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SPIN ASSIGNMENTS FOR THE GROUND STATE OF ^{149}Tb AND EXCITED STATES OF ^{149}Gd

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The assignments of spins and parities of the ground state of ^{149}Tb and the excited states of ^{149}Gd are made here from theoretical considerations of the relative intensities of their α-spectra.

1. Introduction

The spin and parity of the ground state of ^{149}Tb have been assigned to be either $5/2^+$ or $3/2^+$ by Toth[1] and other authors[2-6]. On the other hand, this has been suggested to be $1/2^+$ by Jackson et al.[7] on the presumption that alpha-decay of ^{149}Tb, being of similar nature as that of ^{151}Tb, the ground state spin of the former is expected to be same as that of ^{151}Tb. The spin and parity of this nucleus has been measured by Adelroth et al.[8] and found to be $1/2^+$. Also the spins and parities of two excited levels of ^{149}Gd in the alpha-decay of ^{153}Dy are yet undecided.

In view of this controversy, it will be of interest to find theoretically, the most likely spin and parity values of these nuclear states. It has been shown in a number of papers[9-12] that the penetration ratios calculated by taking the barrier to be the usual Coulomb potential superposed by the effective non local α-nucleus potential suggested by one of the authors[9,10] together with the exchange term are in close agreement with the observed relative intensities (i. e. the ratio of decay constants $\lambda_l/\lambda_{l'}$) for almost all alpha spectra in the rare earth, trans-lead and trans-uranium nuclei. By definition

$$\lambda_l = F_l \times P_l \tag{1}$$

where P_l is the penetrability factor, F_l is the internal transition probability and hence hF_l is the so called reduced width. Apart from such close and consistent agreement of the calculated ratios $P_l/P_{l'}$ with $\lambda_l/\lambda_{l'}$ (as stated above) over such wide ranges of alpha spectra, we also find that the reduced width ratios $F_l/F_{l'}$ thus calculated from Eq. (1) turn out to be $\geqslant 1$ (where l' refers to angular momentum of the maximum intensity α-groups). This finding viz., $F_l/F_{l'}$ is also supported from other sources such as the studies of alpha spectroscopic amplitudes[13]. The above results provide convincing evidences that the barrier in alpha decay comprises the usual Coulomb potential superposed by the aforesaid non local alpha nucleus potential with exchange terms.

In the present paper, we show that the above ideas provide a method of making a new evaluation of the spin values for some α-emitting nuclei, which are yet undecided.

2. *Penetrability factor*

We take as before[9] the interaction kernel in spherically symmetric non-local alpha nucleus potential as

$$J(\vec{r}, \vec{r}') = V(r)\, \delta_b(\vec{r} - \vec{r}') \tag{1.1}$$

where $V(r)$ is the static part of the non local potential and

$$\delta_b(\vec{r} - \vec{r}') = \pi^{-3/2}\, b^{-3} \exp\left[-\left(\frac{\vec{r} - \vec{r}'}{b}\right)^2\right] \tag{1.2}$$

where $\vec{r}'$ is the position vector of the alpha particle other than $\vec{r}$ relative to the product nucleus and b is the range of non locality.

In almost all recent papers in alpha decay static potential has been taken to be that given by Igo[14]. On the other hand Woods-Saxon form of potential has almost always been used in scattering problems. It is, however, not clear when the same nuclear interaction is involved, why the two areas should have different optical model potentials for the static part. This point is discussed in a recent paper[12] and we have shown that Woods-Saxon form of potential is equally well applicable for the static part of the effective nonlocal alpha nucleus potential giving consistently good agreement with the observed relative intensities. Hence, we take[12] for the static part the WS potential

$$V(r) = V_0 f(r)$$

where

$$f(r) = 1 \Big/ \left\{ 1 + \exp\left(\frac{r - r_0 A^{1/3}}{0.5} \right) \right\}. \tag{1.3}$$

r is in fm and $|V_0| = 210$ MeV.

