Solution to the single parametric linear programming problems via simplex-based algorithms: handling the uncertainties in costs, left or right-hand sides

Gizem Temelcan Ergenecosar [1,*], **Hale Gonce Kocken** [2], **Inci Albayrak** [2], and **Mustafa Sivri** [2]

[1] *Department of Software Engineering, Beykoz University, Kavacik, 34010 Beykoz İstanbul, Turkiye E-mail: ⟨temelcan.gizem@gmail.com⟩*

[2] *Department of Mathematical Engineering, Yildiz Technical University, 34210 Esenler İstanbul, Turkiye, E-mail: ⟨{hgonce, ibayrak, msivri}@yildiz.edu.tr⟩*

Abstract. Parametric programming is one of the notable approaches to expressing the uncertainties encountered in real life. Many studies express the parameters of the objective function and right-hand side parametrically, but only a few include the parametric coefficient matrix of the constraints. This paper examines the feasibility and optimality conditions of the simplex table and proposes a simplex-based algorithm (dual-simplex, generalized-simplex, or primal-simplex). In the solution process, each case is considered independently through the mathematical analysis of simplex multipliers. Distinct numerical examples illustrate each case to demonstrate the algorithm's implementation.

Keywords: dual-simplex method, generalized-simplex method, parametric linear programming problem, primal-simplex method

Received: September 25, 2024; accepted: November 29, 2024; available online: June 3, 2025

DOI: 10.17535/crorr.2026.0003

Original scientific paper.

1. Introduction

The essential methods used to model uncertainty in the literature include deterministic, probabilistic, stochastic approaches, fuzzy mathematics, and interval arithmetic. The deterministic approaches contain mainly robust and parametric programming modeling. The parametric programming problem involves modeling uncertainties using parameters. It can be classified according to the location of the parameter (objective function, left-hand side (LHS), and right-hand side (RHS)) as well as the number of parameters (single or multi). Despite significant theoretical advancements in these classifications, it is observed that a limited number of studies have been conducted where the single parameter is on the LHS of the constraints in a Linear Programming (LP) problem.

A widely used algorithm in LP is a known simplex method that finds the optimal solution maximizing or minimizing a linear objective function, subject to linear (in)equality constraints. The method starts at a vertex of the feasible region and then moves from vertex to vertex along the edges of the feasible region. The method persists until it reaches the optimal vertex, where it either maximizes or minimizes the objective function. This method efficiently solves large-scale LP problems and is fundamental in operations research for problems such as resource allocation and production planning.

*Corresponding author.

Sensitivity analysis shows how parameter changes affect the optimal solution and examines how changes in the parameters affect the optimal solution. Ranges on the objective function coefficients and RHS values can describe the sensitivity of an optimal solution. These ranges were valid for one objective-function coefficient or RHS value change, while the remaining problem data are fixed. For instance, shadow prices determine changes in the objective function. Shadow prices, also known as dual prices, indicate how much the objective function would change if there is a one-unit increase in a constraint's RHS, assuming all other factors remain constant [10]. Therefore, a positive shadow price indicates that an increase in the resource will enhance the objective function and help prioritize resources with a higher marginal benefit. In contrast, a zero shadow price indicates a slight increase in the resource does not impact the objective. Sensitivity analysis helps determine the stability of the optimal solution and provides insight into which constraints or variables are most critical.

Khalilpour [6] presented a detailed chronological literature review on parametric optimization introduced in 1953. Here is a summary of recent literature in this field: Zuidwijk [17] provided the characterization of the objective function in terms of the parameter by applying realization theory to parametric sensitivity analysis. Ferris et al. [4] reviewed the sensitivity analysis for the parametric optimization of the objective function and RHS. In the study of Jongen et al. [5], one-parametric optimization problems were discussed, considering the local structure of the semi-infinite singularities. The parametric uncertainties in the objective function and RHS were looked into by Dua and Pistikopoulos [3] using a primal-simplex step to move from one basis to its neighbor. Using some classic results from matrix algebra, Khalilpour and Karimi [6] developed a two-stage iterative method to solve parametric optimization problems with a single parameter on the LHS. Charitopoulos [1] presented a method using Karush-Kuhn-Tucker conditions to solve a general Parametric LP (PLP) problem having uncertainties. In the study of Kolev and Skalna [7], an interval constraint satisfaction technique was proposed by obtaining the interval hull solution to a PLP problem. Yu and Monniaux [16] proposed a method using PLP for computations on polyhedral projection. In the study of Sivri et al. [11], an objective function with one parameter was converted into a linear structure by using a first-order Taylor series approximation. Mehanfar and Ghaffari [8] studied a uni-parametric linear program where an identical parameter perturbed the objective coefficients and the LHS and RHS of constraints linearly. They examined how the variation affects a specific optimal solution and how the optimal value function behaves within its domain.

