YU ISSN 0015-3206
FZKAAA 21 (1) 75 (1989)

UNIFICATION OF INTERACTIONS IN SUPERSYMMETRIC ANOMALY-FREE FERMION-SCALAR CONSTITUENT MODELS

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Received 9 September 1988

UDC 539.12

Original scientific paper

Anomaly free supersymmetric fermion-scalar constituent models are constructed using SU(7) unification group. The simplest model uses just four, strongly mass--split superfields, i. e. SU(7) representations 7, 7*, 35 and 35*.

1. Introduction

Preon models[1-5] which involve both fermions and scalars very naturally lead to the introduction of supersymmetry (SUSY). Moreover, it seems that a SUSY-version of a fermion-scalar preon model can be easily made to be anomaly free.

The main aim of this paper is to investigate group properties and symmetries among preon fields, although those might be badly broken in the real physical world. Presumably such symmetries did exist at some point in the cosmic history, at the unification energy scale. It is of interest to find which are the simplest symmetry schemes capable of accommodating all physical and theoretical constraints associated with the preonic models of elementary particles.

Recently[1] we have investigated a class of fermion-scalar preon models which possess an attractive simplicity. Their main weakness was that they did not satisfy the t'Hooft anomaly consistency condition at the preonic level, so that one had to speculate about double confining mechanism[5] in which scalar preons were made of two even more fundamental fermions.

As will be shown in this paper there exists a supersymmetric theory, based on SU(7) unifying group in which all basic preonic fields are left-handed chiral super-fields, and which is anomaly-free. In order to obtain such theoretical consistency (by which one does not mean merely a t'Hooft condition) one has to pay a certain price. New preonic ficlds have to be introduced in addition to those which were needed to describe quarks and leptons. However, the symmetry between scalars and fermions simplifies the model structure. Various classes of the models are now connected and, as discussed in the next section, there are only two distinct choices of the model superfields.

It turns out that the most economical one does not require more than four superfields. However, the SU(7) symmetry of those superfields has to be badly broken one has to use mass-split representations.

Although the electromagnetic preonic charges are not uniquely fixed by the model building, they can be constrained[1] by the unification of the fundamental interactions. One aims to build the anomaly-free supersymmetric model in which the preonic electromagnetic charges are constrained.

So far the idea of preonic models does not have any direct experimental support. In Ref. 6 one can find an example for a weak indirect evidence. That reference speculates about a contribution to the magnetic moment of the electron which can arise if electron is a composite particle.

2. *Supersymmetric models and interactions*

As before[1,7] fundamental forces in the model are classified by the direct--product gauge group

$$G_p = \mathrm{SU}(4)_H \otimes \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_E \subset \mathrm{SU}(7). \tag{2.1}$$

The $\mathrm{SU}(4)_H$ is the minimal hypercolour group which in a simple way leads to the hypercolour-force which is stronger than the colour-force. The usage of a smaller group as was for example done in Ref. 8 necessitates the introduction of a rather large number of new preon fields.

The properties of quarks and leptons are determined by the colour ($\mathrm{SU}(3)_C$) and the electromagnetic ($\mathrm{U}(1)_E$) interactions. Absent are the weak interactions which in the preonic models, are usually not considered as fundamental interactions. The gravitational interactions are also not included in the preonic models.

In the SUSY-version of the model the classes I and II, defined in Ref. 1 are connected. In Ref. 1 fermionic (α, β) and scalar (x, y) preons were labeled according to the representations of G_p as follows:

Class Ia:

$$\alpha = \left(4, 1, \frac{1}{2} + \delta\right) \qquad \beta = \left(4, 1, -\frac{1}{2} + \delta\right)$$

$$x = \left(\bar{4}, 3, \frac{1}{6} - \delta\right) \qquad y = \left(\bar{4}, 1, -\frac{1}{2} - \delta\right). \tag{2.2}$$

Class IIa:

$$\alpha = \left(4, 3, \frac{1}{6} - \delta\right) \qquad \beta = \left(4, 1, -\frac{1}{2} - \delta\right)$$

$$x = \left(\bar{4}, 1, \frac{1}{2} + \delta\right) \qquad y = \left(\bar{4}, 1, -\frac{1}{2} + \delta\right). \tag{2.3}$$

The classes Ib and IIb are obtained by the exchange $4 \leftrightarrow \bar{4}$ in the formulas (2.2) and (2.3). The charge of preon α is $Q = \frac{1}{2} + \delta$ (expressed in the units of positron charge) etc. (The class IIa differs from the class II of Ref. 7; the changed sign of δ will be useful in the following discussions). It has been already indicated[1] that subclasses α and β have different SU(7) assignements.

