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ON THE COSMOLOGICAL STOCHASTIC MOTION AND THE SCATTER ON HUBBLE DIAGRAM

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Two points important for understanding of the cosmological stochastic motion are given: first, explanation of the meaning of the maximal stochastic velocity and its connection with the mean stochastic velocity and, second, definition of the allowed class of model-functions describing the dependence of maximal stochastic redshift from the expansive redshift.

1. Introduction

The scatter of data on the corrected magnitudes (m) and the measured redshifts (Z) of galaxies and quasars from the theoretical curves for any deceleration parameter (q_0) can be understood if the overall cosmological happening is composed from: 1) the cosmological expansion and 2) the cosmological stochastic motion. Two types of scatter exist: a) ΔZ — the scatter on Z-axis of Hubble diagram (for constant corrected magnitude m) and b) Δm — the scatter on m-axis of Hubble diagram (for constant measured redshift Z).

Here, an attempt is made to elaborate two points, which are important in further understanding of these scatters:

— first, what is the meaning of the maximal stochastic velocity v_s^m, taken to belong to the objects (galaxies and quasars) observed with the maximal or minimal measured redshift Z for the same corrected magnitude m on the Hubble diagram; this point is discussed in part 2 here;

* Dedicated to the memory of my close friend, Prof. Dr. Aleksandar Milojević.

— second, what forms, of model-function are allowed to describe the maximal stochastic redshift Z_s^0 as a function of the expansive redshift Z_e (and the cosmological scale factors); this point is discussed in part 3 here.

Also, in part 4, discussion of cosmological consequences of the cosmological stochastic motion is presented.

2. *On the meaning of the maximal stochastic velocity*

It is taken that the cosmological objects (galaxies, quasars) do participate in the cosmological stochastic motion, which is isotropic and governed by a distribution function $f(v_s)$, where v_s is the »stochastic« velocity of a cosmological object (E) with respect to another one (R), at some cosmological time $t = t_E$. Then, if N is the total number of cosmological objects (in universe), the fraction of those cosmological objects ($E_1, E_2, ..., E_N$) which appear with the stochastic velocities between v_{s1} and v_{s2}, with respect to the cosmological object R, at time t_E, is given as

$$n = N \int_{v_{s1}}^{v_{s2}} f(v_s) \, dv_s \tag{1}$$

and, in order to have $n = N$ if all velocities are included ($v_{s1} \to 0$, $v_{s2} \to \infty$), the normalisation of the distribution function

$$\int_0^\infty f(v_s) \, dv_s = 1 \tag{2}$$

is demanded.

Now, if one takes that $v_{s1} = v_s^m$, which is the maximal stochastic velocity in the sense that it belongs to the last observable object on the tail of the distribution function, and $v_{s2} \to \infty$, then the number of the cosmological objects in that interval of stochastic velocity, according to the expression (1), is given as

$$n = N \int_{v_s^m}^{\infty} f(v_s) \, dv_s. \tag{3}$$

Remembering again that, at v_s^m, the last object at the tail of the distribution $f(v_s)$ toward rising v_s is found, then the choice $n < 1$ means that there is no object found after v_s^m. More precisely, one can take $n = 1 - \varepsilon$, where $0 < \varepsilon < 1$. Then, Eq. (3) takes form

$$\int_{v_s^m}^{\infty} f(v_s) \, dv_s = \frac{1 - \varepsilon}{N}. \tag{3a}$$

To see what Eq. (3a) means, one would have to know the distribution function $f(v_s)$. As yet, it is not known. First thought leads to try the Gaussian function, second one to find a function designed so to be defined, as positive, only for the region $v_s < c$ and limiting the integration domain up to the velocity of light c.

But, leaving these trials for later consideration, perhaps the right thing is to take here a simple integrable function in order to understand qualitatively how Eq. (3a) works. So, let it be

$$f(v_s) = \frac{1}{\langle v_s \rangle} e^{-v_s/\langle v_s \rangle}, \tag{4}$$

where $\langle v_s \rangle$ is the mean value of the stochastic velocity given by

$$\langle v_s \rangle = \int_0^\infty v_s f(v_s)\, dv_s. \tag{4a}$$

Then, Eq. (3a) gives the relation

$$\langle v_s \rangle = \frac{v_s^m}{\ln N - \ln (1 - \varepsilon)}. \tag{5}$$

Now, it is seen that the trivial choice to have $n = 0$, i. e. $\varepsilon = 1$, according to Eq. (5), leads automatically to $\langle v_s \rangle = 0$, which is suitable to the description of situation with no stochastic motion existing. However, the non-trivial choice that $\Delta n < 1$, with $\varepsilon > 0$ and infinitesimally small, leads to

