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ON THE EXISTENCE OF THE ROTATING CHIRAL SOLITON*

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We discuss the conditions for the existence of spherically symmetric chiral soliton. We show that the system has a nontrivial minimum that corresponds to a rotating chiral soliton even after the collective-coordinate quantization has been performed. However, the stability of the soliton corresponds to a question of finitness of the moments of inertia of collective coordinate modes. In the case of the chiral soliton of the Skyrme model the rotational moment of inertia is divergent but the characteristic rotational time is much longer than the characteristic radiation time therefore permitting the semiclassical approximation in which the soliton is treated as a rotating classical body.

1. Introduction

Some time ago it was shown that solitons in SU (N) $\times$ SU (N) chiral models can be interpeted as baryons[1,2]. This is a realization of the old idea of Skyrme[3], for a unified theory of mesons and baryons. It was of equal importance to establish that solitons in chiral σ-models may be fermions[1]. As a consequence, one can calculate many static properties of baryons, asymptotic forms of two and three body nuclear force, mass difference, etc. All these results can be compared with experimentally measured values. Generally, one finds that the calculated values are about 20% off.

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The model has a static solution of finite energy, the soliton. To quantize the soliton, it is necessary to introduce collective coordinates[1]. In Ref. 5 it was pointed out that the soliton solution of the effective Hamiltonian, because of the lack of stabilyzing terms, may be unstable due to emission of soft poins by the rotational degrees of freedom of the soliton. This means that the soliton will lose energy and slow its rotation. In this letter we clarify the problem from the point of view of collective coordinate quantization. We show that the above problem is not connected with the collective coordinate quantization. The effective system, obtained after the collective coordinate quantization, always has a minimum provided that the static classical solution exists. The above-mentioned instability is due to emisison of soft pions. It is dynamical effect present in the model because one works with massless pions and it can be prevented by adding a pion mass term to the system.

This letter is organized as follows. First we briefly derive the effective Hamiltonian of the soliton using the collective coordinate method. For simplicity we use as an example the case of two-dimensional nonlinear theory with soliton solutions[1,6]. A model theory can be taken to be a φ^4 theory or sine-Gordon theory. Next we show that the four-dimensional Skyrme model (SU (2) $\times$ SU (2) nonlinear σ-model) has similar properties. Carefully introducing the collective coordinates, we derive the effective Hamiltonian. Minimizing the system is equivalent to solving the appropriate variational equations of motion. The solution of the equations of motion will always exist. However, the existence of the solution does not guarantee its stability. In principle it is possible that the inertial effects are strong enough to deform the soliton so much that it ceases to exist. This can be seen, for example, by the emission of particles by the soliton configuration. Whether this happens will depend on the specifics of the model.

2. *Soliton in 1 + 1 dimensional scalar theory*

We begin by studying the two-dimensional nonlinear scalar theory that has soliton solutions. Consider the following Lagrangian

$$L = \int \mathrm{d}x \left[\frac{1}{2}\partial_\mu \varphi\, \partial^\mu \varphi - V(\varphi)\right], \tag{2.1}$$

where the potential $V(\varphi)$ is nonlinear in φ, and allows the static soliton solutions $x = \int_0^{\varphi_0} \mathrm{d}\varphi \sqrt{2V(\varphi)}$. The general form of the classical solitary wave solution is $\varphi_0(x, t) = \varphi_0\left(\frac{x - vt - a}{\sqrt{1 - v^2}}\right)$ and has finite energy. It can be interpreted as a particle of mass $M_0 = \int \mathrm{d}x\, \varphi_0'^2$.

To quantize the soliton, one expands around the classical background configuration φ_0. In general, this leads to problems because the propagator of the quantum theory in the soliton sector is the inverse of the operator $-\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{\partial^2 V(\varphi)}{\partial \varphi^2}\Big|_{\varphi_0}$. This implies infrared divergencies due to the existence of the zero frequency eigenvalues of the operator $\frac{d^2}{dx^2} - \frac{\partial^2 V(\varphi)}{\partial \varphi^2}|_{\varphi_0}$. They correspond to the motion of the centre of mass of the classical configuration. Therefore, the standard quantization by expansion around the classical background is not possible[6,4]. To solve the problem, it is necessary to separate the motion of the centre of mass coordinate. This is accomplished by introducing the collective coordinates. We give some details below.

