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LETTER TO THE EDITOR

MESON EXCHANGE CUTOFFS AND CHIRAL SOLITONS

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It is argued that the chiral soliton models provide a natural basis for the otherwise *ad hoc* distance regularization in hadron form factors. We present a proposal of how one can qualitatively connect the cutoffs in the phenomenological boson exchange models with the finite size of chiral solitons and illustrate it with a rough calculation for the case of the nucleon-omega form factor.

The rediscovery of the Skyrme model[1] has greatly contributed to the comeback of descriptions of low-energy phenomenology in terms of meson degrees of freedom[16]. Works of 't Hooft, Balachandran et al. and Witten[2,3] showed that such an effective, purely meson theory may in fact be well-based in QCD, although it makes no explicit reference to quarks and gluons. Whereas in the conventional Skyrme model[1,4] only pions are present, Adkins and Napp[6] replaced Skyrme's stabilizing term by one which stabilizes the chiral soliton by omega-exchange. Subsequently the other light vector mesons were introduced in the chiral soliton model[17,18]. This immediately raises the question of the relation between the Skyrme model and the boson exchange models, which are important for the description of the nucleon-nucleon interaction. The understanding of this relation can be of much benefit to the study of the nucleon-nucleon interaction in the Skyrme model, where many practical and conceptual difficulties still remain. On the other hand, the soliton nature of the nucleon should be incorporated into the boson

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exchange models to remove some arbitrariness which arises from the introduction of short distance cutoffs there. Namely, Skyrmions are extended objects and their finite size provides a natural cutoff for meson exchange as the interbaryon distance goes to zero[16,19]. The purpose of this paper is to sketch a proposal of how one can qualitatively connect meson exchange cutoffs and the finite extension of Skyrmions and illustrate it with a crude calculation of such a cutoff in the omega-stabilized chiral soliton model.

The property of spatial extension of a hadron, which is naturally present in the chiral soliton model, can be crudely related to the parametrization of form factors in the old phenomenological approach. Namely, from the energy due to ω-exchange repulsion we effectively obtain the form factors with roughly the same short distance cutoffs as the ones used in the Bonn potential[10,11].

The form factors reflect the fact that hadrons have finite spatial extension. In the boson-exchange models, for example in the Bonn potential[10], when we calculate interactions between point nucleons, we introduce by hand these form factors parametrized by cutoffs which render short-distance interaction finite and in reasonable agreement with data. Alternatively, in the Paris potential[13] short distance interaction is replaced by a completely phenomenological one from NN scattering data.

Whereas investigators often wish to attribute the cutoffs of boson-exchange interactions at short distances to the quark-gluon substructure of the nucleon, we note that already in the Skyrme model, where the radius of the quark-gluon core has been taken to zero, there will be cutoffs because of the finite extent of the nucleon[19]. There are also hybrid models, topological bags[8], which contain both quarks and meson solitons. In fact, the omega-stabilized topological bag[7], consisting of quark-gluon core and soliton cloud, is closely related to the omega-stabilized Skyrmion discussed here. One can then ask the question what could be the differences from the regularization via a pure ω-stabilized Skyrmion and ω-stabilized two-phase model. We do not expect these differences to be large because the size of the core (which is roughly $R \approx 0.5\,\mathrm{fm}$ for the optimum description of the nucleon in the two-phase model[7,9] is not large. Also, when such quark cores overlap, due to Pauli principle one must get a repulsion, just one gets it from ω-exchange.

Form factors give a transfer momentum dependence to the vertex coupling of a given meson field, α, to the nucleon:

$$g_{NN\alpha}(p^2, p'^2, k^2) = g_{NN\alpha}(m^2\ m_N^2, m_\alpha^2)\, F_\alpha(p^2, p'^2, k^2). \tag{1}$$

Form factors F_α are determined empirically. For instance, Holinde[10] gives the following parametrization for the form factor of the nucleon-nucleon-omega vertex:

$$F_\omega(k^2) = \left(\frac{\Lambda^2 - m_\omega^2}{\Lambda^2 - k^2}\right)^{3/2} \tag{2}$$

and similarly for other mesons. Λ is the so-called cutoff mass parameter which has the same value for all the mesons, and was determined by Holinde[10] from NN scattering phase shifts to be $\Lambda = 1.530$ GeV. (Other authors, like Dominguez and Verwest[12]) have values for that parameter which are somewhat smaller, but generally still of the order of 1 GeV.)

