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THEORETICAL ANALYSIS OF THE THERMOELECTRIC POWER IN n-CHANNEL INVERSION LAYERS OF TERNARY SEMICONDUCTORS IN THE PRESENCE OF A QUANTIZING MAGNETIC FIELD

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An attempt is made to study the thermoelectric power of the electrons in *n*-channel inversion layers on ternary semiconductors under magnetic quantization, without any approximations of weak or strong electric field limits, on the basis of fourth order in effective mass theory and taking into account the interactions of the conduction, light-hole, heavy-hole and split-off bands. It is found, taking *n*-channel inversion layers on *n*-$Hg_{1-x}Cd_xTe$ as an example, that the thermoelectric power decreases with increasing surface electric field and decreasing magnetic field, respectively, in an oscillatory manner. The band non-parabolicity significantly enhances the same power in both the cases.

1. Introduction

In recent years there has been considerable interest in the study of semiconductor inversion layers which are formed at the surfaces of semiconductors under the influence of a sufficiently strong electric field applied perpendicular to the sur-

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face by means of a large gate bias[1]. In these layers, the carriers form a 2D electron gas and are free to move parallel to the surface which their motion is quantized in a direction perpendicular to it leading to the formation of electric subbands. In the presence of a quantizing magnetic field parallel to the direction of electric field, this free motion becomes quantized and 3D quantization occurs. Though considerable work has already been done, there still remain scopes in the investigations made while the interest for further researches of the different other aspects of such systems is becoming increasingly important. One such important parameter is the thermoelectric power of the electrons in semiconductors (hereafter referred to as TPM) and has extensively been investigated under various physical conditions[2–5]. The TPM is independent of scattering mechanism and in the case of spherical energy surfaces, the shape of the conduction band can be determined from its experimental determination[2]. Nevertheless, it appears from the literature, that the generalized expression for the TPM in n-channel inversion layers of Kane type semiconductors has yet to be theoretically investigated under magnetic quantization without any approximations of strong or weak electric field limits. This is very important since various aspects of inversion layers on narrow-gap semiconductors in the presence of a quantizing magnetic field are being increasingly studied for their peculiar electronic properties[1]. In particular such studies for ternary semiconductors under magnetic quantization having Kane type energy bands would be more interesting since the compound $Hg_{1-x}Cd_xTe$ is a very important optoelectronic material because its bandgap can be varied to over the entire spectral range from 0.8 μm to over 30 μm by adjusting the alloy composition[6]. Its use as infrared detector material has spurred the $Hg_{1-x}Cd_xTe$ technology for the production of high-mobility single crystal with specially prepared surface layers[7]. Furthermore, the same material is ideally suited for narrow-gap subband physics, because the relevant physical parameters are within easy experimental reach[7].

In what follows, we shall first derive the model expression for the TPM in inversion layers on ternary semiconductors under magnetic quantization without any approximations of weak or strong electric field limits and taking into account the interactions of conduction, light-hole, heavy-hole and split off bands. We shall also derive the same result by using the well-known two-band Kane model for the purpose of comparison. We shall study the surface electric field and magnetic field dependences of TPM in accordance with both the models taking n-channel inversion layers on $Hg_{1-x}Cd_xTe$ as an example.

2. *Theoretical background*

The $E - \vec{k}_z$ dispersion relation of the conduction electrons in ternary semiconductors having Kane-type energy bands in the presence of a quantizing magnetic field H along the z-direction can be expressed[8] on the basis of the fourth-order effective mass theory and taking into account the interactions of the conduction, light-hole, heavy-hole, split off band, and the influence of spin-splitting as

$$E = \left[-\tau + \hbar\,\omega_c \left(n + \frac{1}{2} \right) + \frac{\hbar^2 k_z^2}{2m^*} \pm \left(\frac{m^*}{m} \right) \frac{\hbar\,\omega_c g_0^*}{4} \pm \right.$$

$$\pm K_0 (\hbar \omega_c/E_g) \hbar \omega_c \left(n + \frac{1}{2}\right) \pm (\hbar \omega_c/E_g)(\hbar^2 k_z^2/2m^*) -$$

