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KINEMATIC VISCOSITY EVIDENCE OF TRANSPORT PHENOMENON AND PHASE TRANSITION

DURGA RAY*

Service de Chimie Physique II, Campus Plaine U. L. B., Bruxelles, Belgium

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Kinematic viscosity is expressed by a new formula depending on pressure at constant temperature which agrees fairly well with the experimental results at high pressure. Kinematic eddy viscosity obtained by a similar height dependent formula give agreement with the observed results in the atmosphere, also justifies the reversal of circulation in fluid convection. An interpretation regarding the rotation of the molecules and an analogy with the mesophase of liquid crystals are drawn according as the kinematic viscosity changes from decrease to increase or increase to decrease followed by a phase change involved in the new formula and the corresponding formula for kinematic eddy viscosity.

1. *Introduction*

A new formula for kinematic viscosity has been developed and is written in the form as

$$\nu = \nu_0 + \gamma \cos \lambda p \qquad p < P/2$$
$$\nu = \nu_0 - \gamma \cos \lambda p \qquad p > P/2 \tag{1}$$

* Permanent adress: Department of Physics and Astrophysics, University of Delhi, Delhi — 110007, India.

where at pressure p, kinematic viscosity attained is defined as the ratio η/ϱ, η and ϱ being the viscosity and the density of the fluid, ν_0 is the minimum kinematic viscosity at a pressure $P/2$, $\lambda = \frac{\pi}{P}$, and γ is a characteristic parameter for the problem in hand. With the help of formula (1) kinematic viscosity of fluids at high pressure and at constant temperature are calculated. The results are comparable with the results obtained by Enskog's formula which is written as,

$$\eta = \eta_0 \iota \varrho (\iota \varrho \varkappa)^{-1} + 0.8 + 0.7614 \iota \varrho \varkappa. \qquad (2)$$

η_0 is the coefficient of viscosity for a normal gas. He assumed the molecules are rigid elastic spheres and thus $\iota = 2\pi\sigma^3/3m$, where σ and m are the diameter and mass of the molecule, $\varkappa$ a factor related to the collision probability. Eq. (2) shows that it has a minimum corresponding to, $\iota\varrho\varkappa = 1.146$ and thus (2) reads as,

$$2.545 \eta/\varrho = (\eta/\varrho)_{\min} (\iota\varrho\varkappa)^{-1} + 0.8 + 0.7614 \iota\varrho\varkappa. \qquad (3)$$

Neither of these formula are independent theoretically as the values of $\iota\varrho\varkappa$ together with $(\eta/\varrho)_{\min}$, are to be taken from experimental results.

In atmosphere kinematic eddy viscosity is calculated by a similar height dependent formulae given as,

$$\begin{aligned} K &= K_0 - \gamma \cos \lambda z \qquad 0 < z < H/2 \\ K &= K_0 + \gamma \cos \lambda z \qquad H/2 < z < H, \end{aligned} \qquad (4)$$

where K_0 is the maximum value of K at height $H/2$, H being the gradient height, where viscosity is effective. γ is a characteristic parameter. Thus K values obtained by formula (4) are comparable with the results observed by Mildner[3]. Graham[4] has made an interesting suggestion that the difference in circulation pattern in the convection experiment may be a result of the fact that the kinematic viscosity varies with temperature in opposite manner in liquids and gases. Tippelskirich[5] observed that in the case of liquid sulphur kinematic viscosity decreases with an ascending motion and increases with a descending motion according as the temperature is less or greater than 153 °C. Thus Graham's suggestion was confirmed. Following (4), if the form of dependence of kinematic viscosity of uniformly heated fluid is written as,

$$\begin{aligned} \nu &= \nu_0 \mp \gamma \cos \lambda z \qquad 0 < z < H/2 \\ \nu &= \nu_0 \pm \gamma \cos \lambda z \qquad H/2 < z < H. \end{aligned} \qquad (5)$$

ν_0 is the optimum kinematic viscosity attained at height $H/2$, H the height of the fluid contained and γ is a characteristic parameter then ν given in (5) together with above argument of Graham[4], justifies the reversal of circulation of the fluid at mid height.

2. *Calculations of kinematic viscosity and kinematic eddy viscosity*

Tables 1—6 are constructed for kinematic viscosity of nitrogen at 50 °C[6] and 75 °C[7], argon at 50 °C[8] and 75 °C[8] and CO_2 at 31.1 °C[9] and 35 °C[9], respectively. Those tables show that formulae (1) give a good agreement with the experimental results. Table 7 is constructed for the kinematic eddy viscosity K ($= A/\varrho$), where the first two rows give the data observed by Mildner[2]. It shows that eddy viscosity becomes maximum and takes a value 500 at height 240 m, which should be $H/2$ if formulae (4) is used. Thus a column for height 480 m is inserted in the table and the corresponding eddy viscosity is written as 70 which

TABLE 1.

