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COMPARATIVE TOPOLOGICAL DISTRIBUTIONS IN NEUTRON AND DEUTERON TARGETS

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The paper presents a detailed study of the characteristics of topological distribution of *pd* and *pn* interaction at 400 GeV/c and comparison with the different scaling laws and with other data. Also the energy dependence of $\langle n \rangle$ in *pd* and *pn* interactions has been studied using available data and compared with the predictions of various popular particle production models.

1. *Introduction*

The deuteron as a target is particularly attractive because it constitutes the simplest multi-nucleon system. It thus enables us to investigate the transition of particle production phenomenon from hadron-hadron to hadron-nucleus interactions.

Scaling effects of multiplicity distributions and the mean multiplicity $\langle n \rangle$ in *pp* collisions have been studied exhaustively for a wide range of energy. It has thus been observed[1] that the energy dependence of $\langle n \rangle$ can be described well by functions of the form

$$a + b \log S + C (\log S)^2$$

or

$$a + b \log S + CS^{-1/2}.$$

pp multiplicity distribution are found to obey the Koba-Nielsen-Oleson scaling law over the entire accelerator energy region[2]. The absence of such detailed studies in the case of pd and pn interaction provided the main motivation for the present work.

In this paper we investigate the characteristics of multiplicity distribution of pd and pn interactions at 400 GeV/c and compare them with the different scaling laws. We also study the energy dependence of mean multiplicity $\langle n \rangle$ in pd and pn interaction and its fit with the predictions of popular particle production models.

2. *Multiplicity scaling*

The KNO scaling for multiparticle distribution at infinite energies was derived by Koba, Nielsen and Oleson[3] starting from Feynman's scaling for all many-particle inclusive cross-sections and by neglecting quantities of the order of $1/\log S$. The KNO scaling prediction may be written in the following form

$$\sigma_n/\sigma_{inel} \xrightarrow[s \to \infty]{} \langle n \rangle^{-1} \, \psi(n/\langle n \rangle)$$

where σ_n is the partial cross-section for the production of n particles of a certain type at c. m. energy $\sqrt{s}$, σ_{inel} is the total inelastic cross-section of the reaction, $\langle n \rangle$ is the average number of charged particles produced at that energy, and ψ is an energy-independent scaling function.

A simple empirical modification of Eq. (1) was proposed by Bura's et al.[4] to extend KNO scaling at low energy. The modified KNO scaling law has the form

$$P(n) = [1/(\langle n \rangle - a)] \, \psi(Z')$$

where $Z' = (n - a)/(\langle n \rangle - a)$ is constant and is independent of energy but may depend on the type of interactions. The function $\psi(Z')$ has the form

$$\psi(Z') = A(Z' + B) \exp(CZ' + DZ'^2). \qquad (1)$$

It has been shown[4] that the function

$$\psi(Z') = 2.30(Z' + 0.142) \exp(-0.0586Z' - 0.659Z'^2)$$

can describe all the pp interaction data in the momentum range (5.5—400) GeV/c with $a = 0.9$.

A functional form of $\psi(Z)$ has been defined by Slattery[2] given by

$$\psi(Z) = (AZ + BZ^3 + CZ^5 + DZ^7) \exp(EZ). \qquad (2)$$

This KNO scaling hypothesis has been verified for pp interaction in the energy range (50—300) GeV/c by Slattery[2] using the function

$$\psi(Z) = (3.79Z + 33.7Z^3 - 6.64Z^5 + 0.332Z^7)\,e^{-3.04Z}.$$

Third is an empirical formula of Czyzewski-Rybicki which is as follows[5]

$$D\sigma_n/\sigma_{inel} = f\left(\frac{n - \langle n\rangle}{D}\right) \tag{3}$$

with

$$f(x) = Zd\,e^{-d^Z} \frac{d^{Z(xd + dZ)}}{(xd + d^Z + 1)}$$

where d is the only parameter required.