In addition to theoretical research in the field of PLP, the use of such problems is also common in application areas. Wakili [13] dealt with an interval LP problem having a parameter in the objective function and provided a case study in the Coca-Cola company. In Chen [2], an LP technique for multidimensional analysis of preference methodology was utilized for addressing multiple criteria group decision-making problems based on Pythagorean fuzzy sets and by determining individual goodness of fit and poorness of fit. Widyan [15] presented a novel parametric method based on the simplex algorithm for decomposing the parametric space of the stochastic multicriteria optimization LP problem. The mathematical approach transformed the stochastic model with a random variable in the objective functions to a deterministic one, then scalarized the problem using the nonnegative weighted sum approach. Mousavi and Wu [9] suggested a framework based on parametric programming that would coordinate the work of the independent system operator and the distribution system operator. This would allow distributed energy resource aggregators to participate in the wholesale market as much as possible while ensuring that distribution grids run safely. Wakili [14] considered three types of bread with estimated profits and formulated a problem that was parameterized into PLP from the collected data. It was found that profit was made at different values of a parameter.

This study focuses on the optimization of LP problems with a single parameter. We propose a unified algorithm to obtain a solution for all cases where the parameter is on the objective function coefficients, the RHS, or the coefficient matrix of constraints. Using dual, generalized, or primal simplex algorithms, the proposed algorithm comes up with a parametric analytical solution based on the conditions of feasibility and optimality. Distinct numerical examples from the literature illustrate each case.

This paper is organized as follows: Section 2 gives the mathematical definition of the PLP problem and the proposed algorithm. After the implementation of numerical illustrations in Section 3, conclusions are interpreted in Section 4.

2. Proposed solution algorithm to PLP problem

This section initially outlines the fundamental framework of a PLP problem. After presenting different algorithms for each case where the parameter in the PLP problem lies in the objective function, LHS, or RHS, a unified algorithm for addressing each of these cases is proposed.

A general PLP problem with a single parameter can be defined as follows:

$$f_\lambda = min\left\{\mathbf{c}_\lambda \mathbf{x} : \mathbf{A}_\lambda \mathbf{x} \leq \mathbf{b}_\lambda, \mathbf{x} \geq \mathbf{0}\right\},$$

where λ is a parameter that reflects the uncertainty, A_λ is the coefficient matrix, $\mathbf{c}_\lambda$ and $\mathbf{b}_\lambda$ are the cost and RHS vectors in terms of λ, respectively.

PLP problems with a single parameter can be classified based on their location. If $\mathbf{c}_\lambda$, $\mathbf{A}_\lambda$, $\mathbf{b}_\lambda$ involve uncertainty, then the following PLP problems can be denoted by:

$$f_\lambda = min\left\{\mathbf{c}_\lambda \mathbf{x} : \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\right\}, \tag{1}$$

$$f_\lambda = min\left\{\mathbf{c} \mathbf{x} : \mathbf{A}_\lambda \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\right\}, \tag{2}$$

or

$$f_\lambda = min\left\{\mathbf{c} \mathbf{x} : \mathbf{A} \mathbf{x} \leq \mathbf{b}_\lambda, \mathbf{x} \geq \mathbf{0}\right\}. \tag{3}$$

While many approaches have been proposed in the literature for the solution of (1) and (3), there are only a few solution approaches for (2). Although we mainly focus on the solution of (2), this algorithm is designed to be effective in solving all types of one-parametric problems (1), (2), and (3).

We present the steps for each case before introducing the unified solution algorithm for PLP problems.

2.1. Solution steps for the LHS-PLP problem

The following steps are given for a PLP problem with the parameters on the LHS.

Step 1: Write the problem with m constraints in n variables in the standard form and construct the initial simplex table.

Step 2: Select the non-basic variables having the best costs in the direction of the objective function and obtain a $\lambda-$parametric simplex table by taking them into the base with $min\{m, n\}$ iteration(s). If the same costs appear in the objective function, break the ties arbitrarily.