There is no spatial distinction between α, β and x, y preons in the SUSY--version of the model, they are both left-handed chiral superfields which can be distinguished only through their G_p labelling. Thus, one finds by exchanging $\alpha, \beta \leftrightarrow x, y$:

$$\begin{aligned} &\text{Class Ia} \equiv \text{Class IIb} = \text{Class I} \\ &\text{Class IIa} \equiv \text{Class Ib} = \text{Class II.} \end{aligned} \tag{2.4}$$

These classes belong[1,7] to the antisymmetric representations (R) 7, 21 and 35 of the SU(7) group. Their decomposition under G_p group, together with the corresponding Adler anomalies[9] is given in Table 1.

In order to accommodate all the preons from the classes I and II, one has to use in addition the conjugate representations (R*): 7*, 21* and 35*. Their anomalies $A(R^*)$ are related to the anomalies listed in Table I through

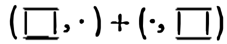
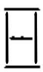
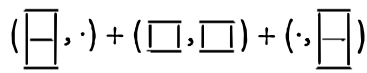
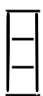
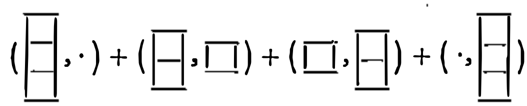
$$A(R^*) = -A(R). \tag{2.5}$$

As usual[8] the first generation of quarks and leptons is constructed from the appropriate scalar and fermion pieces, appearing in the chiral superfields, in the case of the class I for example that means:

$$u = (\alpha x);\ (\alpha x) \qquad \nu_e = (\alpha y);\ (\beta x)$$

$$\alpha = (\beta x);\ (\alpha y) \qquad e^- = (\beta y);\ (\beta y). \tag{2.6}$$

TABLE 1.

	Young scheme	$SU(4) \times SU(3) \times U(1)$ decomposition	anomaly
7		 $(4, 1, Q_1) = (1, 3, Q_2)$	1
21		 $(6, 1, 2Q_1) + (4, 3, Q_1 + Q_2) + (1, 3, 2Q_2)$	3
35		 $(4, 1, 3Q_1) + (6, 3, 2Q_1 + Q_2) +$ $+ (4, 3, Q_1 + 2Q_2) + (1, 1, 3Q_1)$	2

$SU(7)$ representations and anomalies.

Here, the combination inside the first parentheses refers to the class Ia, while the combination in the second parentheses refers to the class IIa. The exchange mentioned above always transforms the first combination in the second one, as for example:

$$(\beta x) \rightarrow (y\alpha).$$

The running coupling constants are calculated in the one-loop approximation

$$\frac{1}{\alpha(q)} = \frac{1}{\alpha(q_L)} + \frac{b_N}{2\pi} \ln \frac{q}{q_L} \tag{2.7}$$

where for the SUSY SU(N) gauge group one has[10]

$$b_N = 3N - \sum n_R T(R)$$
$$T(R)\,\delta_{ab} = \mathrm{Tr}\,(T_a T_b). \tag{2.8}$$

Here n_R denotes the number of representations R.

The unification energy scale M is found from the condition

$$\alpha_M^{-1}(M) = \alpha_C^{-1}(M) = \alpha'^{-1}_E(M). \tag{2.9}$$

Here, the running electromagnetic coupling constant α_E is rescaled by introducing a factor c

$$\frac{1}{\alpha'_E(q)} = c\left[\frac{1}{\alpha_E(q_E)} + \frac{b_E}{2\pi} \ln \frac{q}{q_E}\right]. \tag{2.10}$$

This factor is connected with the SU(7) group generators through an arbitrary traceless charge matrix Q, which is for the representation 7 of the form

$$\operatorname{diag} Q = \Big[\underbrace{\frac{1}{2} + \delta, \dots, \frac{1}{2} + \delta}_{\text{4-times}}, \underbrace{-\frac{3}{2} - \frac{4}{3}\delta, \dots, -\frac{3}{2} - \frac{4}{3}\delta}_{\text{3-times}}\Big]. \qquad (2.11a)$$

Thus for example preons α in (2.2) occupy four states in the representation 7, as indicated in the formula (2.17) below.