3. *Energy dependence of multiplicity*

The behaviour of mean charged multiplicity as a function of energy has been compared by several authors with the predictions of various theoretical models. Some models predict an energy dependence of $\langle n\rangle$ of the $\ln S$ type and others predict a power dependence of a dependence faster than log S[6]. However there is no such detailed study in case of pn or pd interactions.

The predictions of some of the popular models are as follows:

1) Fermi's statistical model and Belenjki and Landau's hydrodynamical model predict $\langle n\rangle \sim S^{1/4}$,
2) the multiperipheral model and the Muller-Regge analysis predict $\langle n\rangle \sim \ln S$,
3) the Cheng-Wu model gives $\langle n\rangle \sim S^B$ $B > 0$,
4) the thermodynamic models predict an energy dependence of $\langle n\rangle$ faster than S.

Obviously this test is far from complete. However we have used the following forms of energy dependence to fit the pd and pn data at all available energy range.

$$\langle n\rangle = AS^B \tag{4}$$

$$\langle n\rangle = A + B\ln S \tag{5}$$

$$\langle n\rangle = A + B(\ln S)^2 \tag{6}$$

$$\langle n\rangle = A + B\ln S + C(\ln S)^2 \tag{7}$$

$$\langle n\rangle = A + B\ln S + CS^D. \tag{8}$$

We use experimental data obtained in our laboratory by scanning of 35 mm Bubble chamber films on pd interactions at 400 GeV., exposed at the 30″ chamber at Fermilab, U. S. A. We calculate the mean pd multiplicity (from Table 1) to be 9.42 ± 0.11.

4. *Experimental details*

A total sample of 33 000 frames were initially scanned. Of these, 43 % were finally double scanned and these data used in the present communication. The chamber density was taken as 0.1364 gm/cm^3 and the fiducial length as 45 cm. The double scanning efficiency was estimated to be $99 \pm 1\%$.

Multiplicity measurements were done for $n_{ch} > 3$. The large density of tracks in the forward direction made prong counting very laborious.

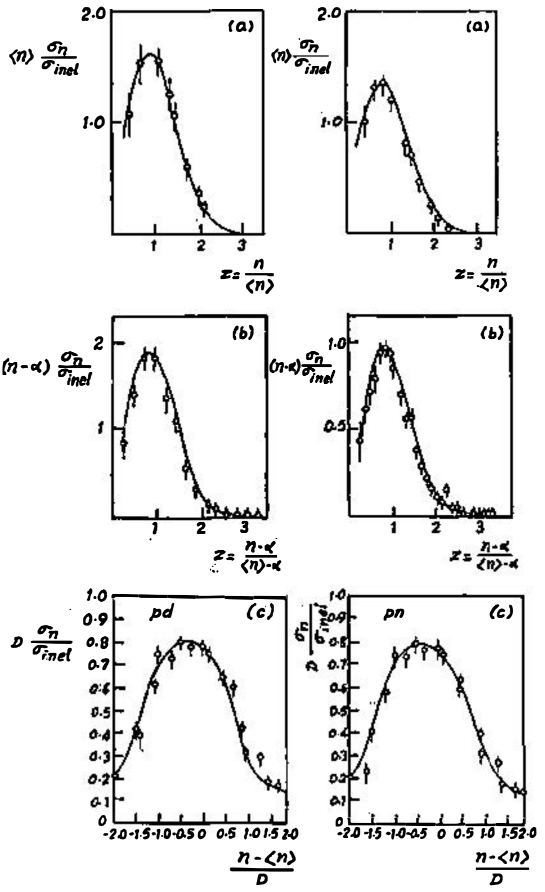

Fig. 1. a) *pd* multiplicity distribution in terms of KNO variable. The curve represents KNO scaling function given by Eq. (1). b) *pd* multiplicity distribution in terms of Bura's variable. The curve represents Bura's scaling function given by Eq. (2). c) *pd* multiplicity distribution in terms of Czyzewski-Rybicki variable. The curve represents Czyzewski-Rybicki functions given by Eq. (4).

