YU ISSN 0015—3206
FZKAAA 20 (3) 395 (1988)

LETTER TO THE EDITOR

A SOLUBLE MODEL FOR OSCILLATIONS IN A MULTICOMPONENT CLASSICAL PLASMA

IVICA CRNKOVIĆ+ and VLADIMIR ŠIPS

Faculty of Science, P. O. Box 152, 41001 Zagreb and Ruđer Bošković Institute, Zagreb, Yugoslavia

+ABB Automation, 72157 Västerås, Sweden

Received 19 July 1988

UDC 533.951

Original scientific paper

The present paper is concerned with the study of optical and acoustic normal modes in a coupled unmagnetized, homogeneous, collisionless classical plasma involving an arbitrary number of different species with disparate frequencies and disparate screening wave vectors.

The simplest model of plasma is an electron plasma. It contains a number of mobile electrons immersed in a uniform background of neutralizing positive charge. The first theory of a single-component plasma was developed by Langmuir and Tonks[1,2]; they determined the electron plasma frequency describing the oscillations of the negative charge about the positive ions.

However, the constituents of a real plasma are generally charges with different specifications. This leads us to the model of a multicomponent plasma. Treating the corresponding dispersion relation by use of numerical methods, Fried and Gould[3] concluded that in addition to the high-frequency Langmuir plasma waves there also exist low-frequency waves which can be appreciably Landau damped. They are called acoustic plasma waves. The origin of this name lies in the fact that, like in the case of lattice acoustic waves, the frequency of plasma acoustic waves goes to zero when the wavelength goes to infinity.

Examining the conditions for the exsistence of wave instabilities in semiconductors composed of light electrons and heavy holes, Pines and Schrieffer[4] determined the long-wavelength solution of the dispersion relation for both the high-frequency optical mode and the low-frequency acoustic mode. In the long-wavelength limit, the normal modes of a three-component plasma were calculated in Ref. 5. Fröhlich[6,7] studied the behaviour of optical and acoustic waves in metallic plasmas consisting of outer s-electrons and inner d-electrons. His calculation was improved by Salustri[8] to take into consideration the role of positive ions. The oscillations in a gaseous two-component electron plasma with essentially different temperatures were examined by Watanabe and Taniuti[9] and by Yu and Shukla[10]. Gary and Tokar[11] and Gary[12] investigated the oscillations of acoustic waves in a three-component plasma involving positive ions, hot electrons and cold electrons.

In the present article we consider an idealized soluble model of non-isothermal classical plasma formed by an arbitrary number of components. Imposing certain conditions on the unperturbed frequencies and screening wave vectors of isolated plasma components, we evaluate plasma wave frequencies in a coupled system. In spite of its simplicity and naiveness, we believe that this model may serve as a qualitative picture for the behaviour of more realistic multicomponent plasmas.

The dispersion relation for the N-component Maxwellian plasma can be written in the form

$$1 + \frac{1}{k^2} \sum_{j=1}^{N} k_j^2 \, W\left(\frac{\omega}{k v_j}\right) = 0 \tag{1}$$

where the response function is given by

$$W(z) = \lim_{\delta \to 0+} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{x \exp(-x^2/2)}{x - z - \mathrm{i}\delta} \, \mathrm{d}x. \tag{2}$$

In Eq. (1) k_j is the Debye-Hückel screening wave vector of the j-th species

$$k_j = \sqrt{\frac{4\pi n_j e^2}{\varepsilon k_B T_j}} \tag{3}$$

and v_j is the corresponding most probable velocity

$$v_j = \sqrt{\frac{2 k_B T_j}{m_j}} \tag{4}$$

ε being the dielectric constant of the medium and k_B the Boltzmann's constant. The concentration, mass and temperature of the j-th species are denoted by n_j, m_j and T_j, respectively.