To derive the effective Hamiltonian in the soliton sector of the model we begin by separating the motion of the centre of mass. This is done by introducing the collective coordinate $a(t)$ describing the trajectory of the centre of mass. The solution of the model in the soliton sector is of the form

$$\varphi(x, t) = \tilde{\varphi}(x - a(t), t) = \tilde{\varphi}(y, t),$$

where $y = x - a(t)$. The Lagrangian (2.1) becomes

$$L = \int dx \left(\frac{1}{2} (\dot{\tilde{\varphi}} - \dot{a}\, \tilde{\varphi}')^2 - \frac{1}{2} \tilde{\varphi}'^2 - V(\tilde{\varphi}) \right).$$

The canonical momenta conjugate to variables φ and $a(t)$ are π and P, respectively. They are given by the expressions

$$\pi = \dot{\tilde{\varphi}} - \dot{a}\, \tilde{\varphi}', \qquad P = - \int dx\, \pi\, \tilde{\varphi}'.$$

Note that the equation defining the moment P is a constraint equation. This signifies that the quantum theory in the soliton sector has gauge symmetry. The static classical solution is easy to find. It will minimize the action. It is given by a configuration

$$P = M_0\, \dot{a}, \qquad a = \text{constant}.$$

$$\tilde{\varphi} = \varphi_c \left(\sqrt{1 + \frac{P^2}{M_0^2}} (x - a) \right), \tag{2.2}$$

where the function φ_c is the solution of the equation

$$\partial_x^2 \varphi_c = \frac{\partial V(\varphi)}{\partial \varphi}.$$

In what follows, we will perform a series of canonical transformations to obtain the effective Hamiltonian in a form suitable for perturbation expansion. The stability of the solution, at this level, is guaranteed by the topology of the space of the configurations. The Hamiltonian of the theory can be extracted from the Lagrangian. It is convenient to write

$$L = P\, \dot{a} + \int dx\, \pi\, \tilde{\varphi}' - H(\pi, \tilde{\varphi}) + \lambda (P + \int \pi\, \tilde{\varphi}'),$$

where λ is the Lagrange multiplier field, and the Hamiltonian is of the standard form

$$H = \int \mathrm{d}x \left(\frac{1}{2}\pi^2 + \frac{1}{2}\tilde{\varphi}'^2 + V(\tilde{\varphi})\right).$$

To continue we will solve the constraint and write the Hamiltonian in terms of independent variables only. Based on the general theory for the quantization of the systems with constraints we need to introduce a gauge condition[7]. We choose the gauge condition to be

$$\int \mathrm{d}x\, f(x)\, \varphi'(x - a, t) = 0,$$

where $f(x)$ is an arbitrary function and the dynamics of the system does not del pend on it. It is convenient to choose $f(x)$ such that the translational zero mode-disappears. It is also convenient to linearize the constraint $P + \int \pi\tilde{\varphi}' = 0$, by making the shift in the canonical momentum π,

$$\pi = -f\frac{P + \int \bar{\pi}(\tilde{\varphi}' - cf)}{\int f\tilde{\varphi}'} + \bar{\pi},$$

where c is some constant which we will determine later. The linearized constraint is given by

$$\int \mathrm{d}x\, f(x)\, \bar{\pi}(x, t) = 0.$$

The Hamiltonian becomes[6]

$$H = \left(\int f^2\right)\frac{1}{2}\frac{[P + \int \bar{\pi}(\tilde{\varphi}' - cf)]^2}{(\int f\tilde{\varphi}')^2} + \int \mathrm{d}x \left[\frac{1}{2}\bar{\pi}^2 + \frac{1}{2}\tilde{\varphi}'^2 + V(\tilde{\varphi})\right]. \quad (2.3)$$