Following the procedure in the previous work, the penetration factor P_l is obtained as

$$P_l = \exp\left[- \left\{ 2 (2\mu)^{1/2}/\hbar \int_{R_l}^{R_0} \left(\frac{2(Z-2)e^2}{r} + \frac{\hbar^2}{2\mu} \frac{l(l+1)}{r^2} - \right. \right. \right.$$

$$\left. - V_{\pm} f(r) K_l(r) - E \right)^{1/2} \mathrm{d}r \Big] \tag{2}$$

where

$$V_{\pm} = |V_0| \, [S + (1 - S)(-1)^l] \tag{2.1}$$

$$K_l(r) = \frac{1}{2} [1 + \mathrm{erf}(z)] \left[1 - \frac{l(l+1) b^2}{4r^2} \right] \tag{2.2}$$

$$z = \frac{r - R_l}{b} \tag{2.3}$$

and

$$R_0 = \frac{2(Z-2)e^2}{E}. \tag{2.4}$$

The subscript $\pm$ corresponds to l even or odd, respectively, S is the mixture proportion of the ordinary part of the potential, E is the decay energy after being corrected for nuclear recoil and electron screening and R_l is the inner turning point.

3. Result and discussion

In Table 1 we present the values of $P_l/P_{l'}$ calculated from Eq. (2) for α-decay of ^{149}Tb taking successively $1/2^+$, $3/2^+$, $5/2^+$ for its ground state spin and parity as suggested by different authors[1-7]. Similarly calculated penetrability ratios for the α-decay of ^{153}Dy $\rightarrow$ ^{149}Gd using different spins and parities for the excited states of the product ^{149}Gd are also shown in the same Table 1. We have taken for calculations, $b = 0.7$ fm and $S = 0.6$ in Eqs. (2)—(2.3) as in previous papers, and $r_0 = 1.4$ for the rare-earth nuclides.

We find from the table that the assignements $1/2^+$, $3/2^+$ for the ground state spin of ^{149}Tb lead to the values of $P_l/P_{l'}$ differing largely from the observed intensity ratio whereas for the assignment $5/2^+$, the calculated $P_l/P_{l'}$ is consistently

TABLE 1.

Parent Nucleus → Product	Spin and parity[a] of Parent	Spin and parity[a] of Product	l of alpha and state of excitation	Decay energy E (MeV)	Penetration factor P_l [Eq. (2)]	Reduced width (MeV) hF_l	Penetration ratio $(P_l/P_{l'})$[c]	Observed relative intensity	Reduced width ratio $(F_l/F_{l'})$
1	2	3	4	5	6	7	8	9	10
$^{149}_{65}$Tb	$(1/2^+)$[7]	$5/2^+$	(2), (g)[d]	3.967	0.118 [−24][b]	0.373	1.00	1.00	1.00
↓	or $(3/2^+)$[2−6]	$11/2^-$	(5), (1st)	3.644	0.984 [−29]	1.503	8.37 [−5]	30.00 [−5]	4.02
$^{145}_{63}$Eu	$5/2^+$	$5/2^+$	0, g	3.967	0.247 [−24]	0.178	1.00	1.00	1.00
		$11/2^-$	3, 1st	3.644	0.630 [−28]	0.210	25.48 [−5]	30.00 [−5]	1.18
$^{153}_{66}$Dy	$7/2^-$	$7/2^-$	0, g	3.464	0.156 [−28]	0.239	1.00	1.00	1.00
↓		$(5/2^-)$[1,7]	(2), (1st)	3.305	0.336[−30]	0.002	214.6 [−4]	2.03 [−4]	0.01
$^{149}_{64}$Gd		$3/2^+$ * or $13/2^+$	3, 1st	3.305	0.294 [−31]	0.026	18.82 [−4]	2.03 [−4]	0.11
		$(3/2^-)$[1,7]	(2), (2nd)	3.000	0.449 [−33]	0.101	28.72 [−6]	12.15 [−6]	0.42
		$9/2^+$	1, 2nd	3.000	0.117 [−33]	0.388	7.5 [−6]	12.15 [−6]	1.62

[a] In columns 2 to 10 the quantities which are shown underlined, refer to the assignments and results of our work.

The values of spin, parity, l etc. which are disputed are shown within braces ( ) with reference numbers as superscripts. The corresponding results in columns 6, 7, 8 and 10 are therefore unacceptable and braces in these columns are omitted. The quantities, which are undisputed are neither underlined nor shown with braces for clarity.