$$\langle v_s \rangle = \frac{v_s^m}{\ln N} \tag{5a}$$

which represents the physical situation with the stochastic motion being present. To clarify things further, one has to remember that the maximal stochastic velocity produces the Doppler effect and, accordingly, the maximal stochastic redshift Z_s^0. Among v_s^m and Z_s^0 there is a Dopplerian relation

$$\frac{v_s^m}{c} = \frac{(1 + Z_s^0)^2 - 1}{(1 + Z_s^0)^2 + 1}, \tag{6}$$

which, introduced into Eq. (5a), leads to

$$\langle v_s \rangle = \frac{c}{\ln N} \frac{(1 + Z_s^0)^2 - 1}{(1 + Z_s^0)^2 + 1}. \tag{7}$$

In two extremal situations

$$\langle v_s \rangle \to \begin{cases} 0, & \text{for } Z_s^0 \to 0, \\ \dfrac{c}{\ln N}, & \text{for } Z_s^0 \to \infty, \end{cases} \tag{8}$$

so, it is evident that the maximal possible value of $\langle v_s \rangle$ is, by far, less than the velocity of light c, since N is a big number. If the galaxies are taken into account only, then an estimation leads to about $N = 10^{12}$, which means that the maximal value of $\langle v_s \rangle$ is about 11 000 km/s. Of course, one may, also, question the idea that N could be the number of all particles (with non-zero mass) in the universe, in which case the maximal value of $\langle v_s \rangle$ comes to about 1 600 km/s.

Eq. (5a) shows the deep connection of the maximal stochastic velocity v_s^m with the mean stochastic velocity $\langle v_s \rangle$ appearing in the distribution function given in Eq. (4). It is to be expected with certainty that such type of parameters for other distribution functions (say, the mean square of stochastic velocity, $\langle v_s^2 \rangle$, for the Gaussian function) are going to show similar behaviour.

3. On the dependence of the maximal stochastic redshift from the expansive redshift

If both the cosmological expansion and the cosmological stochastic motion are taken into account, the measured redshift Z is given here as

$$Z = Z_e + (1 + Z_e) Z_s, \tag{9}$$

where Z_s is the stochastic redshift. The scatter of data on Hubble diagram for any constant magnitude (m) can be given then as

$$\Delta Z = Z_s^0 (1 + Z_e)(2 + Z_s^0)/(1 + Z_s^0) \tag{10}$$

while the scatter of data on Hubble diagram for any constant measured redshift (Z) is given then as

$$\Delta m = 5 \log [A (Z_e^{\pi}, q_0)/A (Z_e^0, q_0)] \tag{11}$$

where

$$A (Z_x, q_0) = Z_x \left[1 + \frac{Z_x (1 - q_0)}{1 + q_0 Z_x + \sqrt{1 + 2 q_0 Z_x}} \right] \tag{12}$$

is valid for $q_0 > 0$[1] and it is easily proved that the *extremal* expansive redshifts are

$$Z_e^0 = \frac{Z - Z_s^0}{1 + Z_s^0}, \quad Z_e^{\pi} = Z + (1 + Z) Z_s^0. \tag{13}$$

Further, if one demands that the scatter Δm has

(a) to satisfy the theoretical criterion to vanish ($\Delta m \to 0$) as $Z \to 0$ and

(b) to follow reasonably well the observational data[2], which roughly demand that Δm rises with the rise of Z, it comes out that the *maximal* stochastic redshift Z_s^0 has to be dependent from the expansive redshift Z_e.

Of course, as yet, there is no sound theoretical background on which one could know the form of this dependence. However, one could operate with a trial function of the form