It is convenient to choose the gauge fixing function to be $cf(x) = \tilde{\varphi}'$. The effective Hamiltonian then takes the simple from

$$H = \frac{1}{2}\frac{P^2}{\int \tilde{\varphi}'^2} + \int \mathrm{d}x \left(\frac{1}{2}\bar{\pi}^2 + \frac{1}{2}\tilde{\varphi}'^2 + V(\tilde{\varphi})\right).$$

It can be easily checked that the classical solution of the system is given by (2.2). This is because all the transformations performed are canonical. The perturbation expansion is obtained by expanding around the classical configuration $\tilde{\varphi} = \varphi_0$, $\bar{\pi} = 0$. Setting (temporarily) the quantum fluctuation fields to zero, one obtains the effective Hamiltonian which is sufficient for studying, in the semiclassical approximation, the quantum mechanics of the particle that represent the soliton. The effective Hamiltonian from which the Schrödinger equation follows is given by

$$H = \frac{P^2}{2M_0} + M_0.$$

Now that we understand the simple example we can proceed to study a more complicated case.

3. The Skyrme model

In this section we introduce the Skyrme model[3] and write the Lagrangian of the model in the form where the kinetic energy is separated from the potential. This is also the general form of the Lagrangian for the models with soliton solutions.

The chiral model for SU (2) group was proposed to be a unified theory of mesons and baryons some twenty years ago[3] by Skyrme. It is a nonlinear σ-model where the term, quartic in currents (but only quadratic in time derivatives), has been added to stabilize the soliton against shrinking to zero size and energy. The Lagrangian can be written in terms of SU (2) valued matrix field $U(x)$

$$L = \int \mathrm{d}^3x \left\{\frac{F^2}{16} \mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger + \frac{1}{32e^2} \mathrm{Tr}\, [(\partial_\mu U) U^\dagger, (\partial_\nu U) U^\dagger]^2\right\}.$$

The soliton solution exists due to the fact that the configuration space of the model (group SU (2)) has a nontrivial third homotopy group; $\Pi_3(SU(2)) = \mathscr{Z}$. The topological, conserved quantum number measures how many times the field $U(x)$ wraps arround the SU (2). It is given by

$$B = \frac{1}{24\pi} \int \mathrm{d}^3x\, \varepsilon^{abc}\, \mathrm{Tr}\, (U^{-1} \partial_a U U^{-1} \partial_b U U^{-1} \partial_c U).$$

If the soliton of the model is interpreted as a nucleon (baryon), then the topological quantum number has a natural interpretation as a baryon number[1]. The static, one soliton solution is the configuration of the form

$$U(\vec{x}) = \exp(\mathrm{i}G(|\vec{x}|)\, \vec{\tau} \cdot \hat{x}), \tag{3.1}$$

with boundary conditions $G(0) = \pi$, $G(\infty) = 0$. Substituting the solution (3.1) into the Lagrangian, gives the following functional for the function G

$$L = -M[G] = -4\pi \int \mathrm{d}r\, r^2 \left\{\frac{F^2}{8}\left[\left(\frac{\partial G}{\partial r}\right)^2 + 2\,\frac{\sin^2 G}{r^2}\right] + \frac{1}{2e^2}\frac{\sin^2 G}{r^2}\left[\frac{\sin^2 G}{r^2} + \right.\right.$$
$$\left.\left. + 2\left(\frac{\partial G}{\partial r}\right)^2\right]\right\},$$

where the constant $F \sim 180$ MeV, is the pion decay constant and the constant $e \sim 5.5$, is a dimensionless number. The unknown function $G(x)$ can be determined by minimizing the functional $M[G]$. The Euler-Lagrange equation satisfied by G (written in terms of the dimensionless variable $y = eFr$) is

$$\left(\frac{y^2}{4} + 2\sin^2 G\right) G'' + \frac{y}{2} G' + \sin 2G\, G'^2 - \frac{1}{4}\sin 2G - \frac{\sin^2 G}{y^2}\sin 2G = 0.$$

