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KOWALEVSKAYA'S EXPONENTS, AUTO-BÄCKLUND TRANSFORMATION AND ON THE NONINTEGRABILITY OF THE YANG-MILLS SYSTEM

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We study the algebraically completely integrable property of the recently suggested Yang-Mills system. Using the Kowalevskaya's exponents as the criteria for the algebraic integrability, we found our results are not contradicting the earlier observations of nonintegrability of this system. Moreover, we found a very small subsystem of this YM theory which is integrable and possesses auto-Bäcklund transformation.

1. Introduction

Exciting and significant discoveries have been made in the nonlinear classical dynamics of dissipative and conservative systems. Numerical, analytical and experimental works of the last two decades or so, give evidence that most of the nonlinear systems exhibit a transformation from »regular« to »chaotic« behaviour. The connection between movable singularities and algebraic integrability of dynamical systems is widely studied in different contexts[1,2,3]. Based on the analytic behaviour of the solutions of nonlinear ordinary differential equations, there are methods like Painlevé property (PP)[4,5], and Kowalevskaya's exponent (KE)[6,7,8] have been developed for the identification of the algebraically completely integrable dynamical systems.

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The names like Kowalevskaya, Fuchs[9], Painlevé and Gambier[10] have been associated with the early developments of the study of algebraic integrability and singulaiities of dynamical systems. It was Sonya Kowalevskaya who first used the algebraic integrability property in 1888 to completely integrate a dynamical system, the rotating rigid body with moment of inertia ratios 1 : 1 : 2. For an algebraically completely integrable system their independent, single valued integrals are rational functions of the phase space coordinates and their tori are part of a compact, complex tori on which the motion is linear. For a given set of similarity invariant first order ordinary differential equations is algebraically integrable, every possible KE must be rational number[8]. The existence of imaginary or irrational KE implies the nonintegrability of the system[9].

In this paper, first we are reviewing the method of evaluating KE of a set of ordinary differential equations. Then we will apply this procedure to study the integrability of a recently suggested Yang-Mills (YM) system.

2. *Kowalevskaya's exponents of a system of differential equations*

First we shall give a brief outline of the method of determination of KEs of an autonomous system of ordinary first order differential equations. This method can be easily extended to nonautonomous system. Let us consider a system of autonomous differential equations:

$$\frac{dx_i}{dt} = X_i(x_1, x_2, ..., x_n), \qquad i = 1, 2, ..., n \tag{2.1}$$

where X_i $i = 1, 2, ..., n$ are rational functions of $x_1, x_2, ..., x_n$. The singularity of the solutions of (2.1) is characterised by a set of exponents, called the KEs. These exponents are determined by a finite procedure, provided the system (2.1) is invariant under a self-similar transformation.

The system (2.1) is said to be self-similar invariant, when it is invariant, under a set of following transformation

$$t \to \alpha^{-1} t \tag{2.2}$$

$$x_i \to \alpha^{s_i} x_i, \qquad i = 1, 2, ..., n \tag{2.3}$$

for a constant α. This implies

$$X_i(\alpha^{s_1} x_1, \alpha^{s_2} x_2, ..., \alpha^{s_n} x_n) = \alpha^{s_i+1}, \qquad i = 1, 2, ..., n. \tag{2.4}$$

The choice of the set of rational numbers $s_1, s_2, ..., s_n$ is unique when the functional determinant

$$\det_{1 \leq i,j \leq n} \left| x_j \frac{\partial X_i}{\partial x_j}(x_1, x_2, ..., x_n) - \delta_{ij} X_i(x_1, x_2, ..., x_n) \right| \tag{2.5}$$

is nonzero.