Step 3: Let c_j be the coefficient of each variable in the objective function and j be the value found by multiplying each variable in a column with the corresponding coefficient of the basic solution variable in the objective function and summation of them. Identify λ values making shadow price $c_j - z_j$ for each non-basic variable, and $\mathbf{b}$ values equal to zero. Form a chart analyzing the signs of $c_j - z_j$ and $\mathbf{b}$ values in terms of the parameter λ to obtain possible optimal solutions.

Step 4: Make a joint sign chart having λ values in the columns and $\mathbf{b}$ and $c_j - z_j$ in the rows. To find the optimal basis for λ values, check the conditions for feasibility and optimality in each column of the joint sign chart and then apply one of the steps below:

Step 4a: If the RHS is negative and the optimality condition is satisfied, apply the dual-simplex algorithm.

Step 4b: If the RHS is negative and the optimality condition is not satisfied, apply the generalized-simplex algorithm.

Step 4c: If the RHS is positive and the optimality condition is not satisfied, apply the primal-simplex algorithm.

Step 5: State the final optimal basis for each interval.

2.2. Solution steps for the RHS-PLP problem

The solution to a PLP problem with parameters on the RHS is introduced through the following steps:

Step 1: Write the problem with m constraints in n variables in the standard form and construct the initial simplex table.

Step 2: Determine the entering variable by controlling the shadow prices, the $c_j - z_j$ values, taking into account the direction of the objective function.

Step 3: While determining the leaving vector, the minimum ratio test is applied by considering the row yielding all the parametric $\mathbf{b}$ values being non-negative.

Step 4: Save λ values, making $\mathbf{b}$ nonnegative in each iteration of the simplex method.

Step 5: Make a joint sign chart having λ values in the columns and $\mathbf{b}$ and final-basic variables in the rows. To find the optimal basis for λ values, check the conditions for feasibility and optimality in each column of the joint sign chart and then do one of the steps below:

Step 5a: If the RHS is negative and the optimality condition is satisfied, apply the dual-simplex algorithm.

Step 5b: If the RHS is negative and the optimality condition is not satisfied, apply the generalized-simplex algorithm.

Step 5c: If the RHS is positive and the optimality condition is not satisfied, apply the primal-simplex algorithm.

Step 6: State the final optimal basis for each interval.

2.3. Solution steps for the PLP problem with cost parameters

The solution to a PLP problem with parameters in the objective function is presented as follows:

Step 1: Write the problem with m constraints in n variables in the standard form and construct the initial simplex table.

Step 2: Considering the direction of the objective function (maximization or minimization), calculate the parametric shadow prices (if max, $c_j - z_j \geq 0$; if min, $c_j - z_j \leq 0$) for each vector that is not in the basis. Determine the entering variable by controlling the parametric $c_j - z_j$ values.

Step 3: Determine the λ values based on the parametric $c_j - z_j$ values concerning the direction of the objective function in each iteration of the simplex technique.

Step 4: Determine the leaving vector by applying the minimum ratio test.

Step 5: Make a joint sign chart having λ values in the columns, cost c_j of final basic variables, and shadow parametric prices $c_j - z_j$ variables in the rows. Considering the feasibility and optimality conditions in each column of the joint sign chart, apply one of the following steps to find the optimal basis relative to λ values:

Step 5a: If the RHS is negative and the optimality condition is satisfied, apply the dual-simplex algorithm.

Step 5b: If the RHS is negative and the optimality condition is not satisfied, apply the generalized-simplex algorithm.

Step 5c: If the RHS is positive and the optimality condition is not satisfied, apply the primal-simplex algorithm.

Step 6: State the final optimal basis for each interval.

2.4. Unified algorithm

The following steps can be expressed as a unified algorithm for solving all types of single PLP problems, such as (1), (2) or (3):

Step 1: Write the problem in the standard form and construct the initial simplex table.

Step 2: Select the non-basic variables having minimum costs in the objective function and obtain a $\lambda-$parametric simplex table by taking them into the base.

Step 3: Find the λ values that make $c_j - z_j$ equal to zero for each non-basic variable. Then, make a chart that shows how the sign of $c_j - z_j$ changes with λ to find possible optimal solutions.

