

VALIDITY RANGE OF THE MAGNETIC

AHARONOV — BOHM EFFECT

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Received 8 December 1987

UDC 530.182

Original scientific paper

The magnetic Aharonov-Bohm effect is analyzed. Using basic properties of quantum mechanics, it is shown that the effect exists in a case where the energy spectrum of the source of the magnetic flux does not belong to a continuum. On the other hand, the effect disappears in an example of a source whose energy spectrum is continuous. It is shown that in all cases the system demonstrates the ordinary relations between classical physics and quantum mechanics.

1. Introduction

Predictions of the Aharonov-Bohm (AB) effects are known for a rather long time^{1,2)}. Unlike other quantum mechanical predictions, these effects have been considered a controversial issue³⁾. The main reason for the sceptical attitude is probably the uneasiness associated with the claim saying that the effects take place in a field-free region where there exists no classical analogue.

Two different kinds of the AB effect are discussed in the literature. The electric AB effect uses an electric field which takes finite values at a region that is inaccessible to the moving electron whose interference is measured. The magnetic effect uses a magnetic flux for this purpose. A claim made in the original derivation of the AB effects^{1,2)} says that the two kinds of the effect stem from a common principle. This claim relies on a covariant form of the phase shifts $\oint (V dt - \mathbf{A} \cdot d\mathbf{l})$,

where V denotes the 0-component of the 4-potential and the integration is carried out along a closed trajectory in space-time^{1,2)}. However, this claim is not self-evident because no acceptable transformation casts the setup of one experiment into that of the other.

The last argument can be proved as follows. There are two kinds of transformations that relate electric and magnetic fields. The first kind consists of coordinate transformations in space-time like the Lorentz ones. These transformations do not affect invariants of the system. In particular, the invariant $F^{\mu\nu}F_{\mu\nu}$ created from the fields' tensor remains constant. This invariant is positive at a point where a pure magnetic field exists and takes a negative value in the case of a pure electric field. The opposite signs of this invariant prove that no coordinate transformation can cast the setup of the electric AB effect into that of the magnetic one.

A second kind of possible transformations are duality transformations. An example of this kind of transformations is the following $E \rightarrow B; B \rightarrow -E; e \rightarrow g; g \rightarrow -e$, where e and g denote electric charges and magnetic monopoles, respectively. Since neither the electric AB effect nor its magnetic version uses monopoles, duality transformations are also inapplicable for proving an equivalence of the two kinds of the AB effect. Relying upon the statement proved above, one concludes that the negative arguments published recently against the electric AB effect^{4,5)} have no direct implication on the existence of its magnetic version.

The previous discussion explains the need for having an unbiased analysis of the magnetic AB effect. Following this approach, it is shown that the effect exists in a case where the source of the magnetic flux is a quantized system whose energy spectrum is discrete. On the other hand, a corresponding analysis similar to that of Ref. 5 shows that the effect does not exist in another case where the energy spectrum of the source of the magnetic flux belongs to a continuum.

The present work relies upon basic properties of quantum mechanics. One of its conclusions is that even when the effect exists, it bears the well known ordinary relations between classical physics and quantum mechanics. Therefore, the results derived in the following analysis are relevant to the underlying reasons for the controversy aroused by the predictions of the AB effects.

The second section describes general properties of this work. It is proved in the third section that the magnetic AB effect does not exist in a case where the source of the magnetic flux has a continuous energy spectrum. An analysis proving that the effect exists in an example where the energy spectrum of the source is discrete is presented in the fourth section. Concluding remarks are the contents of the last section.

2. General properties of the paper

The quantum mechanical wave function of a closed system is generally written as a sum of terms, each of which is a function of the coordinates of all the particles of the system. The present work discusses the interference pattern of two coherent electronic beams. The wave function of the system is written as a sum of two terms

$$\Psi = \Phi_L \psi_L + \Phi_R \psi_R \quad (1)$$

where ψ_L and ψ_R respectively denote two beams of the electron whose interference pattern is measured. Φ denotes the part of the wave function that depends on the coordinates of the rest of the system.