For any self-similar transformation invariant system (2.1), there exists a particular solution

$$x_i = c_i\, t^{-s_i}, \qquad i = 1, 2, \ldots, n. \tag{2.6}$$

The complex valued constants $c_1, c_2, \ldots, c_n$ are to be found out from the set of following simultaneous algebraic equations,

$$X_i\,(c_1, c_2, \ldots, c_n) = -s_i c_i, \qquad i = 1, 2, \ldots, n. \tag{2.7}$$

The Kowalevskaya's matrix K_{ij} is defined as

$$K_{ij} = \frac{\partial X_i}{\partial x_j}\,(c_1, c_2, \ldots, c_n) + \delta_{ij}\, s_i. \tag{2.8}$$

The KEs are the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of the Kowalevskaya's matrix (Eq. (2.8)), obtained by finding the roots of the characteristic polynomial $K(\lambda)$ defined as

$$K(\lambda) = \det_{1 \leqslant i,j \leqslant n} |\lambda\, \delta_{ij} - K_{ij}|. \tag{2.9}$$

For a given self-similar transformation invariant system (2.1) is algebraically integrable, the every possible KE must be rational number. Existence of an irrational or imaginary valued KE implies the nonexistence of integrability.

Recently[8] this procedure is extensively studied and applied to some well-known nonintegrable system and established as a powerful tool for showing non-integrability. This method we are applying to a recently suggested Yang-Mills (YM) system.

3. Yang-Mills system

Beseyan et al.[11] have suggested a YM system and discovered some particular solutions of a special case of this system. Recently, Nikolaevskii and Shur[12] proved that above special case of the system is intrinsically nonintegrable. This observation is confirmed by Chang[13] who studied the same class of solutions using adiabatic invariance and found solution is chaotic. We are considering both the special case of the YM system, that the earlier workers are studied and the general case of the system. We are also studying another special case of this system.

For convenience we use the notations and conventions of Ref. 13. Specifically, the Lagrangian under consideration is

$$L = -(1/4)\, F^a_{\mu\nu}\, F^a_{\mu\nu}, \qquad \mu, \nu = 0, 1, 2, 3. \tag{3.1}$$

The field equations are

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, \varepsilon^{abc}\, A^b_\mu\, A^c_\nu \tag{3.2}$$

$$\partial_\mu F^a_{\mu\nu} + g\,\varepsilon^{abc} A^b_\mu F^c_{\mu\nu} = 0. \tag{3.3}$$

Using the ansatz,

$$A^a_0 = 0, \tag{3.4}$$

$$A^a_i = A^a_i(t) \tag{3.5}$$

Eqs. (3.2) and (3.3) yield

$$F^a_{0i} = \partial_0 A^a_i \tag{3.6}$$

$$F^a_{ij} = g\,\varepsilon^{abc} A^b_i A^c_j. \tag{3.7}$$

Setting $\nu = 0$ in (3.3), we get the Gauss law

$$g\,\varepsilon^{abc} A^b_i F^c_{i0} = 0, \tag{3.8}$$

or equivalently

$$A^b_i \partial_0 A^c_i - A^c_i \partial_0 A^b_i = 0. \tag{3.9}$$

Setting $\nu = i$, in (3.3), we obtain the remaining field equations,

$$\partial^2_0 A^a_i - g^2 \varepsilon^{abc} \varepsilon^{cde} A^b_j A^d_j A^e_i = 0, \tag{3.10}$$

or

$$\partial^2_0 A^a_i + g^2 (A^a_i A^b_j - A^a_j A^b_i) A^b_j = 0. \tag{3.11}$$

When

$$A^a_i = O^a_i f_a(t), \tag{3.12}$$

where the index a is not summed and O^a_i are constant orthogonal matrices obeying

$$O^a_i O^b_i = \delta^{ab}/g^2, \tag{3.13}$$

we get a reduced set of equations of motion from (2.10):

$$\frac{d^2 f_1}{dt^2} + f_1 (f_2)^2 + f_1 (f_3)^2 = 0, \tag{3.14}$$

$$\frac{d^2 f_2}{dt^2} + f_2 (f_1)^2 + f_2 (f_3)^2 = 0, \tag{3.15}$$

and

$$\frac{d^2 f_3}{dt^2} + f_3 (f_1)^2 + f_3 (f_2)^2 = 0. \tag{3.16}$$

One can extend the solutions of (3.14)—(3.16) to an arbitrary non-abelian gauge theory straightforwardly.