The matrix Q (2.11a) is a generator of the SU(7) group. It can be written as a linear combination of the standard Gell-Mann type matrices:

$$\operatorname{diag} F_{24} = \frac{1}{2\sqrt{10}}[1, 1, 1, 1, -4, 0, 0]$$

$$\operatorname{diag} F_{35} = \frac{1}{2\sqrt{15}}[1, 1, 1, 1, 1, -5, 0] \qquad (2.12)$$

$$\operatorname{diag} F_{48} = \frac{1}{2\sqrt{21}}[1, 1, 1, 1, 1, 1, -6]$$

$$\operatorname{Tr}(F_{xy})^2 = \frac{1}{2}.$$

One finds

$$Q = A + 2\delta A$$

$$A = \frac{2\sqrt{10}}{15}F_{24} + \frac{14\sqrt{15}}{45}F_{35} + \frac{2\sqrt{21}}{9}F_{48}. \qquad (2.11b)$$

This matrix is not normalized in the same way as F_{xy}. One has

$$\operatorname{Tr} Q^2 = \frac{1}{3}7(1 + 2\delta^2)^2. \qquad (2.13)$$

Since the group SU(7) (2.1) is a gauge group containing electromagnetic interactions it seems natural to require that Q should be normalized in the same way as SU(7) generators (2.12) i. e. to introduce the modified charge matrix Q which satisfies

$$Q' = \sqrt{c}\,Q$$

$$\operatorname{Tr} Q'^2 = \frac{1}{2}, \quad c = \frac{3}{14(1 + 2\delta)^2}. \qquad (2.14)$$

However this is still an arbitrary choice, which defines a particular preon model by fixing a way in which the charge, being a physical quantity, is associated with the SU(7) group generators. As discussed in Ref. 1, this situation shows some remote similarity with the description of quark charges and the SU(3)-flavour group properties.

The unification scale can be obtained from the formula

$$M = q_H^{\frac{b_H}{b_H - b_C}} \, q_C^{\frac{b}{b_C - b_H}}. \tag{2.15}$$

The scales $q_{H,C}$ were selected as follows[1]:

$$\alpha_C^{-1}(q_c) = 1 \qquad q_C \approx 0.5 \text{ GeV}$$

$$\alpha_H^{-1}(q_H) = 1 \qquad q_H = 10^3, 10^4, 10^5, 10^6 \text{ GeV}. \tag{2.16}$$

In order to find b-factors appearing in (2.10) one has to decide which preons appear in the model.

For the class-I models one has to use the following SU(7) representations[1]

$$\alpha = \left(4, 1, \frac{1}{2} + \delta\right) = (\Box, \cdot) \subset 7$$

$$\beta = \left(4, 1, -\frac{1}{2} + \delta\right) = \left(\begin{smallmatrix}\Box\\ \Box\\ \Box\end{smallmatrix}, \cdot\right)^* \subset 35^*$$

$$x = \left(\bar{4}, 3, \frac{1}{6} - \delta\right) = \left(\Box, \begin{smallmatrix}\Box\\ \Box\end{smallmatrix}\right)^* \subset 35^*$$

$$y = \left(\bar{4}, 1, -\frac{1}{2} - \delta\right) = (\Box, \cdot)^* \subset 7^*. \tag{2.17}$$

Here all preons, i. e. α, β, x and y are left-handed chiral fields. The decomposition (third column) is under the G_p (2.1) group. According to Table 1 and (2.5) the sum of the anomalies for the representations $7 \oplus 7^* \oplus 35^*$ is (-2). These anomalies can be cancelled by adding some other preonic states where anomalies would be $(+2)$. The simplest possibilities would be either two 7-representaties (say α' and α'') or one 35-representation (say $\beta'^* \oplus x'^*$). The results are summarized in the Table 2.

TABLE 2.

	Preons $\alpha \oplus \beta \oplus x \oplus y = (a)$ only	$(a) \oplus (\alpha') \oplus \alpha''$	$(a) \oplus (\beta'^* \oplus x'^*)$
b_H	9	8	7
b_C	7	7	5
b_E	$-\frac{10}{3} - 24\delta^2$	$-\frac{16}{3} - 8\delta - 32\delta^2$	$-\frac{14}{3} + 8\delta - 28\delta^2$

Coefficients b for the class-I models.