Henceforth, the electron whose interference pattern is measured is called the moving electron. Other charges are sometimes called the source. For simplicity, it is assumed that the moving electron and the charges of the source with which the magnetic flux is associated, are fermions of different kinds. Hence, no anti-symmetrization of the single particle wave function of the moving electron with other single particle wave functions is required. This property simplifies notation and does not affect the main results.

It is shown in this article that the kind of energy spectrum of the source of the magnetic flux is of a particular importance. A source whose energy spectrum belongs to a continuum is hereafter called a classical source. A source having a discrete energy spectrum is called a quantized source. These terms are used just for the simplification of notation and no claim is made concerning their rigorosity.

It is assumed that the moving electron is macroscopically far from the other charges of the system and that the classical limit holds for its interactions. It is also assumed that the velocities of all particles are small and that the nonrelativistic limit is effective. Generally, in this case, it is accustomed to use the Schrödinger equation. This approach is not adopted here because it turns out that the linearity of the Dirac equation is very helpful in the following analysis. It is well known that the Schrödinger equation is valid in cases that are within the range of applicability of the Dirac equation. Hence, the approach of this work is legitimate.

Units where the speed of light and Planck's constant take the value $\hbar = c = 1$ are used throughout the work. The magnetic flux of the source is confined within an infinitely long cylinder whose axis coincides with the z -axis. Most expressions are written in cylindrical coordinates (r, φ, z) and \mathbf{r} , $\vec{\varphi}$ and \mathbf{z} denote unit vectors in the corresponding directions. Cartesian coordinates are also used and \mathbf{i} , \mathbf{j} and \mathbf{k} denote unit vectors in the x , y and z -directions, respectively.

3. Systems with a classical source

Consider a test of the magnetic AB effect with an infinitely long cylinder (see Fig. 1). R denotes the radius of the rotating layer (the subscript R denoting quantities related to the right hand slit of Fig. 1 should not cause confusion). The moment of inertia of the rotating layer is very large and, for all practical purposes, its energy spectrum is assumed to be continuous. The absolute value ρ of the charge density per unit length in the z -direction is the same for the two layers.

For the purpose of having results obtained under different conditions, the experiment is carried out twice. The interference pattern that takes place while the inner cylinder rotates in a uniform angular velocity ω is compared with the corresponding results obtained in the case of a motionless cylinder. In the rest of this work, the former case is called the rotating experiment and the latter one is called the null experiment.

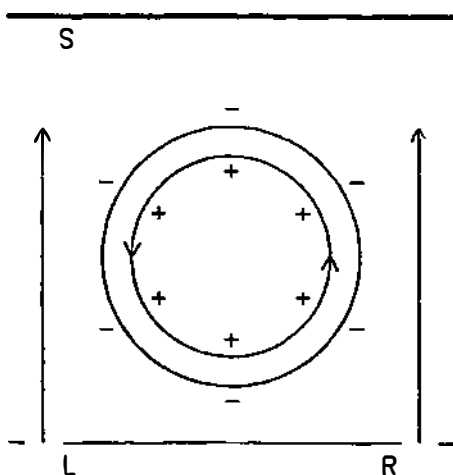


Fig. 1. The cross section of the (x, y) plane with an infinitely long cylinder made of two layers of an insulating material is depicted. The outer layer is covered uniformly with negative charges and is held fixed. The inner layer, which is covered uniformly with positive charges, rotates around the cylindrical axis. P_L and P_R denote two coherent beams of an electronic wave function. These beams pass the slits L and R , respectively, and move along symmetrical trajectories at the two sides of the cylinder. Later, the beams interfere on the screen S .

The electromagnetic fields of the source vanish outside the cylinder. The same is true for the electric potential V associated with them. On the other hand, in the rotating experiment the vector potential A is nonzero. The Dirac Hamiltonian of the system is

$$H = H_c + \vec{\alpha} \cdot (\mathbf{p} + e \mathbf{A}) + \beta m \quad (2)$$

where H_c denotes the Hamiltonian of the cylinder, $-e$ denotes the electronic charge and the rest of the quantities are the usual ones of the Dirac Hamiltonian of a single electron. The vector potential $A(r)$ used in (2) is the sum of the contributions of all the moving charges at the cylinder