Step 4: Considering the feasibility and optimality conditions in each column of the sign chart, apply one of the following to investigate the basis changes with λ values:

Step 4a: If the RHS is negative and the optimality condition is satisfied, apply the dual-simplex algorithm.

step 4b: If the RHS is negative and the optimality condition is not satisfied, apply the generalized-simplex algorithm.

Step 4c: If the RHS is positive and the optimality condition is not satisfied, apply the primal-simplex algorithm.

Step 5: State the final parametric solution for each interval.

The flowchart of the proposed algorithm is presented in Figure 1.

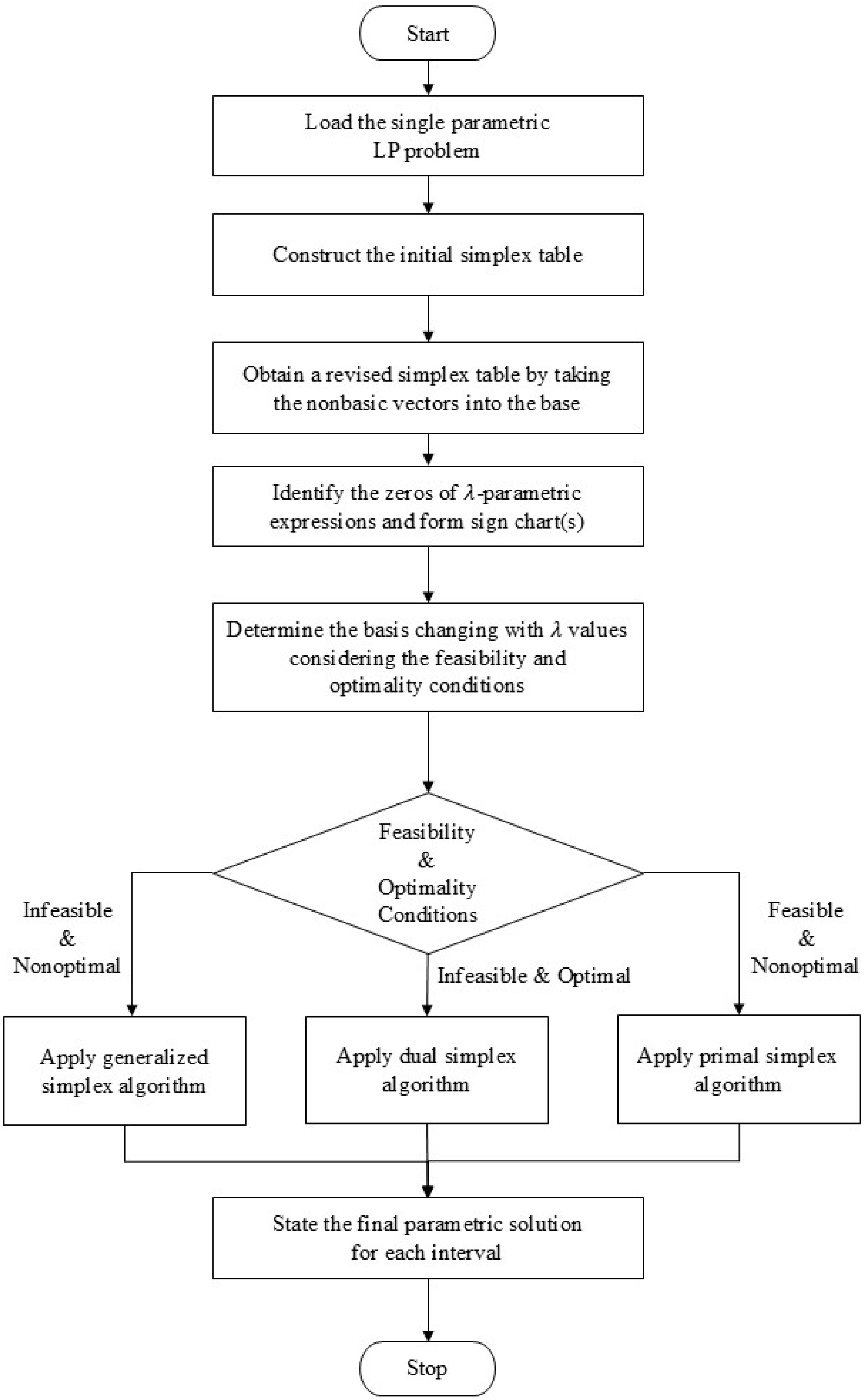

Figure 1: *Flowchart of the proposed algorithm.*

3. Numerical examples

This section specifically illustrates the solution algorithms for each case through numerical examples.