Fig. 2. a) *pn* multiplicity distribution in terms of KNO variable. The curve represents KNO scaling function given by Eq. (1). b) *pn* multiplicity distribution in terms of Bura's variable. The curve represents Bura's scaling function given by Eq. (2). c) *pn* multiplicity distribution in terms of Czyzewski-Rybicki variable. The curve represents Czyzewski-Rybicki functions given by Eq. (4).

TABLE 1.

Interaction.	χ^2/NDF		
	KNO Eq. (1)	BURAS Eq. (2)	CZYZEWSKI-RYBICKI Eq. (3) with $d = 1.5$.
pd	0.95	1.01	1.05
pn	0.90	1.01	1.04

Table 1. Results of the fit of *pd* and *pn* data to different scaling functions.

TABLE 2.

Formula used	*pd*			*pn*		
	A	B	χ^2/NDF	A	B	χ^2/NDF
$\langle n \rangle = AS^B$	1.00 ±0.01	0.24 ±0.01	0.46	1.00 ±0.01	0.24 ±0.01	0.70
$\langle n \rangle = A + B \ln S$	−0.67 ±0.03	0.39 ±0.01	0.26	−0.64 ±0.03	0.38 ±0.01	0.60
$\langle n \rangle = A + B (\ln S)^2$	0.25 ±0.01	0.018 ±0.01	0.40	0.18 ±0.01	0.012 ±0.001	0.65

Table 2. Results of the fit to *pd* and *pn* multiplicity data with different predictions.

TABLE 3.

Interaction	400 GeV	
	pd	*pn*
$\langle n \rangle$	9.41537	8.45037
$\langle n^2 \rangle$	113.2380	93.09738
$\langle n^3 \rangle$	1612.6455	1226.1133
$\langle n^4 \rangle$	26111.0235	18577.5243
$\langle n^5 \rangle$	468734.553	316782.823
$\langle n \rangle (\langle n^2 \rangle - \langle n \rangle^2)^{-1/2} = \langle n \rangle / D$	1.8988	1.814516
$D = (\langle n^2 \rangle - \langle n \rangle^2)^{1/2}$	4.9587	4.6571
g_1	9.4154	8.45037
g_2	103.9226	84.647
g_3	1291.7622	963.7219
g_4	17624.2765	12194.2135
f_3	28.5057	24.68797
f_2	15.1735	13.23817
f_4	69.3750	62.3194

Table 3. Different multiplicity moments for *pd* and *pn* interactions.

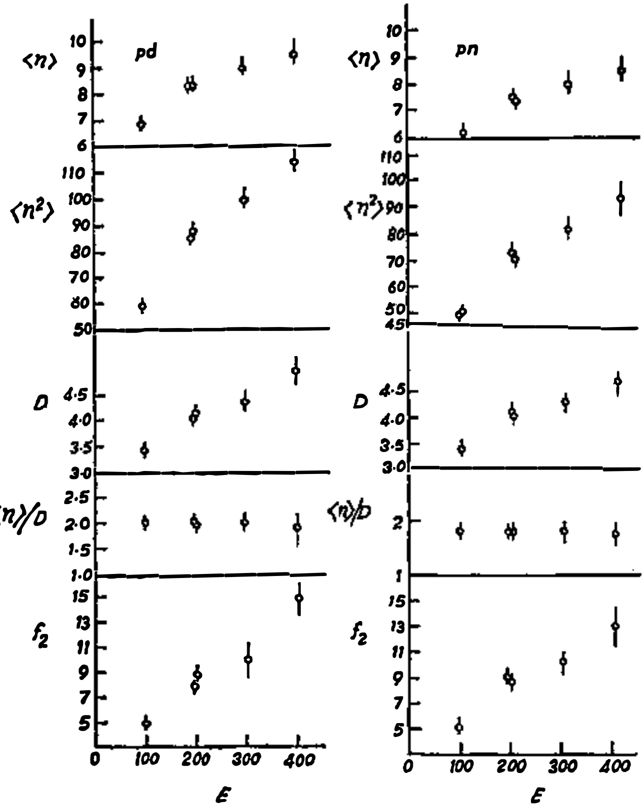

Fig. 3. a) Energy dependence of $\langle n\rangle$, $\langle n^2\rangle$, D, $\langle n\rangle/D$ and f_2 for *pd* interactions. b) Energy dependence of $\langle n\rangle$, $\langle n^2\rangle$, D, $\langle n\rangle/D$ and f_2 for *pn* interaction.