$$\mathbf{A}(\mathbf{k}) = \sum_i e_i \int \Phi^+ (\vec{\alpha}_i / |\mathbf{r} - \mathbf{r}_i|) \Phi \, d^3r_1 \dots d^3r_n \quad (3)$$

where i runs on the cylindrical charges, $\vec{\alpha}_i$ is the velocity operator of the i 'th charge and \mathbf{r} and \mathbf{r}_i are the positions coordinates of the moving electron and of the cylindrical i 'th charge, respectively. Using (3), one can write the Dirac Hamiltonian (2) in the following form

$$H = H_c + \vec{\alpha} \cdot \mathbf{p} + \beta m + \sum_i e e_i \vec{\alpha} \cdot \vec{\alpha}_i / |\mathbf{r} - \mathbf{r}_i|. \quad (4)$$

The form of this Hamiltonian clarifies the dynamics of the system. The first three terms operate either on the cylinder or on the moving electron. The last term is the interaction part which is a sum of two body symmetric operators.

The Hamiltonian (4) yields the phase difference between the two terms of the wave function (1). This difference is calculated for the null experiment and for the rotating one. In the null experiment the expectation value of each α_i vanishes. Hence, the interaction term of (4) makes no contribution and the expression reduces to a sum of operators operating either on the cylinder or on the moving electron. Therefore, in the null experiment, the time-dependent equations of each of the two terms of (1) are

$$\frac{i\partial}{\partial t}(\Phi_L \psi_L) = (H_c \Phi_L) \psi_L + \Phi_L (H_e \psi_L) \quad (5)$$

$$\frac{i\partial}{\partial t}(\Phi_R \psi_R) = (H_c \Phi_R) \psi_R + \Phi_R (H_e \psi_R). \quad (6)$$

Here H_e denotes the single particle Hamiltonian of the moving electron: $H_e = \vec{\alpha} \cdot \mathbf{p} + \beta m$. The corresponding expressions of the rotating experiment are

$$\frac{i\partial}{\partial t}(\Phi_L \psi_L) = (H_c \Phi_L) \psi_L + \Phi_L (H_e \psi_L) + H_I \Phi_L \psi_L \quad (7)$$

$$\frac{i\partial}{\partial t}(\Phi_R \psi_R) = (H_c \Phi_R) \psi_R + \Phi_R (H_e \psi_R) + H_I \Phi_R \psi_R \quad (8)$$

where H_I is the last term of (4).

One can take advantage of the validity of the classical limit as well as that of the nonrelativistic one and simplify the evaluation of the above expressions. The experiment satisfies a basic requirement of the AB effect were the moving electron is not affected by any external field. Hence, using Ehrenfest's theorem, one finds that, in both experiments, the moving electron travels along the same trajectories in space-time. The change of the interference pattern stems from the different contributions of the right hand sides of (5) and (7) and from the corresponding quantities of (6) and (8). These quantities are evaluated below.

It is pointed out above that there is no change in the kinetic energy of the moving electron. It follows that the second term of (5) makes precisely the same contribution as that of (7) and the difference between these two quantities vanishes. The same is true for the second term of (6) and that of (8).

Next, let us evaluate the contribution of the last term of (7). This quantity is

$$(H_I) = \sum_i \int \Phi_L^\dagger \psi_L^\dagger (e e_i \vec{\alpha}_i |\mathbf{r} - \mathbf{r}_i|) \Phi_L \psi_L d^3r d^3r_1 \dots d^3r_n. \quad (9)$$

Using the above mentioned approximations, one integrates on the coordinates of the cylindrical charges and obtains the vector potential $\mathbf{A}(r)$ of (3). A calculation that is well known from classical electrodynamics yields

$$\mathbf{A}(\mathbf{r}) = \rho \omega R^2 \vec{\varphi}/r \quad (10)$$

which holds at the cylindrical outer region.

Consider an electronic wave packet centered at a point \mathbf{r} on a beam. Using (9), (10) and the classical limit, one finds for this wave packet the expectation value of the interaction term

$$(H_I) = v e \rho \omega R^2 \cos \Theta / r \quad (11)$$

where Θ denotes the angle between the unit vector $\vec{\varphi}$ and the direction of the velocity \mathbf{v} of the moving electron.