The above equation has to be solved numerically, for example, by the Runge-Kuta method. One finds that the function G is monotonic and that in the asymptotic region $y \sim 0$, G is linear with a negative slope, and for $y \to \infty$, $G \sim \frac{1}{y^2}$. To quantize the theory in the soliton sector it is necessary to introduce collective coordinates. In order to determine which collective coordinates are needed one has to find all the symmetries of the solution $U(x)$ which give rise to zero frequency eigenvalues of the operator $\frac{\delta^2 M[G]}{\delta G^2}|_{G_0}$. One finds that there are translational, rotational, isorotational and vibrational collective coordinates. Translational collective coordinates correspond to movement of the centre of mass. Rotational and isorotational collective coordinates correspond to spin and the isospin of the configuration and are constrained by the condition $\boldsymbol{J} = \boldsymbol{I}$, where $\boldsymbol{J}$ is the spin operator and $\boldsymbol{I}$ is the isospin operator. Vibrational degrees of freedom correspond to fluctuations of the surface of the soliton. At this point we could continue along the lines of the Ref. 2, but instead it will be more convenient to develop a more general technique for working with collective coordinates. This will also enable us to use the technique of Refs. 6 and 4 to derive the effective Hamiltonian for the system. Before we go to study the general case, it is useful to rewrite the Lagrangian of the Skyrme model in the following form

$$L = \frac{1}{2} \int d^3x\, \dot{U}_{ab}\, K_{abcd}\, \dot{U}_{cd} - \int d^3x\, V(U),$$

where the potential energy is given by

$$V(U) = -\frac{F^2}{16} \operatorname{Tr} \partial_i U \partial_i U^\dagger + \frac{1}{32e^2} \operatorname{Tr} [(\partial_i U) U^\dagger, (\partial_j U) U^\dagger]^2,$$

and the kinetic energy operator $K_{abcd} = K_{cdab}$ is

$$K_{abcd} = -\frac{F^2}{8} U^\dagger_{ab} U^\dagger_{cd} + \frac{1}{4e^2} \{[U^\dagger (\partial_i U) U^\dagger \partial_i U U^\dagger]_{bc} U^\dagger_{da} -$$
$$- [U^\dagger (\partial_i U) U^\dagger]_{bc} [U^\dagger (\partial_i U) U^\dagger]_{da}\}.$$

We turn to the study of the Lagrangians of the general form above in the following section.

4. *Effective Hamiltonian for solitons*

In the previous section we have studied the Skyrme model. We have seen that the Lagrangian of the model can be rewritten in the form which is useful for studying all the collective coordinates at once. We will present the study of the general system with the Lagrangian of the same form in this section. The general Lagrangian is of the form

$$L = \frac{1}{2} \int \dot{\psi} K[\psi] \dot{\psi} - \int V(\psi). \tag{4.1}$$

The Lagrangian of the Skyrme model is the special case of (4.1) with the kinetic and the potential energy operators given in the previous section.

Let $\mathscr{G}$ be the symmetry of the static soliton solution of the model described by (4.1). The soliton solution is a solution of the equation $\frac{\delta V(\psi)}{\delta \psi} = 0$ and is characterized by the existence of the conserved topological quantum number $B = \mathscr{N} [\psi(+\infty) - \psi(-\infty)]$, where $\pm\infty$ are symbolic notation for special boundaries and the constant $\mathscr{N}$ is some appropriate normalization, such that B is an integer. Let $S(\Theta) \in \mathscr{G}$, be an element of the symmetry group of the soliton. In general it is possible to write $S(\Theta) = \exp \vec{\Theta} \cdot \vec{T}$, where Θ_a's are parameters, and T_a's are the group generators. When quantizing the soliton sector of the theory, the parameters Θ_a will become time dependent collective coordinates. Because the $S(\Theta)$ is the symmetry operation of the soliton solution, we have, that if φ is a soliton solution, then $S(\Theta)\varphi$ is a soliton solution too. We can write