$$Z_s^0 = \frac{C \cdot Z_e^{1+a}}{1 + D \cdot Z_e^{\beta}}. \tag{14}$$

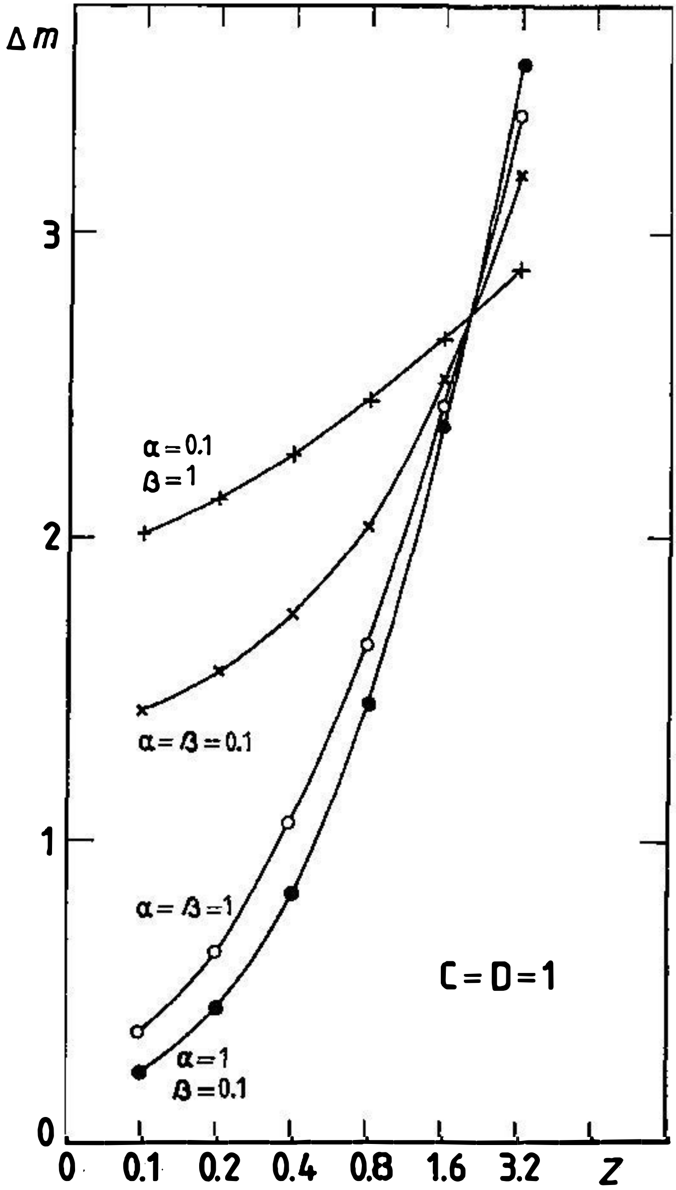

Fig. 1. The dependence of Δm — scatter from α and β (see text).

Then, for any positive α and β, the criterion (a) is satisfied, while, concerning the criterion (b) the choice of α and β is dependent on observational data corrected for *all* astrophysical reasons. In order to get an idea of the dependence of Δm-scatter from α and β, in Fig. 1 the calculated curves with four different choices of α and β are shown. This example was made with fixed $C = D = 1$ and, of course, the variation of C and D could help if a precise fit of observational data is needed.

4. Discussion

The introduction of the cosmological stochastic motion offers new ways of interpreting the scatter of observational data on the corrected magnitudes and redshifts of galaxies and quasars, i. e. the Hubble diagram. However, it has to be stressed again that *all* astrophysical corrections have to be made *before* the data are fitted in the way described in the preceeding paragraph here. Of course, it is well--known that the problem of *all* astrophysical corrections of the magnitudes is not completely settled as yet. Some of these corrections are well-founded[2], while the others are hypothetical. With all reserves, it is interesting to mention here that the limitation to well-founded corrections and the data on the Δm — scatter taken from the reference[2] leads to a possible interpretation of scatter with $C = D = \alpha = \beta = 1$ and $0.5 < q_0 < 2$. If so, it comes out that the maximal stochastic redshift becomes simply dependent from the cosmological scale factors $a = a(t)$ and $a_0 = a(t_{n0w})$, i. e.

$$Z_s^0 = \frac{a}{a_0}\left(\frac{a_0}{a} - 1\right)^2 \tag{15}$$

and, as $t \to \infty$, $Z_s^0 \to Z_e$. Of course, before identifying this kind of cosmology in which, at the beginning, the expansion is, perhaps, caused by highly energetic stochastic motion, one has to think that some of today hypothetical corrections of the magnitude, if proven to be true, may considerably diminish the role of the stochastic motion.

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References

1) J. Terrell, Am. J. Phys. **45** (1977) 869;

2) E. J. Wampler, Astron. Astrophys. **178** (1987) 1.

O KOSMOLOŠKOM STOHASTIČKOM KRETANJU I RASEJANJU NA HABLOVOM DIJAGRAMU

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Razmatrana su dva detalja, važna za razumevanje kosmološkog stohastičkog kretanja: prvo, objašnjenje značenja maksimalne stohastičke brzine i njene veze sa srednjom stohastičkom brzinom i drugo, definicija dozvoljene klase model-funkcija koje opisuju zavisnost maksimalnog stohastičkog crvenog pomaka od ekspanzionog crvenog pomaka.