Example 1. *Consider the following PLP problem with a parameter in LHS, solved in the study [6]:*

$$min\ z = x_1 + 1.5x_2 + 3x_3 \tag{4a}$$

such that

$$x_1 + (1+2\lambda)x_2 + 2x_3 \geq 6,$$

$$(1+\lambda)x_1 + 2x_2 + x_3 \geq 10,$$

$$x_1, x_2 \geq 0.$$

First, the PLP problem is rewritten in the standard form and an initial simplex table is constructed as shown in Table 1.

x_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$\mathbf{b}$
x_6	1	$(1+2\lambda)$	2	-1	0	1	0	6
x_7	$(1+\lambda)$	2	1	0	-1	0	1	10

Table 1: *Initial simplex table*

There are two constraints in the PLP problem, and the variables x_1 and x_2 have the minimum costs in the objective function (4a), which means they should be minimized. Therefore, Table 1 is reformed by taking these variables into the basis using elementary row operations. Thus, Table 2 is constructed, where $\mathbf{b}^R$ is the RHS vector in the reformed simplex table.

x_B	x_1	x_2	x_3	x_4	x_5	$\mathbf{b}^R$
x_1	1	0	$2+\dfrac{(1+2\lambda)^2}{1-3\lambda-2\lambda^2}$	$-1-\dfrac{1+3\lambda+2\lambda^2}{1-3\lambda-2\lambda^2}$	$\dfrac{1+2\lambda}{1-3\lambda-2\lambda^2}$	$6-\dfrac{4+2\lambda-12\lambda^2}{1-3\lambda-2\lambda^2}$
x_2	0	1	$\dfrac{-1-2\lambda}{1-3\lambda-2\lambda^2}$	$\dfrac{1+\lambda}{1-3\lambda-2\lambda^2}$	$\dfrac{-1}{1-3\lambda-2\lambda^2}$	$\dfrac{4-6\lambda}{1-3\lambda-2\lambda^2}$
$c_j - z_j$	0	0	$c_3 - z_3$	$c_4 - z_4$	$c_5 - z_5$	

Table 2: *Reformed simplex table*

In Table 2, $c_j - z_j, (j = 3, 4, 5)$ and $\mathbf{b}$ values depend on the parameter λ. To find the possible optimal solutions, all the zeros of λ-parametric expressions must be found. This creates a sign chart for each reduced cost and RHS value.

Equating $c_3 - z_3$ to zero,

$$3 - \left(2 + \frac{(1+2\lambda)^2}{1-3\lambda-2\lambda^2} + \frac{-1.5-3\lambda}{1-3\lambda-2\lambda^2}\right) = 0 \tag{5}$$

is obtained. The sign chart is constructed in Table 3 by considering zeros of (5), i.e., -1.78, -0.93, 0.27, and 0.28.

λ		-1.78	-0.93	0.27	0.28
$c_3 - z_3$	$+$	$-$	$+$	$-$	$+$

Table 3: *Sign chart for* $c_3 - z_3$

For $c_4 - z_4$ and $c_5 - z_5$, the equations

$$0 - \left(-1 - \frac{1+3\lambda+2\lambda^2}{1-3\lambda-2\lambda^2} + \frac{1.5+1.5\lambda}{1-3\lambda-2\lambda^2}\right) = 0, \tag{6}$$

$$0 - \left(\frac{1+2\lambda}{1-3\lambda-2\lambda^2} - \frac{-1.5}{1-3\lambda-2\lambda^2}\right) = 0, \tag{7}$$

and the sign charts given in Table 4 and Table 5 are obtained, respectively.

λ		-1.78	0.28	0.33
$c_4 - z_4$	$-$	$+$	$-$	$+$

Table 4: *Sign chart for* $c_4 - z_4$

λ		-1.78	0.25	0.28
$c_4 - z_4$	$-$	$+$	$-$	$+$

Table 5: *Sign chart for* $c_5 - z_5$

Moreover, to find the feasible solutions, the RHS column of Table 2 should be positive, i.e., $\mathbf{b}^R \geq 0$. Therefore,

$$6 - \left(\frac{4+2\lambda-12\lambda^2}{1-3\lambda-2\lambda^2}\right) = 0,$$

and

$$\frac{4-6\lambda}{1-3\lambda-2\lambda^2} = 0$$

will be investigated. The sign chart for $\mathbf{b}_1^R$ and $\mathbf{b}_2^R$, the components of the vector $\mathbf{b}$ is presented in Table 6.