$$- (K_2'/E_g) \left[\left(n + \frac{1}{2}\right) \hbar \omega_c + \frac{\hbar^2 k_z^2}{2m^*}\right]^2 \Bigg], \qquad (1)$$

where E is the electron energy in the presence of magnetic quantization as measured from the edge of the conduction band in the absence of any quantization, $\tau = -J_c$, $K_2' = -K_2$, n is the magnetic quantum number and the other notations are the same as in Ref. 8. Thus following the method as given in the literature[9], the modified electron energy spectrum in n-channel inversion layers on ternary semiconductors can be expressed in the presence of a quantizing magnetic field H along z-axis as

$$\frac{2}{3}(G_{,\pm}(\gamma_{n,\pm})^3 - \frac{2}{5}(\gamma_{n,\pm}) = A(i) N_s \qquad (2)$$

where

$$G_{\pm} \equiv (\hbar^2/2m^*)(b_{\pm}/2\Theta),$$

$$b_{\pm} \equiv (\hbar^2/2m^*)\left[1 \pm K_1 (\hbar \omega_c/E_g) - 2K_2 \hbar \omega_c \left(n + \frac{1}{2}\right)(E_g)^{-1}\right]$$

$$\Theta \equiv (K_2/E_g)(\hbar^2/2m^*)^2,$$

$$\gamma_{n,\pm} \equiv [G_{\pm} - \Theta_1 (\overline{H}_{\pm} - \varepsilon_n)^{1/2}]^{1/2},$$

$$\Theta_1 \equiv (\hbar^2/2m^*)/\sqrt{\Theta},$$

$$\overline{H}_{\pm} \equiv [b_{\pm}^2 + 4\Theta a_{\pm}]/4\Theta, \; a_{\pm} \equiv \left[\left(n + \frac{1}{2}\right) \hbar \omega_c - \tau - \frac{K_2}{E_g}\left\{\left(n + \frac{1}{2}\right) \hbar \omega_c\right\}^2 \pm\right.$$

$$\left. \pm \frac{1}{4} \hbar \omega_c g_0^* (m^*/m) \pm K_0 (\hbar \omega_c/E_g)^2 E_g \left(n + \frac{1}{2}\right) \hbar \omega_c\right],$$

$$A(i) \equiv \left\{\frac{2}{3}[S(i)]^{3/2} \hbar^2 e^2 \Theta_1^2/2\hbar \varepsilon_{sc} \sqrt{2m^*}\right\},$$

$S(i)$ are the zero's of the Airy function $A_i(-S(i)) = 0$, $i (= 0, 1, 2, \dots)$ is the electric sub-band index, ε_{sc} is the permittivity of the semiconducting substrate material and ε_n is the electron energy under magnetic quantization as measured from the edge of the conduction band at the surface in the absence of any quantization. For $E_g \to \infty$ and neglecting skin effects equation (2) takes the well-known form[1]

$$\varepsilon_n = \left(n + \frac{1}{2}\right) \hbar \omega_c + S(i) [\hbar e F_s/\sqrt{2m^*}]^{2/3} \qquad (3)$$

where F_s is the normal surface electric field.

Therefore the density-of-states function can be written as

$$D(\varepsilon) = \frac{eH}{2\pi\hbar} \sum_{n=0}^{n_{max}} \sum_{i=0}^{i_{max}} \delta'(\varepsilon - \varepsilon_n) \quad (4)$$

where δ' is Dirac's delta function.