The models of the class II use somewhat different SU(7) representations, i. e.:

$$\alpha = \left(4, 3, \frac{1}{6} - \delta\right) = (\square\!\square, \square\!\square) \subset 21$$

$$\beta = \left(4, 1, -\frac{1}{2} - \delta\right) = \left(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}, \cdot\right)^* \subset 35^*$$

$$x = \left(\bar{4}, 1, \frac{1}{2} + \delta\right) = \left(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}, \cdot\right) \subset 35$$

$$y = \left(\bar{4}, 1, -\frac{1}{2} + \delta\right) = (\square, \cdot)^* \subset 7^*. \qquad (2.18)$$

TABLE 3.

	Preons $\alpha \oplus \beta \oplus x \oplus y = (b)$ only	$(b) \oplus y' \oplus y''$	$(b) \oplus \beta'$
b_H	9	8	$\frac{17}{2}$
b_C	7	7	7
b_E	$-\frac{10}{3} - 24\delta^2$	$-\frac{16}{3} + 8\delta - 32\delta^2$	$-\frac{13}{3} - 4\delta - 28\delta^2$

Coefficients b for the class-II models.

Here the sum of anomalies is $(+2)$ so that they can be cancelled by adding two 7^*-representations $(y' \oplus y'')$ or one 35^*-representation (β'). The corresponding b-factors are listed in Table 3. Parameters δ and c and the unification energy scale M are shown for various q'_Hs and b'_ys, in Tables 4 and 5 which refer to class-I and class-II models, respectively. One q_H-value was always fixed for the preselected $M = 10^{15}$ GeV from the equality $\alpha_C^{-1}(M, q_C = 0.5 \text{ GeV}) = \alpha_H^{-1}(M, q_H = ?)$.

TABLE 4.

q_H	10^3	10^4	10^5	10^6	1.26×10^3
M	3.58×10^{14}	1.13×10^{19}	3.58×10^{23}	1.13×10^{28}	10^{15}
δ_1	−0.759	−0.170	−0.217	−0.257	−0.116
c_1	0.799	0.492	0.669	0.907	0.363
δ_2	−0.109	−0.692	−0.642	−0.600	−0.751
c_2	0.350	1.453	2.656	5.368	0.848

(a)-no new preons

q_H	10^3	10^4	10^5	10^6	40.89
M	1.28×10^{26}	1.28×10^{34}	1.28×10^{42}	1.28×10^{50}	10^{15}
δ_1	−0.248	−0.314	−0.394	no	−0.126
c_1	0.846	1.540	4.732	real	0.383
δ_2	−0.616	−0.548	−0.466	roots	−0.747
c_2	3.995	23.251	652.622		0.878

$(a) + \alpha'^* + \alpha''^*$

q_H	10^3	10^4	10^5	10^6	1.18×10^4
M	1.79×10^{11}	5.66×10^{14}	1.79×10^{18}	5.66×10^{21}	10^{15}
δ_1	0.020	−0.050	−0.103	−0.635	−0.055
c_1	0.155	0.265	0.340	2.935	0.270
δ_2	−0.821	−0.741	−0.682	−0.146	−0.736
c_2	0.520	0.922	0.287	0.428	0.959

$(a) + \beta'^* + \varkappa'^*$

Values of M, δ and c for class-I models, q_H and M are in GeV.

TABLE 5.

q_H	10^3	10^4	10^5	10^6	250×70
M	2.54×10^{18}	1.18×10^{24}	5.47×10^{29}	2.54×10^{35}	10^{15}
δ_1	−0.167	−0.056	−0.275	−0.321	−0.121
c_1	0.483	0.272	1.058	1.672	0.373
δ_2	−0.699	−0.636	−0.585	−0.537	−0.749
c_1	1.353	0.260	7.415	39.132	0.864

$(b) \oplus \beta'$

q_H	10^3	10^4	10^5	10^6	40×89
M	1.28×10^{26}	1.28×10^{34}	1.28×10^{42}	1.28×10^{50}	10^{15}
δ_1					−0.147
c_1	no	real	roots		0.429
δ_2					−0.642
c_2					2.675

$(b) \oplus y' \oplus y''$

Values of M, δ and c for class-II models, q_H and M are in GeV.

Tables 4 and 5 were calculated using only those pieces of the SU(7) representations which were listed in either (2.17) or (2.18). It was supposed that the rest of each multiplet has acquired (through some suitable symmetry breaking) a very large mass. Therefore, it has decoupled and it did not contribute to the *b*-factors.

Obviously the symmetry breaking mechanism is the most important dynamical question. It goes beyond the scope of this paper which is concerned with the sistematization of the group properties of simple preonic models. Detailed example of the required symmetry breaking are worked out in Refs. 8 and 12.