λ		-1.78	0.1	0.28	0.67
$\mathbf{b}_1^R$	$-$	$+$	$-$	$+$	$+$
$\mathbf{b}_2^R$	$-$	$+$	$+$	$-$	$+$

Table 6: *Sign chart for* $\boldsymbol{b}_1^R$ *and* $\boldsymbol{b}_2^R$

Considering individual sign charts given in Table 3-5, the joint sign chart is presented in Table 7. By checking feasibility and optimality conditions for each interval in Table 7, generalized, dual, or primal simplex methods are applied to obtain possible optimal basis vectors. This analysis is summarized in Table 8. For example, for the interval $(-\infty, -1.78]$, the solution is found infeasible and nonoptimal from Table 8. Thus, the generalized-simplex method is applied, and the possible optimal basis vector is $\{x_3, x_2\}$.

λ	$c_3 - z_3$	$c_4 - z_4$	$c_5 - z_5$	$\mathbf{b}_1^R$	$\mathbf{b}_2^R$
$(-\infty, -1.78]$	$+$	$-$	$-$	$-$	$-$
$[-1.78, -0.93]$	$-$	$+$	$+$	$+$	$+$
$[-0.93, 0.1]$	$+$	$+$	$+$	$+$	$+$
$[0.1, 0.25]$	$+$	$+$	$+$	$-$	$+$
$[0.25, 0.27]$	$+$	$+$	$-$	$-$	$+$
$[0.27, 0.28]$	$-$	$+$	$-$	$-$	$+$
$[0.28, 0.33]$	$+$	$-$	$+$	$+$	$-$
$[0.33, 0.67]$	$+$	$+$	$+$	$+$	$-$
$[0.67, \infty)$	$+$	$+$	$+$	$+$	$+$

Table 7: *Joint sign chart for the reduced costs and RHS values*

Interval	Feasibility	Optimality	Simplex Method	Basis Vectors
$(-\infty, -1.78]$	Infeasible	Nonoptimal	Generalized	$\{x_3, x_2\}$
$[-1.78, -0.93]$	Feasible	Nonoptimal	Primal	$\{x_3, x_2\}$
$[-0.93, 0.1]$	Feasible	Optimal	Primal	$\{x_1, x_2\}$
$[0.1, 0.25]$	Infeasible	Nonptimal	Dual	$\{x_4, x_2\}$
$[0.25, 0.27]$	Infeasible	Nonoptimal	Generalized	$\{x_4, x_2\}$
$[0.27, 0.28]$	Infeasible	Nonoptimal	Generalized	$\{x_4, x_2\}$
$[0.28, 0.33]$	Infeasible	Nonoptimal	Generalized	$\{x_4, x_2\}$
$[0.33, 0.67]$	Infeasible	Nonoptimal	Dual	$\{x_1, x_4\}$
$[0.67, \infty)$	Feasible	Optimal	Primal	$\{x_1, x_2\}$

Table 8: *Possible optimal basis vectors based on the feasibility and optimality conditions*

Example 2. *Consider the following PLP problem having a parameter in RHS, solved in the study [1]:*

$$max\ z = 3x_1 + 2x_2 + 5x_3$$

such that $x_1 + 2x_2 + x_3 \leq 40 - \lambda,$

$$3x_1 + 2x_3 \leq 60 + 2\lambda,$$

$$x_1 + 4x_2 \leq 30 - 7\lambda,$$

$$x_1, x_2, x_3 \geq 0; \lambda \geq 0.$$

We first construct the initial simplex table, and then present the reformed simplex table as Table 9.

x_B	x_1	x_2	x_3	x_4	x_5	x_6	$\mathbf{b}^R$
x_2	-1/4	1	0	1/2	-1/4	0	$5 - \lambda$
x_3	3/2	0	1	0	1/2	0	$30 + \lambda$
x_6	2	0	0	-2	1	1	$10 - 3\lambda$
$c_j - z_j$	-4	0	0	-1	-2	0	

Table 9: *Reformed simplex table for the PLP problem including a RHS parameter*

The zeros of λ-parametric expressions are found, and the values making the RHS nonnegative are determined from Table 9. Since the parameter λ is defined as nonnegative, the sign chart is constructed as Table 10, and according to the sign chart, the optimal basis vectors can be presented in Table 11.

