' \neq 0, \quad \eta_{II}' = 0. \tag{3.7}$$

Substitution of these solutions in the second of Eqs. (3.1) gives

$$
\begin{aligned}
&\text{a)} \qquad i\partial'_0 \chi'_I = \varkappa \chi'_I, \\
&\qquad\qquad 0 = -i\partial'_J \sigma^J \chi'_I, \\
&\text{b)} \qquad i\partial'_0 \chi'_{II} = -\varkappa \chi'_{II}, \\
&\qquad\qquad 0 = -i\partial'_J \sigma^J \chi'_{II}.
\end{aligned} \tag{3.8}
$$

The solutions of these equations with the box normalization are

$$
\begin{aligned}
\chi'_I &= \frac{1}{L'^{3/2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} e^{-i\varkappa x^{0'}}, \qquad (w = k_0 = \varkappa), \\
\chi'_{II} &= \frac{1}{L'^{3/2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} e^{i\varkappa x^{0'}}, \qquad (w = -k_0 = -\varkappa).
\end{aligned} \tag{3.9}
$$

The scalar and energy-momentum constants of motion given by

$$
\begin{aligned}
Q' &= e \int \Psi'^{\dagger} \tau_+ \Psi' \, d^3x', \\
P^{\alpha'} &= \int \Psi'^{\dagger} \tau_+ (i\partial'^{\alpha}) \Psi' \, d^3x',
\end{aligned} \tag{3.10}
$$

for these solutions read

$$
\begin{aligned}
&\text{a) } Q' = a, \qquad P^{0'} = \int \Psi_I'^{\dagger} i\partial^{0'} \Psi'_I \, d^3x' = \varkappa, \qquad \left(\int \Psi_I'^{\dagger} \Psi'_I \, d^3x' = 1\right), \\
&\qquad\qquad\qquad P^{l'} = 0, \\
&\text{b) } Q' = -a, \qquad P^{0'} = -\int \Psi_{II}'^{\dagger} i\partial^{0'} \Psi'_{II} \, d^3x' = \varkappa, \qquad \left(\int \Psi_{II}'^{\dagger} \Psi'_{II} \, d^3x' = 1\right), \\
&\qquad\qquad\qquad P^{l'} = 0.
\end{aligned} \tag{3.11}
$$

Now, let us go over into the inertial reference system S moving with velocity $-v_x$ with respect to the system S'. In the new system S the scalar constants remain unchanged and the energy-momentum constants go over into

$$
\begin{aligned}
P^0 &= P^{0'} \operatorname{ch} \varphi + P^{1'} \operatorname{sh} \varphi = \varkappa \operatorname{ch} \varphi, \qquad \operatorname{th} \varphi = v_x, \\
P^1 &= P^{1'} \operatorname{ch} \varphi + P^{0'} \operatorname{sh} \varphi = \varkappa \operatorname{sh} \varphi, \\
P^{2,3} &= 0.
\end{aligned} \tag{3.12}
$$

Denoting

$$\varkappa \operatorname{sh} \varphi = k_x \tag{3.13}$$

they become

$$P^0 = \sqrt{\varkappa^2 + k^2} \equiv k_0$$
$$P^i = k^i, \qquad k^i = (k_x, 0, 0). \tag{3.14}$$

The state functions are transformed into

$$\text{a)}\ \Psi_I = \frac{1}{L'^{3/2}} \frac{1}{\sqrt{\operatorname{ch}\varphi}} \left(\operatorname{ch}\frac{\varphi}{2} - \alpha^1 \operatorname{sh}\frac{\varphi}{2}\right) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{-ik'_0 x^{0'}} =$$

$$= \frac{1}{L^{3/2}} \sqrt{\frac{\varkappa + k_0}{2k_0}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{k_x}{\varkappa + k_0} \end{bmatrix} e^{-ik_\alpha x^\alpha}$$

$$\Psi_{II} = 0, \qquad k^\alpha = (k_0, k_x, 0, 0), \tag{3.15}$$

b) $\Psi_I = 0,$

$$\Psi_{II} = \frac{1}{L'^{3/2}} \frac{1}{\sqrt{\operatorname{ch}\varphi}} \left(\operatorname{ch}\frac{\varphi}{2} - \alpha^1 \operatorname{sh}\frac{\varphi}{2}\right) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{ik'_\alpha x^{\alpha'}} =$$

$$= \frac{1}{L^{3/2}} \sqrt{\frac{\varkappa + k_0}{2k_0}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{k_x}{n + k_0} \end{bmatrix} e^{ik_\alpha x^\alpha},$$