We can present a qualitative argument that such parametrizations and parameter values are consistent with the chiral soliton model. In the Skyrme model with the ω-meson stabilization, the vector meson ω_μ is coupled to the vector baryon current b_μ. Simplifying assumptions (time independence, spherical symmetry) however, leave us with the Lagrangian[6])

$$\mathscr{L} = \frac{1}{2}\left(\frac{\mathrm{d}\omega}{\mathrm{d}r}\right)^2 + \frac{m_\omega^2}{2}\omega^2 - g_\omega \omega \varrho - \frac{F_\pi^2}{8}\left[\left(\frac{\mathrm{d}\Theta}{\mathrm{d}r}\right)^2 + \frac{2\sin^2\Theta}{r^2}\right] - \\ - \frac{1}{4}F_\pi^2 m_\pi^2 (1-\cos\Theta) \tag{3}$$

in which we see that in the rest frame of the soliton the only remaining component of the omega-field, the time component $\omega \equiv \omega_0$, couples to the only remaining component of the baryon current, the baryon density $\varrho(r) = b^0(r) = \Theta' \sin^2\Theta / 2\pi^2 r^2$. In the rest frame of the nucleon this looks exactly like the coupling of a scalar field to a scalar density. In this frame, the energy due to the omega-repulsion of the two Skyrmions separated by a distance R is

$$E_\omega(R) = g_\omega \int \varrho(r) V_\omega(\vec{r}-\vec{R})\,\mathrm{d}^3 r \tag{4}$$

where

$$V_\omega(\vec{r}) = \frac{g_\omega}{4\pi}\int \frac{\exp(-m_\omega|\vec{r}_1-\vec{r}|)}{|\vec{r}_1-\vec{r}|}\varrho(\vec{r}_1)\,\mathrm{d}^3 r_1 \tag{5}$$

if we assume that the presence of one Skyrmion does not change the baryon density distribution of the other, at a distance $\vec{R}$. This assumption is clearly well founded for sufficiently large baryon separations, but in fact becomes more and more suspect as $\vec{R}$ diminishes. Note also that we can study the nucleon-omega form factor while dealing with (unprojected) Skyrmions only and not with real nucleons. The omega-meson is an isoscalar and its zeroth component (whose source is simply baryon number density b^0) is left unaffected by the rotations in the isospin space which project nucleon and delta out of the Skyrmion. The coupling of the omega meson to b^0 of the Skyrmion and to b^0 of the nucleon is then trivially the same, $g_\omega = g_{NN\omega}$.

After some computation we get

$$E_\omega(R) = \frac{g_\omega^2\pi}{m_\omega}\int_0^\infty r_2^2\varrho(r_2)\int_0^\infty r_1\varrho(r_1)\left[\int_{-1}^{1}\frac{\exp\{-m_\omega|r-\sqrt{R^2+r_2^2-2Rr_2x}|\}}{\sqrt{R^2+r_2^2-2Rr_2x}}\mathrm{d}x - \right. \\ \left. - e^{-m_\omega r_1}\frac{\exp\{-m_\omega|R-r_2|\}-\exp\{-m_\omega(R+r_2)\}}{m_\omega R r_2}\right]\mathrm{d}r_1\mathrm{d}r_2. \tag{6}$$

After the model equations are solved, the baryon density $\varrho(r)$ is inserted in the expressions above and the approximate ω-repulsion computed numerically.