The calculation is accomplished after the contribution of the cylindrical self-energy is found. The influence of the moving electron on the cylinder is indeed a relatively small quantity and it does not make a significant change of the cylindrical wave function Φ . However, since the source is a heavy macroscopic apparatus, even a relatively small change of its state can make a significant contribution to the phase.

As stated at the beginning of this section, the energy states of the rotating layer are assumed to belong to a continuum. The change of its self-energy balances the energy transmitted into it. This last quantity equals the time-integral of the power associated with the e. m. f. derived from the electric field of the moving electron. The required power is

$$P = \frac{\rho \omega}{2\pi} \int \mathbf{E}_e \cdot d\mathbf{l}' dz' \quad (12)$$

where \mathbf{E}_e denotes the electric field of the moving electron and the integration on $d\mathbf{l}'$ is carried out on the rotating layer, along circles in the $\vec{\varphi}$ direction. Using Stokes theorem and Maxwell equation, one transforms (12) into an integral on the inner volume of the rotating layer

$$P = - \frac{\rho \omega}{2\pi} \int (\partial B_e / \partial t) d^3r' \quad (13)$$

where B_e is the z -component of the magnetic field of the moving electron.

Integrating the power (13) on the time, one finds the required change of the self-energy of the rotating layer. When the moving electron is far away from the cylinder, its magnetic field at the cylinder is negligible. Hence, the lower limit of the time-integral of (13) vanishes and the energy transmitted into the rotating layer reduces to an integral on its inner volume

$$\varepsilon = - \frac{\rho \omega}{2\pi} \int B_e d^3r' \quad (14)$$

where B_e is taken when the moving electron is at the point \mathbf{r} defined after (10).

This expression is evaluated in the classical limit as well as in the nonrelativistic one. The point \mathbf{r} is described by means of Cartesian coordinates centered at the origin

$$\mathbf{r} = r\mathbf{i}. \quad (15)$$

The velocity of the particle is

$$\mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}. \quad (16)$$

As a matter of fact, the moving electron travels in the (x, y) plane and $v_z = 0$. At a field point \mathbf{r}' , the magnetic field of the nonrelativistically moving electron (whose charge is $-e$) takes the following form⁶⁾

$$\mathbf{B} = -e \mathbf{v} \times (\mathbf{r}' - \mathbf{r}) / |\mathbf{r}' - \mathbf{r}|^3. \quad (17)$$

Substituting the above expressions into (14) and using cylindrical coordinates, one finds

$$\varepsilon = \frac{e \rho \omega}{2\pi} \int_0^R \int_0^{2\pi} \int_{-\infty}^{\infty} \frac{-v_x r' \sin \varphi' + v_y (r' \cos \varphi' - r)}{(r^2 + r'^2 + z'^2 - 2rr' \cos \varphi')^{3/2}} r' dr' d\varphi' dz' \quad (18)$$

The integration on z' yields

$$\varepsilon = \frac{e \rho \omega}{2\pi} \int_0^R \int_0^{2\pi} \frac{-v_x r' \sin \varphi' + v_y (r' \cos \varphi' - r)}{r^2 + r'^2 - 2rr' \cos \varphi'} r' dr' d\varphi'. \quad (19)$$

Integrating on φ' , one obtains⁷⁾

$$\varepsilon = -\frac{e \rho \omega v_y}{r} \int_0^R 2r' dr' = -e \rho \omega v R^2 \cos \Theta / r \quad (20)$$

where Θ is defined after (11).

The last result is precisely the negative value of the interaction energy (11) found above. Adding these two quantities, one finds that, at every instant, the interaction term of (7) cancels the change of the Hamiltonian's expectation value of the source caused by the e. m. f. of the moving electron. The same conclusion holds in the case of the other beam (8).

The results of the above discussion show that the difference between the accumulating phases of (5) and (6) is the same as that of (7) and (8). It follows that the interference pattern of the null experiment is the same as that of the rotating one. Hence, in the example analysed above, where the magnetic flux is associated with a classical source, the magnetic AB effect does not exist. It is interesting to note that the same conclusion holds for the electric AB effect⁵⁾. Notice that also in the case of the electric AB effect, the energy spectrum of the source belongs to a continuum. The conclusions of Ref. 5 and of the present section indicate that the nature of the energy spectrum of the source is significant for the existence of the AB effect.