Dispersion relation (1) can be easily solved under rather strong conditions. Denoting the long-wavelength classical plasma frequency of the isolated j-th component by

$$\omega_j = \sqrt{\frac{4\pi n_j e^2}{\varepsilon m_j}} \tag{5}$$

we assume that the following inequations are fulfilled

$$\omega_1^2 \gg \omega_2^2 \gg \omega_3^2 \gg \ldots \gg \omega_N^2 \tag{6}$$

$$\left(\frac{v_1}{\omega_1}\right)^2 \gg \left(\frac{v_2}{\omega_2}\right)^2 \gg \left(\frac{v_3}{\omega_3}\right)^2 \gg \ldots \gg \left(\frac{v_N}{\omega_N}\right)^2. \tag{7}$$

It is well known that the response function (2) can be expanded into the convergent series[4,13]:

$$W(z) = \mathrm{i} z \sqrt{\pi} \exp(-z^2) + 1 - 2z^2 + \frac{4z^4}{3} - \frac{8z^6}{15} + \ldots \qquad \text{for } z < 1 \tag{8}$$

$$W(z) = \mathrm{i} z \sqrt{\pi} \exp(-z^2) - \left(\frac{1}{2z^2} + \frac{3}{4z^4} + \frac{15}{8z^6} + \ldots\right) \qquad \text{for } z > 1. \tag{9}$$

The new frequencies in the coupled system will be denoted by

$$\Omega_j(k) = \omega_j(k) - \mathrm{i}\gamma_j(k) \tag{10}$$

where the imaginary part $\gamma_j(k)$ measures the intensity of the Landau damping.

The N-component plasma system gives N frequencies, one optical and $N - 1$ acoustic. The frequency of the optical mode is obtained by applying the high-frequency expansion (9) to the all components. Taking into account condition (6) we arrive at:

$$\omega_1^2(k) = \omega_1^2\left(1 + \frac{3k^2 v_1^2}{2\omega_1^2} + \frac{3k^4 v_1^4}{2\omega_1^4} + \ldots\right) \qquad \omega_1 \gg k v_1 \tag{11}$$

$$\gamma_1(k) = \frac{\omega_1^4}{k^3 v_1^3} \sqrt{\pi} \exp[-(\omega_1/k v_1)^2]. \tag{12}$$

The same solution would be obtained if we consider the one-component plasma. Low-frequency components have practically no influence on the high-frequency plasma oscillations.

Note that in the long-wavelength limit the imaginary part of the optical frequency vanishes exponentially, showing that Landau damping can be neglected.

To find the n-th acoustic frequency, we apply the low-frequency expansion (8) to $j = 1, 2, \ldots, n$ and the high-frequency expansion (9) to $j = n + 1, n + 2, \ldots N$. By virtue of conditions (6) and (7) the real and the imaginary part of the acoustic frequencies are:

$$\omega_{n+1}^2(k) = \frac{k^2 v_n^2 \omega_{n+1}^2}{2\omega_n^2\left(1 + \frac{k^2 v_n^2}{2\omega_n^2}\right)} \qquad v_n \gg \frac{\omega_{n+1}(k)}{k} \gg v_{n+1} \tag{13}$$

$$\gamma_{n+1}(k) = \frac{k v_n \omega_{n+1}^2}{4\omega_n^2} \sqrt{\pi}. \tag{14}$$

In the limit of small wave vectors, the ratio between the imaginary and the real component of the frequency becomes

$$\frac{\gamma_{n+1}(k)}{\omega_{n+1}(k)} \approx \frac{\omega_{n+1}}{\omega_n} \ll 1. \tag{15}$$

In other words in the long-wavelength limit the Landau damping of the acoustic modes is negligible.