In all the earlier studies for establishing nonintegrability of YM theory, they have considered the special case $f_3 = 0$, $f_1 \neq 0$ and $f_2 \neq 0$. For this special case the equations of motion (3.14)—(3.16) reduce to a paiı of coupled equations

$$\frac{d^2 f_1}{dt^2} + f_1 f_2^2 = 0 \tag{3.17}$$

and

$$\frac{d^2 f_2}{dt^2} + f_2 f_1^2 = 0. \tag{3.18}$$

In our study we are considering both the set (3.14)—(3.16) and (3.17) and (3.18).

4. *Kowalevskaya's exponents of the Yang-Mills system*

First we shall find the KEs of the system of Eqs. (3.17) and (3.18). Corresponding to this system we can consider following equivalent set of first order equations

$$\frac{dx_1}{dt} = x_1 x_3, \tag{4.1}$$

$$\frac{dx_2}{dt} = x_2 x_4, \tag{4.2}$$

$$\frac{dx_3}{dt} = -(x_2^2 + x_3^2), \tag{4.3}$$

$$\frac{dx_4}{dt} = -(x_1^2 + x_4^2), \tag{4.4}$$

where $x_1 = f_1$, $x_2 = f_2$ and x_3 and x_4 are newly introduced variables which depend on f_1 and f_2 only. Clearly, on differentiating Eqs. (4.1) and (4.2) with ıespect to t and substituting (4.2) and (4.3), yield the set of equations (3.17) and (3.18). We are finding the KEs of (4.1)—(4.4) instead of considering (3.17) and (3.18).

The system (4.1)—(4.4) is invariant under the self similar transformations:

$$t \to \alpha^{-1} t \tag{4.5}$$

$$x_i \to \alpha x_i, \qquad i = 1, 2, 3, 4. \tag{4.6}$$

It is easy to find that the functional determinant (2.5) corresponding to this system is nonvanishing.

The particular solutions of the type (2.6), that are obtained for this system are

$$x_i = C_i t^{-1}, \qquad i = 1, 2, 3, 4 \tag{4.7}$$

and C_1, C_2, C_3 and C_4 are found by solving (2.7). This yields

$$C_1 = C_2 = 2\sqrt{-1}, \tag{4.8}$$

$$C_3 = C_4 = -1. \tag{4.9}$$

The Kowalevskaya's matrix K_{ij} is

$$K_{ij} = \begin{bmatrix} 0 & 0 & 2\sqrt{-1} & 0 \\ 0 & 0 & 0 & 2\sqrt{-1} \\ 0 & -4\sqrt{-1} & 3 & 0 \\ -4\sqrt{-1} & 0 & 0 & 3 \end{bmatrix}. \tag{4.10}$$

The characteristic polynomial $K(\lambda)$ is obtained for the matrix (4.10) as:

$$K(\lambda) = \lambda^2 (\lambda - 3)^2 - 64. \tag{4.11}$$

This yields the exponents

$$\lambda = (3 \pm \sqrt{41})/2, \quad (3 \pm \mathrm{i}\sqrt{23})/2 \tag{4.12}$$

where $\mathrm{i} = \sqrt{-1}$. The imaginary and irrational roots indicate the system of equations (3.17) and (3.18) is not algebraically integrable. Our observation is coinciding with the earlier studies.

Now we shall consider the general case (3.14)—(3.16). The set of first order differential equations that are equivalent to (3.14)—(3.16) are the following:

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = -x_4^2/2, \tag{4.13}$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = -x_5^2/2, \tag{4.14}$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} = -x_6^2/2, \tag{4.15}$$

$$\frac{\mathrm{d}x_4}{\mathrm{d}t} = x_1 (x_2^2 + x_3^2)/x_4, \tag{4.16}$$

$$\frac{dx_5}{dt} = x_2 (x_1^2 + x_3^2)/x_5, \tag{4.17}$$

$$\frac{dx_6}{dt} = x_3 (x_1^2 + x_2^2)/x_6, \tag{4.18}$$

where $x_1 = f_1$, $x_2 = f_2$, $x_3 = f_3$ and x_4, x_5, x_6 are newly introduced variables. Obviously, on differentiating (4.13)—(4.15) with respect to t and substituting (4.16)—(4.18) yield the set (3.14)—(3.16).