Using Eq. (4) and following Tsidilkovskii[2], the TM of the electrons in n-channel inversion layers on ternary semiconductors can be written as

$$S(H) = \frac{1}{eT}\left[\frac{[\sum_{n=0}^{n_{max}} \sum_{i=0}^{i_{max}} \varepsilon_n' F_{-2}(\eta_{ni})]}{[\sum_{n=0}^{n_{max}} \sum_{i=0}^{i_{max}} F_{-2}(\eta_{ni})]} - E_F\right] \quad (5)$$

where ε_n' is the root of equation (2), $\eta_{ni} = (k_B T)^{-1}[E_F - \varepsilon_n']$, k_B is Boltzmann constant, T is temperature, E_F is the Fermi energy in the presence of magnetic quantization as measured from the edge of the conduction band at the surface in the absence of any quantization and $F_j(\eta_{ni})$ is the one parameter Fermi-Dirac integral of order j as defined by Blakemore[10]. Thus to study the surface electric field dependence of TPM we require an expression of electron concentration which in turn can be expressed as

$$n_0 = \frac{eH}{2\pi\hbar} \sum_{n=0}^{n_{max}} \sum_{i=0}^{i_{max}} F_{-1}(\eta_{ni}). \quad (6)$$

3. Results and discussion

Using Eqs. (5) and (6) together with the parameters[11-13]

$$E_g(x) = [-0.304 + 5 \times 10^{-4}\, T + (0.914 - 10^{-3}\, T)\, x]\ \mathrm{eV},$$

$$m^*(x) = [3\hbar^2 E_g(x)/4P^2(x)],$$

$$P(x) = [(\hbar^2/2m)(18 + 3x)]^{1/2},$$

$$\Delta = 0.9\ \mathrm{eV}, \quad H = 2\ \mathrm{T}, \quad T = 4.2\ \mathrm{K}, \quad x = 0.21$$

and

$$\varepsilon_{sc}(x) = [20.262 - 14.812\, x + 5.2795\, x^2]\ \varepsilon_0$$

where we have plotted the normalized TPM as a function of surface electric field as shown in Fig. 1 in which the dotted plot exhibits the same dependence in accordance with parabolic bands. Using the same parameters as used in obtaining Fig. 1 we have further plotted the normalized TPM as a function of H as shown in Fig. 2

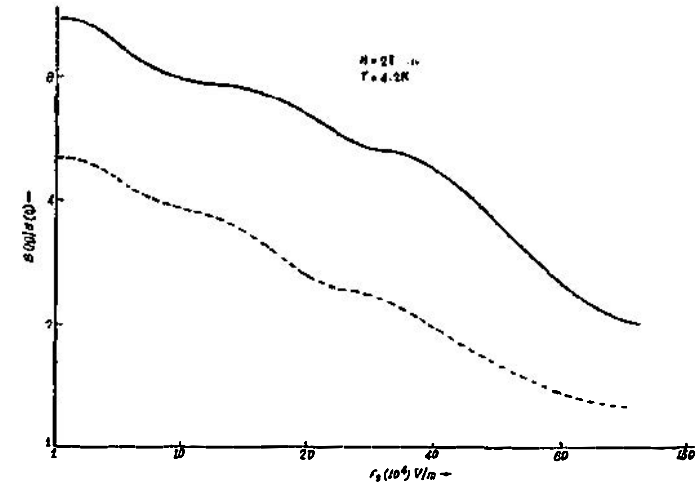

Fig. 1. Plot of $S\,(H)/S\,(0)$ versus surface electric field for n-channel inversion layers on $Hg_{1-x}Cd_xTe$ at 4.2 K. The dashed plot corresponds to inversion layers on a parabolic semiconductors.

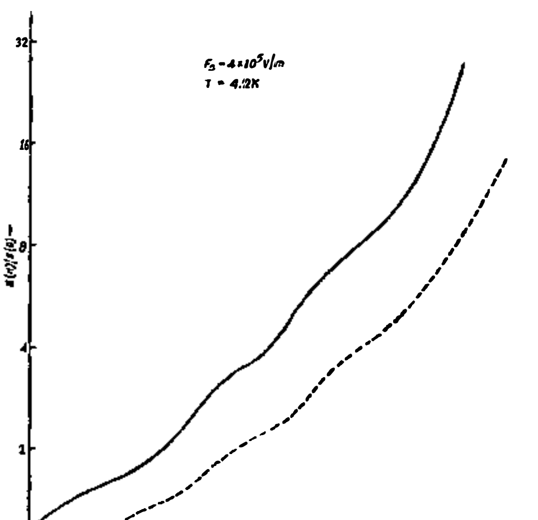

Fig. 2. Plot of $S\,(H)/S\,(0)$ versus the quantizing magnetic field for n-channel inversion layers on $Hg_{1-x}Cd_xTe$ at 4.2 K. The dashed plot corresponds to inversion layers on a parabolic semiconductors.