5. *Results and discussion*

In Fig. 1 (a) we present the *pd* multiplicity distribution in terms of the KNO scaling variable. The curve represents KNO scaling function (Eq. (1)). Fig. 1 (b) shows the similar comparison with Bura's scaling function (Eq. (2)) and Fig. 1 (c) shows comparison with Czyzewski-Rybicki scaling function. We present *pn* multiplicity data in Fig. 2 (a, b, c). The x^2 values are given in Table 1.

The energy dependence of the mean multiplicities $\langle n\rangle$ for *pd* and *pn* interactions has been fitted to the formulas (4, 5, 6). The fits have been performed with standard CERN program MINUIT. The fits wits three parameters have not been performed due to the insufficient number of data points. Table 2 gives the results of the fits to *pd* and *pn* multiplicity data. Table 3 presents the values of different moments of multiplicity distributions. The definitions of f_n and g_n are as follows:

$$g_1 = \langle n\rangle; \quad g_2 = \langle n(n-1)\rangle;$$

$$g_3 = \langle n(n-1)(n-2)\rangle; \quad g_4 = \langle n(n-1)(n-2)(n-3);$$

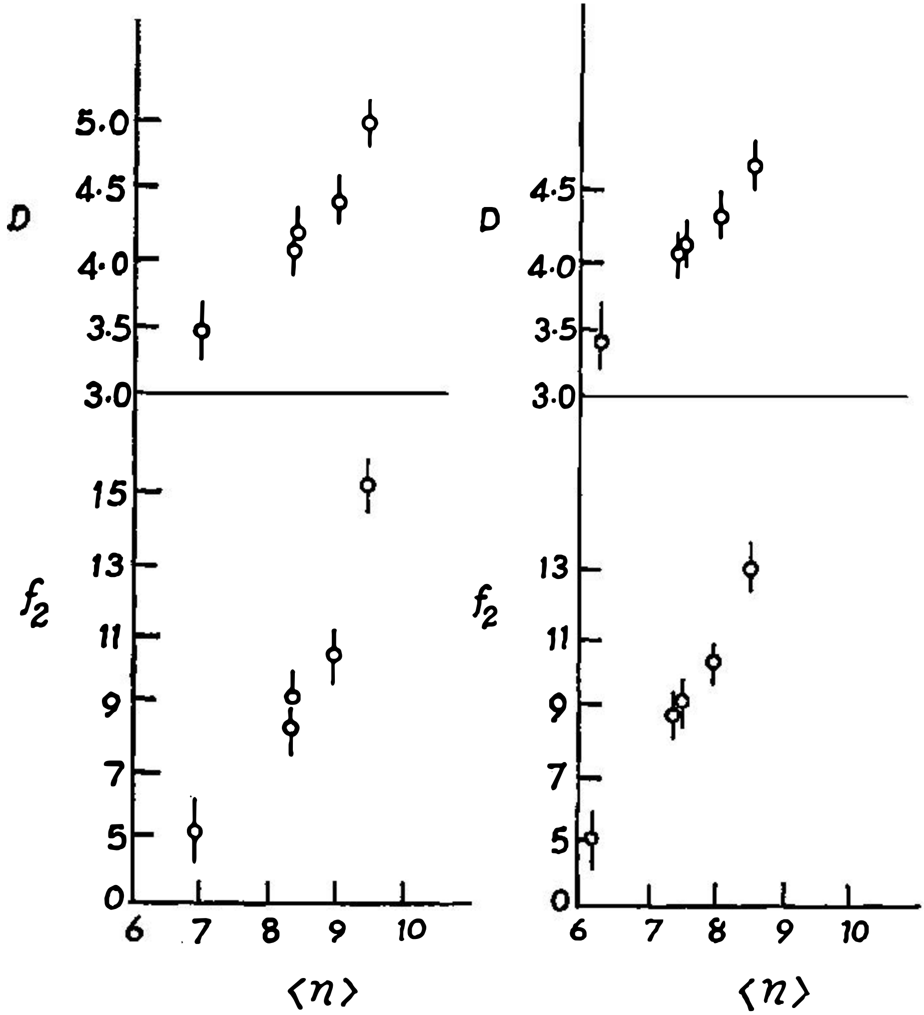

Fig. 4. a) Plot of Muller moment f_2 and multiplicity dispersion D against $\langle n \rangle$ for pd interactions. b) Plot of Muller moment f_2 and multiplicity dispersion D against $\langle n \rangle$ for pn interaction.

$$f_2 = g_2 - g_1^2; \quad f_3 = g_3 - 3g_2g_1 + 2g_1^3$$

$$f_4 = g_4 - 4g_1g_3 + 12g_1^2g_2 - 3g_2^2 - 6g_1^4.$$

Further the energy dependence of $\langle n \rangle$, $\langle n^2 \rangle$, D, $\langle n \rangle / D$ and f_2 for pd and pn interactions are shown in Fig. 3 (a) and 3 (b), respectively. The dependence of f_2 moment which is known as 1-st Muller correlation parameter on E can be represented by:

$$f_2 = -0.95 + 0.45 \ln E + 0.66 (\ln E)^2 \text{ for } pd$$

$$f_2 = -7.80 + 1.50 \ln E + 0.44 (\ln E)^2 \text{ for } pn.$$