[b] The numbers in brackets [ ] in columns 6, 8 and 9 denote power of 10.

[c] l' refers to the maximum intensity groups which also corresponds to the transitions to the ground states, for these cases.

[d] g, 1st and 2nd refer to ground —, first — and second excited states, respectively.

* $3/2^+$ is more probable than $13/2^+$ as discussed in the text.

TABLE 2.

Parent nucleus	Spin and parity[e] of Parent	Spin and parity[e] of Product	l of alpha	Decay energy E (MeV)	Penetration factor P_l [Eq. (2)]	Reduced width hF_l (MeV)	Penetration ratio $(P_l/P_{l'})f$	Observed relative intensity	Reduced width ratio $(F_l/F_{l'})$
1	2	3	4	5	6	7	8	9	10
^{211}Bi	9/2$^-$	1/2$^+$	5	6.774	0.741 [−21]	0.247 [−1]	1.00	1.00	1.00
		3/2$^+$	3	6.394	0.110 [−21]	0.340 [−1]	0.15	0.21	1.36
^{214}Bi	1$^-$	4$^+$	3	5.551	0.185 [−25]	0.522 [−1]	1.00	1.00	1.00
		5$^+$	5	5.282	0.158 [−27]	0.691 [−1]	8.5 [−3]	11.3 [−3]	1.32
^{211}Po	9/2$^+$	1/2$^-$	5	7.583	0.161 [−18]	0.337 [−1]	1.00	1.00	1.00
		3/2$^-$	3	6.696	0.598 [−21]	0.457 [−1]	3.72 [−3]	5.00 [−3]	1.36
^{227}Th	3/2$^+$	1/2$^+$	2	6.146	0.128 [−24]	0.338 [−2]	1.00	1.00	1.00
		(3/2$^+$)[16]	(0)	6.115	0.170 [−24]	0.303 [−3]	1.32	0.12	0.89 [−1]
		3/2$^-$	1	”	0.161 [−25]	0.320 [−2]	0.13	0.12	0.95
		or 1/2$^-$							
^{228}Th	0$^+$	0$^+$	0	5.517	0.159 [−27]	0.299	1.00	1.00	1.00
		1$^-$	1	5.300	0.843 [−30]	0.315	5.29 [−3]	5.58 [−3]	1.05
^{228}Pa	4$^+$	4$^+$	0	6.152	0.896 [−25]	0.320 [−2]	1.00	1.00	1.00
		2$^-$	3	6.195	0.574 [−26]	0.554 [−2]	0.06	0.11	1.72

[e] footnotes a of Table 1 also apply to these cases.

[f] footnotes c of Table 1 applies to these cases (except for ^{228}Pa in which l' refers to maximum in intensity but not the ground state) and value of the parameter r_0 is taken to be 1.37.

close to the observed intensity ratio. Also the calculated reduced width ratio $F_l/F_{l'}$ is found to be 1.18 i. e. greater than 1 though less than that suggested in the formula[13)]

$$F_l/F_0 = (2l + 1).$$

This variation is not unexpected as structure of ^{16}O studied therein is quite different from that of the present case. We therefore conclude that the spin and parity of the ground state of ^{149}Tb is $5/2^+$.

Similarly for the product nucleus ^{149}Gd, we find from the table that the use of $5/2^-$ (Refs. 1 and 7) for the first excited state viz., 159 keV, leads to the value 214.6×10^{-4} for $P_l/P_{l'}$ compared to the observed intensity ratio 2.025×10^{-4}. Also $F_l/F_{l'}$ is found to be too low (0.01). On the other hand if we take $3/2^+$ or $13/2^+$ for the spin and parity of the said excited state, $P_l/P_{l'}$ is 18.82×10^{-4}. Although the agreement in this case is not as close as in other cases, yet the discrepancy between theoretical and observed values is greatly reduced. Since both $3/2^+$ and $13/2^+$ assignments involve the same value of l and hence the same value of $P_l/P_{l'}$ (cf. Table 1), we suggest from the following consideration that $3/2^+$ is more probable than $13/2^+$ assignment for this state.