$k^\alpha = (k_0, -k_x, 0, 0).$

Eqs. (3.14—15) are equal to Eqs. (3.2—3) ($k^i = \pm k_x, 0, 0$). Thus we have obtained the solutions (3.2—3) from the state (3.9) of the particle at rest. We have presented this method because we are going to use it in the selfinteraction problem.

The general solution of the first of Eqs. (3.1) is a linear combination of the solutions (3.2)

$$\Psi(\vec{x}, x^0) = \sum_{\vec{k}} (a_{\vec{k}} \Psi_{k_0 \vec{k}} e^{-ik_\alpha x^\alpha} + b_{\vec{k}} \Psi_{k_0 - \vec{k}} e^{ik_\alpha x^\alpha}). \tag{3.16}$$

Substitution of (3.16) into

$$P^\alpha = \int \Psi'^\dagger \tau_+ \, i\partial^\alpha \Psi \, d^3x$$

gives

$$P^\alpha = \sum_{\vec{k}} |a_{\vec{k}}|^2 k^\alpha + \sum_{\vec{k}} |b_{\vec{k}}|^2 k^\alpha, \tag{3.17}$$

with

$$\sum_{\vec{k}} |a_{\vec{k}}|^2 = 1, \qquad \sum_{\vec{k}} |b_{\vec{k}}|^2 = 1.$$

It is the average value of k^α over the probabilities $|a_{\vec{k}}|^2$ for the particle and $|b_{\vec{k}}|^2$ for the antiparticle.

(2) Let us now come back to the selfinteraction problem. We again expect the existence of the reference system where the Dirac field particle is at rest. Since only the selfinteraction is present, it should also be given by (3.7). Indeed, after substitution of (3.7) into (2.33) we get

$$\begin{aligned}
\text{a)} \quad & i\partial'_0 \chi'_I = (\varkappa + e A'_I) \chi'_I, \\
& 0 = -i\partial'_j \sigma^j \chi'_I, \\
\text{b)} \quad & i\partial'_0 \chi'_{II} = (-\varkappa + e A'_0) \chi'_{II}, \\
& 0 = -i\partial'_j \sigma^j \chi'_{II}.
\end{aligned} \tag{3.18}$$

The second of these equations gives

$$\chi'_I = const \; in \; x^i, \qquad \chi'_{II} = const \; in \; x^i.$$

Taking the box normalization, the energy $e A'_0$ then becomes

$$e A'_0 = \frac{1}{L'^3} \int \frac{e^2}{|\vec{x}' - \vec{x}''|} \, d^3x''$$

and this goes to zero when $L \to \infty$. Consequently, $e A'_0$ can be neglected and the system (3.18) goes over into the system (3.8) with the solution (3.9) and the energy-momentum (3.11). From here we conclude that the Dirac field particles with the selfinteraction have the same stationary states, the energy-momentum, as the particles without selfinteraction.

(3) It is interesting to take the finite, but still large, box L^3. In this case we may apply the following method. Starting from

$$\omega \Psi(\vec{x}) = (H_{ON} + H_{IN}) \Psi(\vec{x}), \tag{3.19}$$

where

$$H_{0N} = H_0 + \delta\,\varkappa\,\tau_+\,\gamma_?^0 H_0 = \begin{pmatrix} -\mathrm{i}\partial_j\,\alpha^j + \varkappa\,\gamma^0 & 0 \\ 0 & -\,\mathrm{i}\partial_j\,\alpha^j - \varkappa\,\gamma^0 \end{pmatrix},$$