λ		10/3	30/7	5	40
$\mathbf{b}_1 = 40 - \lambda$	+	+	+	+	−
$\mathbf{b}_2 = 60 + 2\lambda$	+	+	+	+	+
$\mathbf{b}_3 = 30 - 7\lambda$	+	+	−	−	−
$\mathbf{b}_1^R = 5 - \lambda$	+	+	+	−	−
$\mathbf{b}_2^R = 30 + \lambda$	+	+	+	+	+
$\mathbf{b}_3^R = 10 - 3\lambda$	+	−	−	−	−

Table 10: *Sign chart for the RHS PLP problem*

Interval	Feasibility	Optimality	Simplex Method	Basis Vectors
$[0, 10/3]$	Feasible	Optimal	Primal	$x_2 = 5 - \lambda$ $x_3 = 30 + \lambda$
$[10/3, 30/7]$	Infeasible	Nonoptimal	Generalized	$x_2 = 15/2 - 7/4\lambda$ $x_3 = 30 + \lambda$
$[30/7, 5]$	Infeasible	Nonoptimal	Generalized	No feasible solution
$[5, 40]$	Infeasible	Nonoptimal	Generalized	No feasible solution
$[40, \infty)$	Infeasible	Nonoptimal	Generalized	No feasible solution

Table 11: *Possible optimal solutions for the RHS PLP problem*

Example 3. *Consider the PLP problem with a parameter in the objective function, taken from [12]:*

$$max\ z = (3 - 6\lambda)x_1 + (2 - 2\lambda)x_2 + (5 + 5\lambda)x_3 \tag{9a}$$

such that

$$x_1 + 2x_2 + x_3 \leq 40,$$

$$3x_1 + 2x_3 \leq 60,$$

$$x_1 + 4x_2 \leq 30,$$

$$x_1, x_2, x_3 \geq 0; \lambda \geq 0.$$

The initial simplex table is constructed, and at each step, the vectors maximizing the objective function are taken into the basis. The reformed simplex table is presented in Table 12.

x_B	x_1	x_2	x_3	x_4	x_5	x_6	$\mathbf{b}^R$
x_2	0	1	0	1/4	-1/8	1/8	25/4
x_3	0	0	1	3/2	-1/4	-3/4	45/2
x_1	1	0	0	-1	1/2	1/2	10/3
$c_j - z_j$	0	0	0	$-5 - 13\lambda$	4λ	$2 + 7\lambda$	

Table 12: *Reformed simplex table for PLP problem*

Since the parameter λ is defined as nonnegative, the only root of the expressions in λ found from the reduced cost row is $\lambda = 0$. Also, zeros of the expressions in λ found from the objective function (9a) are $\lambda = 0.5$ and $\lambda = 1$. The sign chart for the PLP problem is constructed as Table 13 and, according to the sign chart, the possible optimal solutions are presented in Table 14.

λ		0.5	1
$c_1 = 3 - 6\lambda$	+	−	−
$c_2 = 2 - 2\lambda$	+	+	−
$c_3 = 5 + 5\lambda$	+	+	+
$c_4 - z_4 = -5 - 13\lambda$	−	−	−
$c_5 - z_5 = 4\lambda$	+	+	+
$c_6 - z_6 = 2 + 7\lambda$	+	+	+

Table 13: *Sign chart for the PLP problem*

Interval	Feasibility	Optimality	Simplex Method	(x_1, x_2, x_3)	Objective Function
$[0, 0.5]$	Feasible	Nonoptimal	Primal	$(0, 5, 30)$	$z = 160 + 140\lambda$
$[0.5, 1]$	Feasible	Nonoptimal	Primal	$(0, 5, 30)$	$z = 160 + 140\lambda$
$[1, \infty)$	Feasible	Nonoptimal	Primal	$(0, 0, 30)$	$z = 150 + 150\lambda$

Table 14: *Possible optimal solutions for the PLP problem*

4. Conclusion

The proposed algorithm provides a systematic approach to PLP problems with a single parameter and generates a solution if the parameter is only in the objective function, only on the RHS, or only in the coefficient of constraints. To the best of our knowledge, the literature lacks a solution approach for all cases, indicating the algorithm's usefulness. Additionally, the study has made a significant contribution to the limited literature in this field, specifically addressing the PLP problem when a parameter falls on the lower bound of the constraints. We propose a traditional approach to real-life problems to mitigate the uncertainty arising from the parameter's location. We can present future work that adapts the proposed approach to multi-parameter PLP problems.