$$\psi(x, t) = S(\Theta)\varphi(x, t). \tag{4.2}$$

This leads to the following transformation property of the kinetic energy operator of the theory

$$K[\psi] = K[S\varphi] = S K[\varphi] S^{-1}.$$

Taking the time derivative of (4.2), $\dot{\psi} = \dot{S}\varphi + S\dot{\varphi}$, and the fact that $S^{-1}\dot{S} = S^{-1}\frac{\partial S}{\partial \Theta_a}\dot{\Theta}_a = T_a m_{ab} \dot{\Theta}_b$, where the last equality holds because the combination $S^{-1}\dot{S}$ is the element of the algebra of the symmetry group $\mathscr{G}$, we can write the Lagrangian (4.1) in the following form.

$$L = \frac{1}{2} \int (\dot{\varphi} - T m \varphi \dot{\Theta}) K[\varphi] (\dot{\varphi} + T \mathrm{m} \varphi \dot{\Theta}) - \int V(\varphi).$$

The canonical moments conjugate to variables φ and Θ are

$$\pi = K(\dot{\varphi} + T m \dot{\Theta}) = (\dot{\varphi} - T m \dot{\Theta}) K,$$

$$P = \int \pi T m \varphi = - \int \varphi T m \pi.$$

The equation for the momentum P is the constraint equation which signifies the existence of the gauge symmetry in the soliton sector of the theory. The equations of motion of the static configuration are easily derived:

$$\frac{1}{2}\dot{\Theta}_a \frac{\delta I_{ab}}{\delta \varphi} \dot{\Theta}_b - \frac{\partial V}{\partial \varphi} = 0, \tag{4.3}$$

or more explicitly,

$$-(\dot{\Theta}\, m\, T (K[\varphi]\, \dot{\Theta}\, m\, T\varphi)) + \frac{1}{2}(\dot{\Theta}\, m\, T\varphi)\frac{\partial K}{\partial \varphi}(\dot{\Theta}\, m\, T\varphi) - \frac{\partial V}{\partial \varphi} = 0.$$

The equation above has a solution. It is a static soliton. In the general case, the solution is the distorted classical solution. However, the question of the stability of the solution comes at this point. The solution of the equation (4.3), in general is not the same as the solution of $\partial V/\partial\varphi = 0$. The two solutions may not even be related in a simple way as in the two-dimensional example of Section 2. This is understandable considering the fact that the collective coordinates imply the rotation and vibration of the static configuration. In other words, there will be additional forces changing the shape of the soliton from the initial static, very symmetric configuration. In general, we will conclude that the soliton exists, not by finding the solution of the equation (4.3) but by calculating the moments of inertia corresponding to collective coordinates and by finding them finite. In other words, if the moment of inertia, $I_{ab} = \int (m\, T\varphi)_a K (m\, T\varphi)_b$, is finite it means that the action of the inertial force is stabilized and that the soliton will be deformed. If the moment of inertia I_{ab} is infinite, it means that there is nothing that can stabilize the inertial force and that the soliton will become more and more deformed and eventually vanish. In this case we conclude that the soliton is unstable. The estimate of the decay time can be obtained from the effective Hamiltonian which we derive next.

The canonical Hamiltonian is given by

$$H = \frac{1}{2}\int \pi\, K^{-1}[\varphi]\, \pi + \int V(\varphi). \tag{4.4}$$

We would like to write the Hamiltonian (4.4) in terms of independent variables only. In order to do that we need to introduce the gauge condition[7]. However, since we are satisfied with the semiclassical quantization of the soliton of the theory and because we will neglect all the fluctuations of the configuration fields φ and π, the explicit form of the gauge condition is not needed for in the remainder of this work. However, for completeness, we specify the following useful gauge condition: $\int f\varphi' = 0$, where the gauge fixing function f is arbitrary. With the appropriate choice of the gauge fixing function f, in the specified gauge condition it is possible to decouple all zero modes. It is also useful to linearize the constraint, $P - \int \pi\, Tm\cdot\varphi = 0$, by the transformation of the momentum π