The system (4.13)—(4.18) is invariant under the self-similar transformation

$$t \to \alpha^{-1} t, \tag{4.19}$$

$$x_i \to \alpha x_i, \qquad i = 1, 2, ..., 6. \tag{4.20}$$

It is easy to find that the functional determinant (2.5) corresponding to this system is nonvanishing.

The set of particular solutions of the type (2.6) that are obtained for this system:

$$x_i = C_i t^{-1}, \qquad i = 1, 2, ..., 6 \tag{4.21}$$

and

$C_1, C_2 \dots C_6$ are

$$C_1 = C_2 = C_3 = \sqrt{-1}, \tag{4.22}$$

$$C_4 = C_5 = C_6 = \sqrt{2i}, \tag{4.23}$$

where $i = \sqrt{-1}$. The Kowalevskaya's matrix K_{ij} is obtained:

$$K_{ij} = \begin{bmatrix} -1 & 0 & 0 & -\sqrt{2i} & 0 & 0 \\ 0 & -1 & 0 & 0 & -\sqrt{2i} & 0 \\ 0 & 0 & -1 & 0 & 0 & -\sqrt{2i} \\ -\sqrt{2/i} & -\sqrt{2/i} & -\sqrt{2/i} & 0 & 0 & 0 \\ -\sqrt{2/i} & -\sqrt{2/i} & -\sqrt{2/i} & 0 & 0 & 0 \\ -\sqrt{2/i} & -\sqrt{2/i} & -\sqrt{2/i} & 0 & 0 & 0 \end{bmatrix}. \tag{4.24}$$

The characteristic polynomial $K(\lambda)$ is obtained for this matrix

$$K(\lambda) = \lambda^2 (\lambda + 1)^2 (\lambda^2 + \lambda + 2). \tag{4.25}$$

Clearly, the third factor in (4.25) gives a pair of complex roots

$$\lambda = (-1 \pm 7\sqrt{-1})/2, \tag{4.26}$$

and hence two of the KEs are imaginary valued. So we can conclude that the general YM system (3.14)—(3.16) is nonintegrable.

It is interesting to note that although the general YM system ((3.14)—(3.16)) is non-integrable, there exists a subsystem which is integrable. This subsystem we are developing from the set of equations (3.14)—(3.16).

Corresponding to each second order differential equations one can expect two first order differential equations and so there exists a set of six first order differential equations with respect to the set of three second order differential equations (3.14)—(3.16). Let us consider the following three first order differential equation

$$\frac{dx_1}{dt} = i\, x_2 x_3, \tag{4.27}$$

$$\frac{dx_2}{dt} = i\, x_1 x_3, \tag{4.28}$$

$$\frac{dx_3}{dt} = i\, x_1 x_2, \tag{4.29}$$

where $x_1 = f_1$, $x_2 = f_2$, $x_3 = f_3$. On differentiating and solving, Eqs. (4.27)—(4.29) give the set of second order differential equations (3.14)—(3.16). Clearly, this system (4.27)—(4.29) is only a very small subset of the original YM system.

This system (4.27)—(4.29) is invariant under the self-similar transformation

$$t \to \alpha^{-1} t, \tag{4.30}$$

$$x_i \to \alpha\, x_i, \qquad i = 1, 2, 3. \tag{4.31}$$

It is easy to verify that the functional determinant (2.5) for this system of transformation is nonvanishing. The set of special type ((2.6)) of particular solutions are obtained:

$$x_i = C_i\, t^{-1}, \qquad i = 1, 2, 3 \tag{4.32}$$

where $C_1 = C_2 = C_3 = \pm \sqrt{-1}$.

The KE matrix K_{ij} is

$$K_{ij} = \begin{pmatrix} 1 & +1 & \pm 1 \\ \pm 1 & 1 & \pm 1 \\ \pm 1 & \pm 1 & 1 \end{pmatrix}. \tag{4.33}$$

The characteristic roots or KEs λ are obtained, corresponding to the matrix (4.33)

$$\lambda = -1, 2, 2. \tag{4.34}$$

All the KEs are real and rational numbers, implies the system (4.27)—(4.29) can be algebraically integrable, unlike the general system.