References

[1] Charitopoulos, V. M., Papageorgiou, L. G. and Dua, V. (2017). Multi-parametric Linear Programming under Global Uncertainty. AIChE Journal, 63(9), 3871-3895. doi: 10.1002/aic.15755

[2] Chen, Ting-Yu (2019). Multiple Criteria Group Decision Making Using a Parametric Linear Programming Technique for Multidimensional Analysis of Preference Under Uncertainty of Pythagorean Fuzziness. IEEE Access, 7, 174108-174128. doi: 10.1109/ACCESS.2019.2957161

[3] Dua, V. and Pistikopoulos, E. N. (2009). Parametric Linear Programming: Cost Simplex Algorithm. Springer. doi: 10.1007/978-0-387-74759-0_502

[4] Ferris, M. C., Mangasarian, O. L. and Wright, S. J. (2007). Linear programming with MATLAB. Society for Industrial and Applied Mathematics. doi: 10.1137/1.9780898718775

[5] Jongen, H. T., Rückmann, J. J. and Stein, O. (2009). Parametric Global Optimization: Sensitivity. Encyclopedia of Optimization, 4, 273-278.doi: 10.1007/978-0-387-74759-0_501

[6] Khalilpour, R. and Karimi, I. A. (2014). Parametric optimization with uncertainty on the left-hand side of linear programs. Computers & Chemical Engineering, 60, 31-40. doi: 10.1016/j.compchemeng.2013.08.005

[7] Kolev, L. and Skalna, I. (2018). Exact Solution to a Parametric Linear Programming Problem. Numerical Algorithms, 78(4), 1183-1194. doi: 10.1007/s11075-017-0418-6

[8] Mehanfar, N. and Ghaffari Hadigheh, A. (2021). Advances in Induced Optimal Partition Invariancy Analysis in Uni-parametric Linear Optimization. Journal of Mathematical Modeling, 9(2), 145-172. doi: 10.22124/jmm.2021.4667

[9] Mousavi, M. and Wu, M. (2022). ISO and DSO Coordination: A Parametric Programming Approach. arXiv preprint arXiv:2201.07433. doi: 10.1109/PESGM48719.2022.9916749

[10] Roos, C., Terlaky, T. and Vial, J.P. (2005). Parametric and Sensitivity Analysis. Interior Point Methods for Linear Optimization. Springer, 361-399. doi: 10.1007/0-387-26379-9_19

[11] Sivri, M., Albayrak, I., Alan, K. S. and Temelcan, G. (2020). A Solution Approach for a Class of Parametric Linear Programming Problems. Journal of the Institute of Science and Technology, 10(4), 2901-2906. doi: 10.21597/jist.690650

[12] Taha, H. A. (2006). Operations Research: An Introduction (8th Edition). Prentice-Hall, Inc., USA. doi: 10.1016/s0898-1221(04)90098-3

[13] Wakili, A. (2013). Application of parametric linear programming in Coca-Cola Company using a developed algorithm, Advancement in Scientific and Engineering Research, 1(1), 17-21.

[14] Wakili, A. (2022). Linear Programming on Bread Production Using Uncertainty Approach. Academic Journal of Applied Mathematical Sciences, Academic Research Publishing Group, 8(2), 27-29. doi: 10.32861/ajams.82.27.29

[15] Widyan, A. M. (2019). A Parametric Technique Based on Simplex for Treating Stochastic Multicriteria Linear Programming Problem. Science, 6(1), 1-5.

[16] Yu, H. and Monniaux, D. (2019). An efficient parametric linear programming solver and application to polyhedral projection. In Static Analysis: 26th International Symposium, SAS 2019, Porto, Portugal, October 8–11, 2019, Proceedings 26 (pp. 203-224). Springer International Publishing. doi: 10.1007/978-3-030-32304-2_11

[17] Zuidwijk, R. A. (2005). Linear parametric sensitivity analysis of the constraint coefficient matrix in linear programs. ERIM report series reference number: ERS-2005-055-LIS, Erasmus Research Institute of Management (ERIM). no: ERS-2005-055-LIS