$$\pi = f\frac{P - \int \bar{\pi}(T\, m\, \varphi - c\, K^{-1} f)}{\int f\, T\, m\, \varphi} + \bar{\pi}, \tag{4.5}$$

where c is some suitably chosen constant. The zero mode is the zero frequency eigenvector of the soliton stability equation $\frac{\delta^2 L}{\delta \varphi^2}|_{\varphi_0} = 0$. Upon the quantization, the parameters describing the zero modes become collective coordinates. It is given by $\varphi_0^a = \frac{\partial \varphi}{\partial \Theta_a}|_{\Theta_a = 0} = \frac{\partial S}{\partial \Theta_a}|_{\Theta = a - 0}\, \varphi_c$, where φ_c is the classical solution of the model, $\frac{\delta V(\varphi)}{\delta \varphi}|_{\varphi c} = 0$. After the transformation (4.5) the linearized constraint is

$$c \int \bar{\pi} K^{-1} f = 0.$$

The effective Hamiltonian expressed in terms of the fields φ and π, the collective coordinate Θ and its conjugate momenta P reads

$$H = \frac{1}{2} (\int f K^{-1} f) \frac{(P - \int \bar{\pi} (T m \varphi - K^{-1} f))^2}{(\int f T m \varphi)^2} + \frac{1}{2} \int \bar{\pi} K^{-1} \bar{\pi} + \int V(\varphi). \tag{4.6}$$

It is convenient to choose the gauge fixing function $f = K\, T\, m\, \varphi / c$, so that the factor in front of the first term in (4.6), $\int f K^{-1} f = 1$. Ignoring the quantum fluctuations of the configuration fields φ and $\bar{\pi}$, the Hamiltonian that can be used to study the quantum mechanics of the collective coordinates is

$$H = P_a \left(\frac{1}{I} \right)_{ab} P_b + M,$$

where, I_{ab} are the moments of inertia, and M is the mass of the soliton. The eigenstates of the Hamiltonian are the quantum states of the soliton. However, the Hamiltonian above makes sense only if all the moments of inertia are finite. Only if the moments of inertia are finite, the minimum soliton configuration is given as the solution of the equation (4.3). To proceed we study the specific model.

We have seen in the above that the stability of the soliton solution becomes the question of the finiteness of the moments of inertia corresponding to collective coordinates. This question is very specific because the answer depends on the model under study. In what follows, we will give an equation for calculating the life time of the soliton under the assumption that some of the moments of inertia are infinite. Note that this is different from stability of the solitons under the quantum fluctuations of the configuration field. Also, we will illustrate the above general considerations on the example of the Skyrme model soliton which was introduced in Section 3.

Our general procedure is designed to treat all the collective coordinates. It is useful at this point to give an example. We will illustrate the general discussion on the example model of Section 3. First, let us quickly examine the two-dimensional case of Section 2. In the two-dimensional model the symmetry group $\mathscr{G}$ is the translation group. The generator T is the translation operator $-\mathrm{i} \frac{\mathrm{d}}{\mathrm{d}x}$ and the moment of inertia is simply the mass of the soliton. The solution of the equation (4.3) in this case is given by (2.1). Using this solution, one can recalculate the moment of inertia (M) and one finds that it is finite. Therefore, the soliton of our two-dimensional example is stable.