In the particular case of a three-component plasma the real parts of the acoustic frequencies can be expressed as:

$$\omega_2(k) = \frac{\omega_2 \dfrac{k}{k_1}}{\sqrt{1 + \left(\dfrac{k}{k_1}\right)^2}} \tag{16}$$

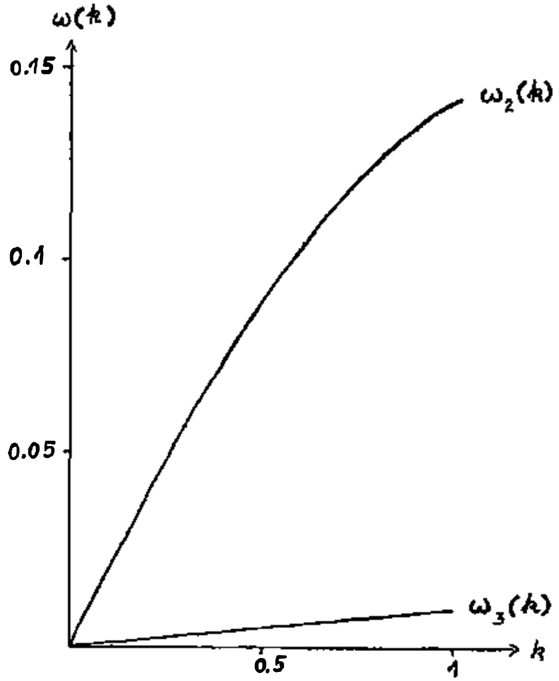

Fig. 1. Plot of real parts of upper and lower acoustic frequencies vs. wave number in a three-component plasma for

$$\frac{\omega_1}{\omega_2} = \frac{\omega_2}{\omega_3} = 5, \qquad v_1 = 25\, v_2.$$

Frequency is expressed in ω_1 and wave vector in k_1.

$$\omega_3(k) = \frac{\omega_1 v_2 \omega_3 k}{\omega_2 v_1 k_1 \sqrt{1 + \left(\frac{\omega_1 v_2 k}{\omega_2 v_1 k_1}\right)^2}}. \tag{17}$$

We have performed numerical calculation by choosing

$$\frac{\omega_1}{\omega_2} = \frac{\omega_2}{\omega_3} = 5, \qquad v_1 = 25\, v_2.$$

The results are presented on Fig. 1. It is observed from the figure that the acoustic plasma frequency behaves like the acoustic phonon frequency. In the region of small wave vectors the dispersion curve shows the proportionality between ω and k and slightly increases for larger values of k.

References

1) I. Langmuir, Proc. Nat. Acad. Sci. **14** (1926) 627;
2) L. Tonks and I. Langmuir, Phys. Rev. **33** (1929) 195, 990;
3) B. D. Fried and R. W. Gould, Phys. Fluids **4** (1961) 139;
4) D. Pines and J. R. Schrieffer, Phys. Rev. **124** (1961) 1387;
5) V. Šips, Nuovo Cimento **30** (1963) 947;
6) H. Fröhlich, Phys. Lett. **26** A (1968) 169;
7) H. Fröhlich, J. Phys. **C 1** (1968) 544;
8) C. Salustri, Phys. Stat. Sol. (b) **94** (1978) 517;
9) K. Watanabe and T. Taniuti, J. Phys. Soc. Jpn. **4** (1977) 1819;
10) M. Y. Yu and P. K. Shukla, J. Plasma Phys. **29** (1983) 409;
11) S. P. Gary and R. L. Tokar, Phys. Fluids **28** (1985) 2439;
12) S. P. Gary, Phys. Fluids **30** (1987) 2745;
13) S. Ichimaru, *Plasma Physics*, The Benjamin/Cummings Publ. Comp. 1986, p. 279.

RJEŠIV MODEL TITRANJA VIŠEKOMPONENTNE KLASIČNE PLAZME

IVICA CRNKOVIĆ+ i VLADIMIR ŠIPS

Prirodoslovno-matematički fakultet, p. p. 152, 41001 Zagreb i Institut Ruđer Bošković, Zagreb, Jugoslavija

+ABB Automation, 72157 Västerås, Sweden

UDK 533.951

Originalni znanstveni rad

Određene su frekvencije optičkog i akustičkog titranja u višekomponentnoj, nemagnetiziranoj, homogenoj, bezkolizionoj klasičnoj plazmi, pri čemu je pretpostavljeno da se u izoliranim komponentama vlastite frekvencije i valni vektori zasjenjenja bitno razlikuju.