4. *Systems with a quantized source*

As an example of a quantized source, let us examine an infinitely long cylinder which is a single domain of a ferromagnetic material whose spins are aligned parallel to the cylindrical axis.

The rotating experiment carried out in the previous section is substituted by an analogous one where the above mentioned magnet replaces the two cylindrical layers. In all experiments, the moving electron travels in the same nonsimply connected field-free region. The magnetic flux of the magnet at the inaccessible domain inside the cylinder takes the same value as in the rotating experiment of Sec. 3. However, it is shown in this section that, in the case of a quantized source, the interference pattern changes and a nonzero magnetic AB effect shows up.

Here, like in the previous case, the interference patterns of two experiments are examined. The null experiment used here is precisely the same as that of Sec. 3. The other experiment (henceforth called the active experiment) uses the cylindrical magnet described above. The right hand sides of (5), (6), (7) and (8) should be evaluated in the present cases. Obviously, the results of (5) and (6) are the same as in the previous section because the null experiments are identical.

In the active experiment discussed here, like in the rotating one of the previous section, the moving electron travels along the same trajectory and »sees« the same vector potential A . Therefore, the expectation values of the second and the last terms of (7) and (8) are the same as those found in Sec. 3.

Let us evaluate the self energy of the cylindrical magnet. Its magnetic spins »see« the magnetic field of the moving electron. However, unlike the case of the classical source discussed above, the magnetic state of the single domain is unaffected by the very weak magnetic field of the macroscopically far electron. In other words, no spin-flip is induced by the magnetic field of the moving electron. This argument means that in the present experiment $\Phi_L = \Phi_R$ at every instant. Thus, the different contributions made by the interaction terms of (7) and (8) are not compensated and a magnetic AB effect shows up. The size of this effects is precisely the same as the one obtained in its original derivation^{1, 2)}.

The existence of the magnetic AB effect has been confirmed recently by convincing experiments⁸⁾. These experiments use a toroidal magnetic source whose properties are analogous to those of the magnet discussed in this section. On the other hand, no experiment with a classical source like the one analysed in Section 3, has been carried out. Hence, it can be cautiously stated that the results of this work are not incompatible with experiment.

5. *Concluding remarks*

A comprehensive comparison of the points of the present article as well as those of Refs. 4 and 5, with the variety of the corresponding ones used in other discussions of the AB effects, is beyond the scope of the present work. The following remarks compare the foundations and the conclusions of these papers with several ideas found in the literature.

The analysis carried out above takes the ordinary quantum mechanical structure. It uses wave functions and operators. The electromagnetic interactions are written in terms of potentials, as required. The interrelations between the quantum mechanical expressions and the classical ones take the ordinary form. Moreover, properties of the classical limit are used in the evaluation of several expressions. In particular, the electromagnetic interactions have the ordinary local property used in classical as well as in quantum physics. In this sense, the above discussion does not put forward any new argument that differs from the established relations between quantum mechanics and classical physics. This aspect of the analysis carried out above is different from the one claimed in the original presentation of the AB effects^{1,2)}.

In this context, it is interesting to examine the interaction term of the Hamiltonian (4) of the system. This term is a symmetric expression of quantities related to the moving electron and to the other charges of the system. Its expectation value is

$$(H_I) = \int \Phi^\dagger \psi^\dagger \sum_i e e_i \vec{\alpha} \cdot \vec{\alpha}_i / |\mathbf{r} - \mathbf{r}_i| \Phi \psi \, d^3 r_1 \dots d^3 r_n. \quad (21)$$

This expression is evaluated separately for every term of the wave function (1). The integration on the coordinates of the cylindrical charges and the summation on them, cast (21) into the well known interaction of the moving electron with the cylindrical potential

$$(H_I) = \int \psi^\dagger e \vec{\alpha} \cdot \mathbf{A} \psi \, d^3 r. \quad (22)$$

Evidently, (22) takes the form of an expectation value of a single particle operator, operating on the wave function of the moving electron. Inspecting (7), (8) and (22), one gets the impression that the rate of phase accumulation is associated with the moving electron and looks like a property of a single particle.