For an integrable system we can expect an auto-Bäcklund transformation (ABT). In the following section we are deriving ABT for the possibly integrable subset of YM system (4.27)—(4.29), using the Painlevé property.

5. Painlevé property and auto-Bäcklund transformation of the Yang--Mills system

Ward[14] has extended the study of Painlevé property (PP), well known in the context of ordinary differential equations, to the partial differential equations (PDEs). A system of PDEs in n independent variables are considered in the complex domain, the coefficients being analytic on C^n. If S is an analytic non-characteristic complex hypersurface in C^n, then every solution of the PDE which is analytic on S is meromorphic in C^n. A weaker form of the PP was suggested by Weiss et al.[15] while studying the Lorentz series expansion of the solutions in the neighbourhood of a movable singularity. They found that the PP of PDEs can be used to develop the ABT of the system. In this section we are using this procedure to develop ABT for the system of ordinary differential equations (4.27)—(4.29).

For a Painlevé type system, the series expansions

$$x_1 = \Phi^{\alpha} \sum_{j=0}^{\infty} U_j(t)\, \Phi^j(t), \tag{5.1}$$

$$x_2 = \Phi^{\beta} \sum_{j=0}^{\infty} V_j(t)\, \Phi^j(t), \tag{5.2}$$

$$x_3 = \Phi^{\eta} \sum_{j=0}^{\infty} W_j(t)\, \Phi^j(t), \tag{5.3}$$

where $U_j(t)$, $V_j(t)$, $W_j(t)$ and $\Phi(t)$ are analytic functions, can be truncated for a finite value of j and the resultant form will give the required ABT.

On substituting (5.1)—(5.3) in (4.27)—(4.29) and by a leading order analysis we get

$$\alpha = \beta = \eta = 1. \tag{5.4}$$

The recursion relations for $U_j(t)$, $V_j(t)$ and $W_j(t)$ are found to be

$$U_{j-1,t} + (j-1)\, U_j \Phi_{,t} = \mathrm{i} \sum_{m=0}^{j} V_{j-m} W_m, \tag{5.5}$$

$$V_{j-1,t} + (j-1)\, V_j \Phi_{,t} = \mathrm{i} \sum_{m=0}^{j} U_{j-m} W_m, \tag{5.6}$$

and

$$W_{j-1,t} + (j-1) W_j \Phi_{,t} = \mathrm{i} \sum_{m=0}^{j} U_{j-m} V_m. \tag{5.7}$$

Collecting the terms involving U_j, V_j and W_j it is found that

$$(j-1) U_j = F_1 (V_j, W_j, U_{j-1}, \ldots) \tag{5.8}$$

$$(j-1) V_j = F_2 (U_j, W_j, V_{j-1}, \ldots) \tag{5.9}$$

and

$$(j-1) W_j = F_3 (U_j, V_j, U_{j-1}, \ldots). \tag{5.10}$$

We note that the recursion relations (5.8)—(5.10) are not defined when $j = 1$. This value of j is called the »resonance« of the recursion relation and corresponds to this value of j we can introduce arbitrary functions in the expansions (5.1)—(5.3).

We find from (5.5)—(5.7)

$$j = 0,\ U_0 = V_0 = W_0 = \pm \sqrt{-1}\, \Phi_{,t} \tag{5.11}$$

$$j = 1,\ U_1 = V_1 = W_1 = -\sqrt{-1}\, \Phi_{,tt}/2\Phi_{,t}, \tag{5.12}$$

$j = 2$, the compatibility condition,

$$\frac{\partial}{\partial t} [\Phi_{,tt}/\Phi_{,t} - 1/2 (\Phi_{,tt}/\Phi_{,t})^2] = 0 \tag{5.13}$$

and

$$U_j = V_j = W_j = 0 \text{ for all } j > 2.$$

Thus the YM subsystem (4.27)—(4.29) possesses the Painlevé property.