$$H_{IN} = H_I - \delta\,\varkappa\,\tau_+\,\gamma_?^0 H_I = \begin{pmatrix} e\,A_\alpha\,\gamma^0\,\gamma^\alpha & 0 \\ 0 & e\,A_\alpha\,\gamma^0\,\gamma^\alpha \end{pmatrix} \tag{3.20}$$

and making the expansion

$$\Psi(\vec{x}) = \sum_n a_n\,\Psi_n^0(\vec{x}), \tag{3.21}$$

where

$$H_{0N}\,\Psi_n^0 = \omega_n^0\,\Psi_n^0, \tag{3.22}$$

we get

$$a_m(\omega - \omega_m^0) - \sum_n a_n\,H_{IN\,mn} = 0. \tag{3.23}$$

This set of equations gives eigenvalues ω_l and corresponding eigenfunctions Ψ_l.

The energy of the state Ψ_l, according to (2.30), is given by

$$P_l^0 = \omega_l + \int\left[-A^0 j^0 + \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)\right]_{\Psi_l} \mathrm{d}^3x. \tag{3.24}$$

Up to this moment $\delta\,\varkappa$ is an arbitrary parameter. We now define this parameter by requirement that P_l^0 becomes ω_l^0. Writing (3.24) in the form

$$P_l^0 = \omega_l^0 + (\omega_l - \omega_l^0) + \int\left[-A^0 j^0 + \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)\right]_{\Psi_l} \mathrm{d}^3x, \tag{3.25}$$

it means that

$$\omega_l - \omega_l^0 + \int\left[-A^0 j^0 + \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)\right]_{\Psi_l} \mathrm{d}^3x = 0. \tag{3.26}$$

Eq. (3.26) defines $\delta\,\varkappa$. It depends on the states Ψ_l. For small $\delta\,\varkappa$ in comparison to $\varkappa$ the left-hand side of (3.26) can be expanded into the powers of $\delta\,\varkappa$ and keep only the first two terms. Denoting the left-hand side by $f(\delta\,\varkappa)$ it is

$$f(0) + (\partial f|\partial\delta\,\varkappa)_{\delta\varkappa=0}\,\delta\,\varkappa + \ldots = 0$$

and from here

$$\delta\,\varkappa \approx -\frac{f(0)}{(\partial f/\partial\delta\,\varkappa)_{\delta\varkappa=0}} \tag{3.27}$$

Applying the perturbation method for ω_i and Ψ_i and keeping the first terms, the left-hand side becomes

$$f(\delta\varkappa) = H_{Iii} - \delta\varkappa I_{ii} + \int [-A^0 j^0 + \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)]_{\Psi_i^0} \mathrm{d}^3x,$$
$$H_{Iii} = \int \Psi_i^{0\dagger} \tau_+ H_I \Psi_i^0 \mathrm{d}x^3, \qquad I_{ii} = \int \Psi_i^{0\dagger} \gamma^0 \Psi_i^0 \mathrm{d}^3x, \tag{3.28}$$

and from here

$$f(0) = \{H_{Iii} + \int [-A^0 j^0 + \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)]_{\Psi_i^0} \mathrm{d}^3x\}_{\delta\varkappa=0},$$
$$(\partial f/\partial\delta\varkappa)_{\delta\varkappa=0} = \{-I_{ii} + (\partial\{\ldots\}/\delta\varkappa)\}_{\delta\varkappa=0}. \tag{3.29}$$

The approximation of $\delta\varkappa$ up to the first order of H_I is then

$$\delta\varkappa = \frac{1}{I_{ii}}\{H_{Iii} + \int [-A^0 j^0 + \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)]_{\Psi_i^0} \mathrm{d}^3x\}_{\delta\varkappa=0}. \tag{3.30}$$

For the particle state $\Psi^i_{\vec{k}=0}$ it is

$$\delta\varkappa = \frac{1}{8\pi}\int \vec{E}^2_{\Psi^i_{\vec{k}=0}} \mathrm{d}^3x = \frac{\mathrm{e}^2}{L^6}\int \frac{1}{|\vec{x} - \vec{x}'|} \mathrm{d}^3x\, \mathrm{d}^3x', \tag{3.31}$$