It is also fair to mention that the symmetry breaking mechanism is not yet really fundamentally explained even for such a well established symmetry as SU(3)-flavour.

TABLE 6.

	Class I 7, 7*, 35*, 7, 7	7, 7*, 35*, 35	Class II 35, 7*, 21, 35, 35*	35, 7*, 21, 35*, 7*, 7*
b_H	5	1	−6	−2
b_C	2	−2	−9	−5
b_E	$-\frac{98}{3}(1+2\delta)^2$	$-\frac{154}{3}(1+2\delta)^2$	$-84(1+2\delta)^2$	$-63(1+2\delta)^2$

Coefficients *b*. Full *SU* (7) representations.

It is of some interest to study the unification assuming that all states in the respective SU(7) representations (and not only those involved in building quarks and leptons) should contribute. The corresponding values of *b*-factors, displayed in Table 6, are quite different from the values shown in Tables 2 and 3.

With complete representations 7, 7*, 35*, 7, 7, one finds positive b_H and b_C coefficients which lead to the asymptotic freedom only for the class-I models. As can be seen in Table 7 $q_H = 10^3, 10^4, 10^5, 10^6$ GeV lead to too small values of the mass scale M. Only for the quite large $q_H \approx 10^9$ GeV one can obtain a reasonable value $M \approx 10^{15}$ GeV. These values are quite different from the corresponding broken symmetry case $((a) + \alpha' + \alpha'')$ which is shown in Table 4. However, this case is somehow singled out as it gives acceptable results with both broken and complete SU(7) representations.

TABLE 7.

q_H	10^3	10^4	10^5	10^6	7.59×10^8
M	1.59×10^5	7.37×10^6	3.42×10^8	1.59×10^{10}	10^{15}
δ_1	0.215	0.102	−0.021	−0.115	−0.064
c_1	0.105	0.191	0.233	36107	0.282
δ_2	−1.214	−1.102	−0.979	−0.885	−0.936
c_2	0.105	0.191	0.233	0.361	0.282

Values of M, δ and c for class-I models using SU(7) representations 7, 7*, 35*, 7, 7

Another attractive choice would be the model which requires the smallest number of preonic representations, as for example class I (7, 7*, 35* ⊕ 35). Somewhat less attractive is the model class II (7*, 21, 35, 35* ⊕ 35*) which has two distinct 35* representations. In both of these model variants the interactions can be unified only if the fundamental SU(7) symmetry is badly broken.

3. *Conclusion*

The main point in this paper is that supersymmetrization of scalar-fermion constituent models leads to an anomaly-free and thus theoretically consistent theory.

All details like the usage of the SU(7) unifying group or the specific SU(7) transformation properties of superfields appear here only as an illustration of the usefullness and the naturalness of the SUSY-approach to the preon-model building. The choice of the $SU(4)_H$ group is quite convenient, but not unique. There exists a model[8] which uses a smaller group. It is also possible[8] that the unification energy scale M might be smaller than it was assumed in this paper.

One cannot select any definitive choice of SU(7) representations either. Esthetically, an attractive combination is the one which contains 7, 7*, 35 and 35*. The important point is that all SU(7) representations have to be strongly mass-split. But this is hardly surprising, as it is a common feature of many preon models and unified theories in general.

While the only physical indication for the existence of preonic level is found in the reoccurence of generations, it would be premature to elaborate on other model details, such as the mass-splitting mechanism or the predictions for decays. At the most, one can hope that some features of preons have been correctly envisaged here, so that they might become part of the definitive preon model when and it the existence of preons is proved.

Acknowledgement

This work has been supported in part by joint Yugoslav—American NSF Project JFP-526 and JFP-683. One of us (S. F.) gratefully acknowledges Alexander von Humboldt Foundation for support and the hospitality of the Max-Planck-Institut für Physik, Munich, where this work was begun.

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UJEDINJENJE MEĐUDJELOVANJA U SUPERSIMETRIČNIM KONSTITUENTNIM MODELIMA FERMIONA I SKALARA BEZ ANOMALIJA

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Originalni znanstveni rad

Konstruirani su supersimetrični konstituentni modeli fermiona i skalara bazirani na unifikacijskoj grupi SU(7). Najjednostavniji model upotrebljava samo četiri superpolja jakog masenog rascjepljenja, tj. SU(7) reprezentacije 7, 7*, 35 i 35*.