TABLE 4.

Interaction	A	B	χ^2/NDF
pd	0.55 $\pm$.02	0.45 $\pm$0.02	1.2
pn	0.55 $\pm$0.02	0.03 $\pm$0.001	1.4
pp [Ref] p-Em.	0.62	1.16	

Table 4. Result of fit of multiplicity dispersion data for pd and pn interactions along with pp and p-emulsion data from Ref. 11.

TABLE 5.

Interaction	Y_2
pd	0.30
pn	0.30
pp	-
p-Emulsion	0.29

Table 5. Result of fit of Muller moment f_2 for pd and pn interactions along with pp and p-emulsion data from Ref. 11.

Lastly in Fig. 4 (a, b) we present the dependence of D and f_2 on $\langle n \rangle$ for pd and pn interactions, respectively.

For D a linear dependence on $\langle n \rangle$ is observed in both cases. The parameter for the linear fit $D = A \langle n \rangle - B$ is shown in Table 4. Also the corresponding values in case of pp and p-emulsion interactions (which have been taken from Ref. 11) are shown in the table for the sake of comparison. In the case of f_2 we have fitted with the expression $f_2 = - \langle n \rangle (1 - y_2 \langle n \rangle)$ and the results are shown in Table 5.

Thus this paper presents an extremely comprehensive analysis of the characteristics of the multiplicity distribution and their energy dependence in pd and pn interactions using all available data including the new data of ours at 400 GeV/c.

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USPOREDNE TOPOLOŠKE RASPODJELE U NEUTRONSKIM I DEUTERONSKIM METAMA

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Dana je detaljna analiza karakteristika topoloških raspodjela *pd* i *pn* međudjelovanja kod 400 GeV/c te usporedba s različitim zakonima skaliranja i drugim eksperimentalnim podacima. Proučavana je energetska ovisnost $\langle n \rangle$ kod *pd* i *pn* međudjelovanja uz korištenje dostupnih podataka te uspoređena s predviđanjima različitih popularnih modela produkcije čestica.