Equations (4.12)—(5.13) yields

$$U_{1,t} = \mathrm{i}\, V_1 W_1, \tag{5.14}$$

$$V_{1,t} = \mathrm{i}\, U_1 W_1, \tag{5.15}$$

$$W_{1,t} = \mathrm{i}\, U_1 V_1, \tag{5.16}$$

where $\mathrm{i} = \sqrt{-1}$. Clearly Eqs. (5.14)—(5.16) are equivalent to the system of given differential equations (4.27)—(4.29) provided we set

$$U_1 = x_1, \tag{5.17}$$

$$V_1 = x_2, \tag{5.18}$$

$$W_1 = x_3, \tag{5.19}$$

form a solution of (4.27)—(4.29).

On substituting (5.11)—(5.13) in (5.1)—(5.3) we get

$$x_1 = \Phi^{-1} [\pm \sqrt{-1}\, \Phi_{,t} + U_1 \Phi], \tag{5.20}$$

$$x_2 = \Phi^{-1} [\pm \sqrt{-1}\, \Phi_{,t} + V_1 \Phi], \tag{5.21}$$

$$x_3 = \Phi^{-1} [\pm \sqrt{-1}\, \Phi_{,t} + W_1 \Phi], \tag{5.22}$$

or equivalently

$$x_1 = \pm \sqrt{-1} \frac{\partial}{\partial t} (\ln \Phi) + U_1, \tag{5.23}$$

$$x_2 = \pm \sqrt{-1} \frac{\partial}{\partial t} (\ln \Phi) + V_1, \tag{5.24}$$

$$x_3 = \pm \sqrt{-1} \frac{\partial}{\partial t} (\ln \Phi) + W_1, \tag{5.25}$$

where $x_1 = U_1$, $x_2 = V_1$ and $x_3 = W_1$ is a set of particular solutions of Eqs. (4.27)—(4.29) and Φ is determined from (5.13). Eqs. (5.13) can be expressed as a »Schwarzian derivative«[16]:

$$\{\Phi, t\} = 0. \tag{5.26}$$

Eqs. (5.24)—(5.27) together define the ABT for this subsystem (4.27)—(4.29). Whenever a particular solution $(U_1 V_1 W_1)$ is known, then another solution (x_1, x_2, x_3) can be determined from the above ABT.

6. *Discussion*

Kowalevskaya's exponents conditions for an algebraic integrability of a dynamical system is only a necessary condition for the existence of integrability, whereas the appearance of a complex or irrational valued Kowalevskaya's exponents is a sufficient condition for the nonexistence of algebraic integrability[8]. By nonexistence of algebraic integrability we shall mean the absence of sufficient number of first integrals that make the system as completely integrable one[3].

In the above study we have considered only the time dependent class of solutions of YM system by suppressing the space variables dependence. For a most general YM system one should consider the time dependence as well as the three space variables dependence. Moreover, in this study, we have considered the class of non-selfdual reduction of the YM system. The PP of selfdual condition of the YM is studied recently[17], and found, that it is integrable. Now, one has to consider whether the selfdual conditions are the only allowed conditions for integrable reduction of the YM system. Recently[18], Savvidy used the property of constant negative curvature to establish the instability and hence pointed that the classical YM system as a Kolmogrov *K*-system is as interesting result to note. The chaotic nature of the YM system has very high implications to the stability of the fundamental particles and their interactions. The confinement mechanism in the Quantum Chromodynamics (QCD) may be able to explain using this property.

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EKSPONENTI KOWALEVSKAJE, AUTO-BÄCKLUNDOVA TRANSFORMACIJA I NEINTEGRABILNOST YANG-MILLSOVA SISTEMA

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Proučavana su svojstva algebarske potpune integrabilnosti nedavno predloženog Yang-Millsova sistema. Koristeći eksponente Kowalevskaje kao kriterije algebarske integrabilnosti nađeno je da naši rezultati nisu u kontradikciji s ranijim opažanjima o neintegrabilnosti tog sistema. Što više, nađen je vrlo mali podsistem ove YM teorije koji je integrabilan i posjeduje auto-Bäcklund transformaciju.