On the other hand, we can try to write the ω-interaction energy from the point of view of the phenomenological approach and compare it with the above result. We will not do it rigorously but only roughly because the aim of our discussion is only qualitative anyway. Since in the rest frame of the Skyrmion our omega-meson is equivalent to a scalar meson, we can crudely write the repulsive potential energy of two nucleons due to ω-exchange as the Fourier transform

$$U_\omega(\vec{R}) = \frac{1}{(2\pi)^3} \int e^{i\vec{k}\cdot\vec{R}} gNN_\omega F_\omega(k^2) \frac{1}{\vec{k}^2 + m_\omega^2} g_{NN\omega} F_\omega(k^2)\, d^3k. \tag{7}$$

For F_ω as defined in (2),

$$U_\omega(R) = \frac{g^2_{NN\omega}}{4\pi} \left\{ \frac{e^{-m_\omega R}}{R} - \frac{e^{-\Lambda R}}{R} \left[1 + \frac{R(\Lambda^2 - m_\omega^2)}{2\Lambda} + \right.\right.$$
$$\left.\left. + \frac{R(\Lambda^2 - m_\omega^2)^2}{8\Lambda^3} + \frac{R(\Lambda^2 - m_\omega^2)^2}{8\Lambda^2} \right] \right\}. \tag{8}$$

Now we face a big problem: Which value of the omega coupling constant should we use? The values favoured by various authors range from small couplings obtained by SU (3) arguments[14] over intermediate values from NN scattering[15] to quite large[10] coupling required by meson exchange models. We would like to use the values in the intermediate range

$$\frac{g^2_{NN\omega}}{4\pi} = 10 - 12 \tag{9}$$

which we also used in the previous work[7], but Holinde[10] as well as the newest review of the Bonn meson exchange model, Machleidt et al.[10], favuor a much larger value, $g^2_{NN_\omega}/4\pi \approx 20$. Obviously, we need a better determined value of the omega-nucleon coupling if we want to make any quantitative statements here. Fortunately, it turns out that some qualitative observations still can be made.

For the chosen values of $g^2_{NN_\omega}$, we evaluate the repulsive interaction energy due to omega-exchange for various values of the baryon separation R_i. To this potential energy $E_\omega(R)$ we try to fit the energy $U_\omega(R)$ by varying Λ in order to minimize the sum of the squares

$$\Delta = \sum_i (E_\omega(R_i) - U_\omega(R_i))^2. \tag{10}$$

The value of Λ yielding the best fit is roughly $\Lambda = 1.6$ GeV for $g^2_{NN_\omega}/4\pi \sim 10$. However, the fit is only crude and qualitative. On the other hand, using in (8) roughly twice as large coupling constant, $g^2_{NN_\omega}/4\pi \approx 20$, as in Ref. 10 the fit yields $\Lambda \approx 1.4$ GeV, a comparable result.

To conclude: Since Holinde[10] obtained the value $\Lambda = 1.530$ GeV through his phase-shift analysis, and since some other researchers favour lower values[12] although still of the order of 1 GeV, we see that on the basis of the vector-meson stabilized chiral soliton model we can qualitatively understand the cutoffs in more phenomenological boson exchange models. We have a check of the qualitative consistency between the QCD-based topological soliton model and the old phenomenological approach through cutoffs and form factors. There is little doubt that an analogous analysis of some other quantity would again yield a similar result although a more accurate determination of meson-nucleon coupling constants is evidently very much needed for any kind of quantitative comparison.

Therefore, the picture of the nucleon as an extended meson soliton, which is well based in low energy QCD, may serve to render boson exchange interactions finite and well behaved.

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REGULARIZACIONI PARAMETRI MEZONSKE IZMJENE I KIRALNI SOLITONI

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U članku se argumentira mišljenje da kiralni solitonski model predstavlja prirodnu osnovu za inače »ad hoc« regularizaciju za male udaljenosti u hadronskim formfaktorima. Predlažemo način kako se kvalitativno mogu povezati regularizacioni parametri u fenomenološkim modelima bozonske izmjene s konačnom veličinom (protegnućem) kiralnih solitona, te ilustriramo to s grubim proračunom za slučaj nukleon-omega formfaktora.

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