We now turn to the Skyrme model. For simplicity, we will only consider the rotational collective coordinate. The static soliton solution is given by (3.1). Due to the rotational symmetry, we can obtain another soliton solution of the same energy from (3.1) by applying the rotation. Therefore, there is a degeneracy in the soliton sector. The rotation operator of the soliton becomes a collective coordinate. Therefore, we should consider a configuration

$$\tilde{U}(\vec{x}, t) = U(R(t)\vec{x}) = \exp(\mathrm{i}G(|\vec{x}|)\,\vec{\tau}\cdot R(t)\,\vec{x}).$$

In the configuration above, the matrix $R(t)$ is the rotation matrix and it becomes a collective coordinate. Inserting the configuration U into the Lagrangian of the Skyrme model one obtains

$$L = -M[G] + \frac{\vec{\omega}^2}{2} I[G].$$

The effective Hamiltonian that is used to find the soliton quantum states is obtained using the recipe given above. It reads

$$H = M + \frac{\vec{J}^2}{2I},$$

where the moment of inertia I is given by

$$I[G] = \frac{8\pi}{3e^2 F}\int_0^\infty \mathrm{d}y\, y^2 \sin^2 G\left[1 + 4\left(\left(\frac{\partial G}{\partial y}\right)^2 + \frac{\sin^2 G}{y^2}\right)\right],$$

and the mass $M[G]$ is given in Section 3. The soliton configuration G satisfies the equation

$$-\frac{\delta M}{\delta G} + \frac{\omega^2}{2}\frac{\delta I}{\delta G} = 0.$$

Defining the constant $b^2 = 4\omega^2/3e^2 F^2$, the equation satisfied by the configuration G reads

$$-(y^2 + 8\sin^2 G)\,G'' - 2y\,G' - 4\sin 2G\,G'^2 + \sin 2G + 4\frac{\sin^2 G}{y^2}\sin 2G -$$

$$-b^2\,[y^2 \sin 2G + 8\sin^2 G \sin 2G - 4y^2\sin^2 G\,G'' - 8y\sin^2 G\,G'] = 0.$$

The solution to the equation is obtained numerically. For $y \to 0$, the function G is linear, with negative slope and intercept π which defines the boundary condition at $y = 0$, and for $y \to \infty$, the function G approaches zero as $j_2(by)$ and $n_2(by)$, where j_2 and n_2 are the spherical Bessel and Neumann functions of order 2. This means that the solution G has an asymptotic tail that behaves roughly as $1/y$. Going back now and recalculating the moment of inertia I, one finds that for large values of y, the integral that defines I is dominated by the asymptotic tail $1/y$ of the solution G, making it linearly divergent. Thus is the result of Ref. 5. More specifically, one has ($R \to \infty$)

$$I \sim \int^{R} \mathrm{d}y \, y^2 \sin^2 \frac{1}{y^2} \approx \int^{R} \mathrm{d}y = R.$$

In other words, as the soliton is rotating, it slows down because its moment of inertia grows. The reason for the increase in the moment of inertia is in the rotation which squashes the soliton. In other words, the soliton emits particles (massless pions) which carry away the energy, making the soliton more extended in space. Effectively, the soliton becomes surrounded by a cloud of mesons. In such a situation, one can think of the soliton as a rotating body in a viscuous fluid. The rotation is slowed down by the drag force, and will eventually stop. We can estimate the ratio of the characteristic rotation and radiation times using simple arguments. The drag force acting on the rotating soliton in a fluid composed of massless pions is proportional to the viscosity of the medium (pion bath) and the area of the soliton and the angular frequency of the soliton rotation. One can write, $F \sim \mu a^2 \omega$, where $\mu \approx \omega/a^2$, is the viscosity, the momentum lost by the soliton per unit area, a is the linear dimension of the soliton, and ω is the rotation frequency. The power radiated by the soliton is then proportional to the negative of the product of the drag force and the linear velocity of the soliton, $P \sim -a \omega^3$. Since the frequency of the radiating pion field is ω, the typical radiation time is given by the ratio of the power and the energy of radiation, $\tau_{rad} \approx \frac{1}{a \omega^2}$. The typical rotation time of the soliton rotating with frequency ω is $\tau_{rot} \approx 1/\omega$. Therefore we have the following ratio of the radiation and the rotation characteristic times

$$\frac{\tau_{rad}}{\tau_{rot}} = \frac{\omega}{\omega^2 a} = \frac{1}{\omega a}.$$

Since the rotation frequency is inversly proportional to the coupling constant, $\omega \approx 1/N$, and the effective radius of the soliton is of the order $a \approx N^0$, we find that the ratio of the characteristic radiation and the rotation times is proportional to the coupling constant,

$$\frac{\tau_{rad}}{\tau_{rot}} = N.$$

In the soliton sector, the coupling constant N is large, and therefore, we find that the radiation characteristic time is much larger than the rotation characteristic time. Therefore, the soliton will make many rotations before the radiation takes away enough energy to slow it down. This is in complete agreement with Ref. 8 where the same result was achieved by different method.