The shell model sequence of levels[15)] indicates that for the 85th neutron in ^{149}Gd, the successive levels for its occupation are in the range of 2 $f_{7/2}$ and above. In this region 1 $i_{13/2}$ is evidently too high a level to be available to the 85th neutron for occupation unless excitation is abnormally high. Hence we suggest that spin and parity of the first excited state is $3/2^+$.

For the second state of 464 keV of ^{149}Gd, the assignment[1,7)] of $3/2^-$ leads to the value 28.72×10^{-6} for $P_l/P_{l'}$ whereas the observed intensity ratio is $12.15 \times$ $\times 10^{-6}$. On the other hand, using $9/2^+$ for the spin and parity of this state, $P_l/P_{l'}$ is found to be 7.5×10^{-6} which is much closer to the observed intensity ratio than the value with $3/2^-$. Hence, our results indicate that the spin and parity of the second excited state of ^{149}Gd is most probably $9/2^+$. Because, it is experimentally found that the nuclei with odd neutron numbers (83, 85 and 87) occupy $2f_{2/7}$ level each which is therefore depressed below $1h_{9/2}$ in the general sequence of levels. It is also seen that except for very light nuclei, $p_{3/2}$ level gets depressed below $f_{5/2}$. Our results indicate that $p_{3/2}$ is further depressed below $1h_{9/2}$. Then the sequence of levels for the 85th neutron would be as follows:

$2f_{7/2}$ (ground), $3p_{3/2}$ (1st excited) and $1h_{9/2}$ (second excited) levels. Additional justification for the said sequence of the levels is a matter for further theoretical work.

This method of using Eq. (2) to obtain closest agreement of $(P_l/P_{l'})$ with observed relative intensities thus provides a method of evaluation of spin and parity values of nuclei emitting α-spectra. We have discussed above only the disputed cases of spins. It is, however, desirable to extend the present calculations to some other nuclei whose spins are undisputed, and hence verify whether the calculated $(P_l/P_{l'})$ using the known spins and parity values are also in agreement with the observed $(\lambda_l/\lambda_{l'})$. Accordingly, calculations for several nuclides are given in Table 2. It is significant that the results in columns 8 and 9 of Table 2 show consistent and close agreement of $(P_l/P_{l'})$ with observed $(\lambda_l/\lambda_{l'})$, except for one case i. e. for the spin value $(3/2)^+$ for the product of ^{227}Th. For this case both assignments $3/2^-$ and $1/2^-$ lead to very close agreement with the observed value. Since spin

3/2 agrees with that in Ref. 16 and parity differs, whereas for $1/2^-$ both spin and parity differ, we think $3/2^-$ is most likely spin and parity value for this state of ^{223}Ra.

It need be mentioned that calculations of $P_l/P_{l'}$ using a pure Coulomb barrier or that superposed by a static α-nucleus potential for the different spin values lead to much higher discrepancies with the observed $\lambda_l/\lambda_{l'}$ in all these cases. For reason of space, we have not included the calculations for Coulomb barrier or that with a pure static α-nucleus potential.

Regarding the semi-empirical rule

$$F_l = (\lambda_l)_{\text{observed}}/(P_l)_{\text{theoretical}} \tag{1}$$

we wish to recall that one purpose of this paper is to provide reliable values of reduced width ($\delta_l^2 = hF_l$) to check the theoretical formulae deduced from shell model. Since in Eq. (1), neither P_l nor F_l is experimentally determinable, there is no way to ascertain F_l or P_l without making assumptions on one of them. Hence the need for the semi-empirical rule. We have preferred to ascertain P_l because internal structure parameters involved in F_l (or δ_l^2) are more uncertain.

On the other hand the assumptions about the barrier (namely Coulomb potential superposed by an α-nucleus potential with non local and exchange terms, cf. Eq. (2)) are justified, from experimentally found momentum dependence and exchange character of nuclear force in scattering and other phenomena, apart from the evidences shown in the previous papers[9-12].

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ODREĐIVANJE SPINA OSNOVNOG STANJA ^{149}Tb I UZBUĐENIH STANJA ^{149}Gd

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Određivanje spinova i pariteta osnovnog stanja ^{149}Tb i uzbuđenih stanja ^{149}Gd učinjeno je na osnovu razmatranja teorijskih vrijednosti relativnih intenziteta njihovih alfa spektara.