In this presentation, the moving electron travels in a nonsimply connected field-free region where the vector potential is nonzero. Topological properties of this presentation are claimed to be significant for the AB effect^{1,2)}. The following lines prove that this assertion is unjustified.

It is evident that the form of (22) is not unique. According to the requirements of the experiment, the moving electron is excluded from the cylindrical volume^{1,2)}. Hence, the integrand of (21) is a regular function (because there is no point in space where both Φ and ψ are nonzero) and the order of integrations and summation can be interchanged. Integrating on the coordinates of the moving electron first, one finds the following form of the interaction energy

$$(H_I) = \sum_i e_i \int \Phi^\dagger \mathbf{A}_e \cdot \vec{\alpha}_i \Phi \, d^3 r_1 \dots d^3 r_n \quad (23)$$

where \mathbf{A}_e denotes the vector potential associated with the moving electron.

In (23) the charges at the source of the magnetic flux interact with the vector potential \mathbf{A}_e of the moving electron. (In the case of magnetic spins, it takes the non-relativistic form $-\mu \cdot \mathbf{B}_e$, where \mathbf{B}_e is the magnetic field of the moving electron.)

The classical meaning of (23) is evident because the fields associated with the moving electron are nonzero at the locations of the cylindrical charges. It is evident that in (21) as well as in (23) one finds the well known relations between classical physics and quantum mechanics. In these expressions one sees the fields used in classical equations of motion and the potentials used in quantum mechanics. The claim saying that the quantum mechanical expressions of the AB effect has no classical analogue^{1,2)} relies upon the form of (22) where the moving electron „sees“ no fields while traveling in a variable potential. The identical value of (21), (22) and (23) shows that this assertion has no profound meaning. The last claim is evident because a profound meaning of an effect should not disappear if the order of integrations and summation is interchanged.

The following argument proves that the topological structure of the region where the moving electron travels is not a decisive property of the AB effect. Indeed, the motion of the moving electron and the potential at its location are precisely the same in the two experiments discussed in Secs. 3 and 4. The same is true for the topological structure of the region to which the moving electron is confined. However, the results of these experiments are different. The AB effect vanishes in the case discussed in the third section whereas it shows up in that of the fourth section. Therefore, the existence of the AB effect does not emerge from the topological structure of the region where the moving electron travels. Moreover, these results show that the above ingredients are not sufficient for a complete physical description of the system and that the quantum mechanical structure of the source of the magnetic flux along the infinite cylinder is an indispensable element of this description.

Another point pertains to nonlocality inherent in quantum mechanics. An application of an operator to a term of the entire wave function yields an expectation value associated with the term. In particular, the expectation value of the Hamiltonian is associated with the phase that accumulates on each term. This phase is a property of the entire term and one cannot split it into phases of the different factors of the term considered. Thus, in the experiments discussed above, the phase of the interfering electron is the sum of the contributions derived from the application of the Hamiltonian to all the factors of each term of (1). This conclusion holds even if, at the interference time a part of the system, namely the source, is far from the screen.

The previous argument shows a nonlocal aspect of the quantum mechanical analysis of the experiments. Each term of the wave function (1) depends on the degrees of freedom of all the particles of the system. A phase accumulates on each term even if the particles of the system are far apart. This nonlocality is analogous to the same features of the Einstein-Podolski-Rosen^{9,10)} paradox which is confirmed in several experiments^{11,12)}. In these experiments, correlated results are obtained from measurements of spin projections carried out at two space-like separated events. No other aspect of nonlocality is applied or derived from the analysis carried out in this article.

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PODRUČJE VALJANOSTI MAGNETSKOG AHARONOV-BOHM
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UDK 530.182

Originalni znanstveni rad

Analizira se magnetski Aharonov-Bohm efekt. Koristeći pravila kvantne mehanike, pokazuje se da efekt postoji u slučaju kada izvor magnetskog toka pripada diskretnom spektru energije. Efekat nestaje u slučaju izvora čiji je spektar energije kontinuirani. Pokazuje se da sistem u svim slučajevima demonstrira uobičajenu vezu između klasične i kvantne mehanike.