The energy of the particle in this state is

$$P_i^0 = \varkappa + \delta\varkappa_i \approx \varkappa + \frac{1}{8\pi}\int \vec{E}^2_{\Psi^i_{\vec{k}=0}} \mathrm{d}^3x \equiv \varkappa_{0b}. \tag{3.32}$$

For $L \to \infty$, $\varkappa_{0b} \to \varkappa$. Further analysis is analogous to that given in (1). From here we conclude that the selfinteraction changes the particle mechanical energy by its electrostatic energy evaluated over the particle space probability distribution given by the particle state function (the mass renormalization). This is what we have expected. But now, in comparison to the classical electrodynamics and the quantum field theory, it is without any difficulty. Thus the new theory gives a consistent treatment of the selfinteraction.

The general solution of the equation

$$\mathrm{i}\partial_0\Psi = (H_0 + H_I)\Psi$$

is not a linear combination of the stationary solutions, because of its inhomogeneity with respect to Ψ. But the expansion

$$\Psi(\vec{x}, x^0) = \sum_i a_i(x^0)\, \Psi_i(\vec{x})\, e^{-i\omega_i x^0} \tag{3.33}$$

is possible evetually.

The coefficients a_i are determined by

$$i\dot{a}_j = \sum_i a_i H_{ iji}. \tag{3.34}$$

The energy of the state (3.33), according to (2.30), is given by

$$P^0 = \sum_i \omega_i |a_i|^2 + \sum'_{i,j,k,l} \int [-A^0_{ij} j^0_{kl} + \frac{1}{8\pi}(\vec{E}_{ij}\vec{E}_{kl} + \vec{H}_{ij}\vec{H}_{kl})]\, d^3x +$$
$$+ \sum_i |a_i|^2 (|a_i|^2 - 1) \int [-A^0_{ii} j^0_{ii} + \frac{1}{8\pi}(\vec{E}_{ii}\vec{E}_{ii} + \vec{H}_{ii}\vec{H}_{ii})]\, d^3x, \tag{3.35}$$

where

$$j^0_{kl} = e\Psi^+_k \tau_+ \Psi_l \text{ etc.} \tag{3.36}$$

The apostrophe at the sum means that $i = j = k = l$ is excluded.

Let us stress that $|a_i|^2$ is not generally time independent. We see that the energy is not an average of stationary states energies any more. The energy contains also the contributions of various transitions among stationary states as well as some changes in stationary states. It requires further attention.

4. Conclusions

The selfinteraction in the new Dirac field theory has a close relation to the classical picture of this problem. The essential difference comes from the description of the particles. The new theory gives a consistent and clear treatment of the self-interaction. No infinities appear.

The mass renormalization effects depend on localization of the particle. When the localization volume goes to infinity the electromagnetic correction of the mass goes to zero. In many cases one may expect that these effects are small. The mass renormalization comes from interaction of the particle with its own field. Let us point out the essential difference in this sense in comparison to the conventional quantum field theory.

We may expect satisfactory explanations of all the other effects which comes from this interaction.

Finally, the obtained results strenghten the new Dirac field theory very much. Using Schwinger's words*, a fatal error has indeed been done in the history of the quantum physics by treating the Dirac field function as the Lagrange's variable instead of as a canonical or linear combination of the canonical variables.

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SAMOMEĐUDJELOVANJE U NOVOJ TEORIJI DIRACOVOG POLJA

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Originalni znanstveni rad

U radu je analiziran problem elektromagnetskog samomeđudjelovanja u novoj teoriji Diracovog polja. Pokazano je da se ovaj problem u novoj teoriji može dovesti u blisku vezu sa klasičnom teorijom samomeđudjelovanja. Nasuprot klasičnoj teoriji utvrđeno je da se ne pojavljuju beskonačnosti i da je teorija u sebi konzistentna. Razmatranje je provedeno za Diracovo polje bez vanjskih izvora. Time je dobivena dobra osnova i za teoriju elektromagnetskog samomeđudjelovanja za više čestica (ili sa vanjskim izvorima). Usporedba sa konvencionalnom kvantnom teorijom također je data.

* J. Schwinger in preface of the book *Quantum Electrodynamics* writes: *But, we may ask is there a fatal fault in the structure of field theory*?.