in which the same dependence has also been plotted for n-channel inversion layers on parabolic semiconductors for the purposes with increasing surface electric field in an oscillatory manner and the band non-parabolicity enhances the normalized TPM. With varying magnetic field, each time a Landau level crosses a Fermi level, a change is reflected in the magnetic TPM through redistribution of the carriers among the Landau levels. It may be noted that the 3D quantization in the absence of broadening of Landau levels leads to discrete energy levels somewhat like atomic energy levels in steps produces very sharp changes. This follows from the inherent feature of the 3D quantization of the 2D electron gas dealt with here. Under such quantization, there remains no free electron state in between any two successive Landau levels unlike that found for 3D electron gases of semiconductors under 2D quantization in $\vec{k}$ space in the presence of a quantizing magnetic field. Consequently, the crossing of the Fermi level by the Landau level under 3D quantization would have much greater impact on the redistribution of the electrons amount the available states, as compared to found for 2D quantization. It is basically this impact which results in the increased sharpness of the oscillatory spike for both type of band models. It appears from Fig. 2 that the normalized magnetic TPM increases with increasing quantizing magnetic field in an oscillatory manner and the band non-parabolicity parameter enhances the value of TPM in the whole range of the magnetic field considered. We must note that the generalized forms of the magneto TPM and the surface electron concentration in n-channel inversion layers on parabolic semiconductors will be given by Eqs. (5) and (6) where the quantity $\varepsilon_n' = \varepsilon_n$ is expressed through Eq. (3). The above facts are true only for 2D electron gases under magnetic quantization. However, the present numerical computations are valid only for $x > 0.17$, since for $x < 0.17$ the band gap in $Hg_{1-x}Cd_xTe$ becomes negative leading to semi-metallic state through the analytical formulations can be used for n-channel inversion layers on Kane type semiconductors. Though in a more rigorous treatment the broadening effects, the hot-electron effects and the many body effects should be taken into account but the present simplified analysis exhibits the basic features on the magneto TPM in inversion layers. Though the experimental verification of the basic content of the present paper is not available to the best of our knowledge but it may be noted that as far as determination of the effective mass under degenerate electron distribution at the surface is concerned, measurement of magneto TPM as compared to the conductivity or cyclotron resonance would not be less advantageous regarding the experimental facilities required or accuracies achieved. It may finally be noted that the basic purpose of the work is not solely to investigate theoretically the magneto TPM but also to formulate the magneto dispersion relation in n-channel inversion layers on Kane type semiconductors without any approximations of weak or strong electric field limits since the different electronic properties of 2D semiconductor devices are based on the appropriate energy spectra in such materials.

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TEORIJSKA ANALIZA TERMOELEKTRIČNE SNAGE U *n*-KANALNIM INVERZIONIM SLOJEVIMA TERNARNIH POLUVODIČA U PRISUSTVU KVANTIZIRAJUĆEG MAGNETSKOG POLJA

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Učinjen je pokušaj da se prouči termoelektrična snaga elektrona u *n*-kanalnim inverzionim slojevima ternarnih poluvodiča pod magnetskom kvantizacijom. To je učinjeno na osnovi četvrtog reda u teoriji efektivne mase bez ikakvih aproksimacija za slabo ili jako električno polje. Pri tome su uzete u obzir interakcije vodljive vrpce, vrpce lakih i teških šupljina te rascijepljene vrpce. Uzevši za primjer *n*-kanalne inverzione slojeve n-$Hg_{1-x}Cd_xTe$ nađeno je da termoelektrična snaga opada s porastom površinskog električnog polja odnosno opadanjem magnetskog polja na oscilirajući način.