5. *Conclusions*

In this paper we have discussed the condition for the existence of the soliton after the semiclassical quantization is performed. In particular, we have used the Skyrme model soliton as an example. In general, the equation of motion for the soliton will always be solvable. However, this by itself does not guarantee the stability of the soliton against the emission of the elementary quanta of the theory. In the massless Skyrme model, the chiral soliton can emit massless pions and loose energy. The existence of such effects is not possible to determine from the generic soliton model because they will depend on the specifics of the dynamics of the model under study. For example, in the Skyrme model, the massless pions can take away the energy from the energy from the soliton and slow its rotation. We have shown by heuristic arguments that the characteristic radiation time is much longer than the typical rotation time. The constant of the proportionality is the coupling constant of the model, which in general is large in the soliton sector of the model.

Keeping in mind the practical value of the Skyrme model and it's soliton, we have found that the radiation time is large enough (compared to the Skyrmion rotation time) to make sense of the perturbation expansion of the effective theory arround the static rotating soliton. The further improvements of the model, like including the pion mass, and making the soliton a more realistic representation of the Baryons by including the effects of valence quarks, will certainly remove the problems that the bare soliton of the Skyrme model is experiencing. The further study of the Skyrme model should certainly take that direction.

Addendum

I would like to make here some comments about the recent work of Ref. 9, which was brought to my attention by the referee, and in which a claim opposite to the conclusion of this paper as well as Ref. 5, 8 and 10 was made. The Ref. 9 makes a statement that the consistent treatment of the quantized Skyrmion leads to an effective equation of motion for the configuration function $F(r)$ which developes no tail of the form $1/y$ thus making the Skyrmion moment of inertia finite (compare with an example at the end of the Section 4). Their claim is based on the axial current conservation law. There are two problems with such an approach. First, after the quantization, the axial current has a tendency to develope the anomaly (they observe that but they fail to calculate the anomaly of their quantum current and incorporate it into the quantum equation of motion). With some additional analysis, one can see that it is this anomaly which makes the Skyrmion unstable. Secondly, they forget to solve the equation of motion for the soliton rotation velocity ω. On the other hand, the approach taken in this paper, as well as the approach of Ref. 5, 8 and 10, assumes nothing about the anomalous divergence of the axial current (we don't even use the current to derive the equations of motion). The only assumption is that the Skyrmion rotates slowly. Then, the conclusion that the Skyrmion looses energy by rotation and that the characteristic radiation time is large compared to the characteristic rotation time is also a consistency check on the assumption made about the rotation velocity. This makes the procedure self consistent. One way or the other, what matters most is that the Skyrmion is a sensible first approximation.

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O POSTOJANJU KIRALOG SOLITONA

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U ovom članku dana je diskusija o uvjetima postojanja sferno simetričnog kiralnog solitona. Pokazano je da sistem ima netrivijalni minimum koji odgovara rotirajućem kiralnom solitonu nakon što je provedena kvantizacija sistema koristeći kolektivne koordinate. Stabilnost solitona postaje problem konačnosti momenata inercije koji odgovaraju kolektivnim stupnjevima slobode. U slučaju Skyrmovog modela, rotacioni momenat inercije postaje divergentan, ali karakteristično vrijeme rotacije je mnogo dulje od karakterističnog vremena radijacije. Zbog toga je poluklasičan opis sistema gdje je soliton tretiran kao rotirajuće klasično tijelo, dobra prva aproksimacija.