Pressure atm	ν Exp.	$\nu \times 10^6$ Calc. from Eq. (1)	$\nu \times 10^6$ Calc. from Eq. (3)	$\nu \times 10^6$ Calc. from Eq. (2)
15.37	0.01179		11152	11768
57.60	0.003274		3141	3323
104.5	0.001928		1893	2004
212.4	0.001148		1084	1026
320.4	0.000952	899.4	925	977
430.2	0.000887	886.7	873	924
541.7	0.000866	871.7	859	908
630.4	0.000859	859	862	910
742.1	0.000870	874.9	873	924
854.1	0.000889	889.6	889	940
965.1	0.000909	902	910	962

The kinematic viscosity of nitrogen at 50°C. Data taken from Ref. 6. ν_0 s taken as 0.000859 attained at a pressure $P/2$ given by 630.4 atm (1 atm = 101 325 Pa) and γ takes the value 0.000058.

TABLE 2.

Pressure atm	ν Exp.	ν Calc. from Eq. (1)
15.37	0.01344	
57.61	0.003734	
104.5	0.002181	
212.4	0.001266	
320.3	0.001033	0.0009214
430.2	0.000937	0.0009137
541.7	0.000900	0.0009042
630.3	0.000890	0.000895
742.0	0.000885	0.000885
854.1	0.000896	0.000896
965.7	0.000913	0.0009063

The kinematic viscosity of nitrogen at 75°C. Data taken from Ref. 7. ν_0 is taken as 0.000885 attained at a pressure $P/2$ given by 742.0 atm and γ takes the value 0.0000468

TABLE 3.

Pressure atm	$\nu \times 10^4$ Exp.	$\nu \times 10^4$ Calc. from Eq. (1)	$\nu \times 10^4$ Calc. from Eq. (3)	$\nu \times 10^4$ Calc. from Eq. (2)
345.2	7.743	7.312	8.014	12.386
379.0	7.532	7.296	7.746	12.444
415.1	7.388	7.279	7.539	12.561
444.4	7.316	7.265	7.462	12.684
473.0	7.267	7.25	7.368	12.818
503.4	7.230	7.234	7.287	12.971
532.2	7.211	7.219	7.255	13.132
561.3	7.204	7.204	7.219	13.299
600.7	7.211	7.223	7.203	13.525
660.7	7.247	7.256	7.208	13.879
722.9	7.310	7.287	7.239	14.253
818.3	7.435	7.329	7.326	14.819
941.8	7.611	7.37	7.474	15.495
1104	7.903	7.394	7.684	16.441

Kinematic viscosity of argon at 50°C. Data taken from Ref. 8. ν_0 is taken as 0.0007204 attained at a pressure $P/2$ given by 561.3 atm and γ takes the value 0.000019.

TABLE 4.

Pressure atm	$\nu \times 10^4$ Exp.	$\nu \times 10^4$ Calc. from Eq. (1)	$\nu \times 10^4$ Calc. from Eq. (3)	$\nu \times 10^4$ Calc. from Eq. (2)
427.1	7.801	7.508	8.005	12.735
468.4	7.638	7.492	7.793	12.834
468.4	7.645	7.479	7.665	12.936
500.0	7.564	7.479	7.665	12.936
534.6	7.488	7.465	7.572	13.067
569.5	7.446	7.45	7.508	13.214
601.0	7.413	7.437	7.465	13.356
631.7	7.408	7.423	7.424	13.460
631.7	7.407	7.423	7.423	13.46
677.1	7.404	7.404	7.409	13.720
733.2	7.418	7.428	7.403	14.010
805.7	7.474	7.459	7.424	14.385
905.4	7.579	7.499	7.49	14.895
1056	7.774	7.55	7.636	15.677
1253	8.084	7.588	7.861	16.66

Kinematic viscosity of argon at 75°C. Data taken from Ref. 8. ν_0 is taken as 0.0007404 attained at a pressure $P/2$ given by 677.1 atm and γ takes the value 0.000019.

is the residual value at the gradient level as observed by Mildner. To obtain the kinematic eddy viscosity $K\,(= A/\varrho)$, density at these heights are calculated from the density-height relation given as

$$\varrho = \varrho_0 \left(1 + \frac{\alpha h}{T_0}\right)^{-mg/K_B\alpha} \approx \varrho_0 (1 - mgh/K_B T_0). \qquad (6)$$

TABLE 5.

Pressure atm	$\nu \times 10^3$ Exp.	$\nu \times 10^3$ Calc. from Eq. (1)
37.6	1.901	1.171
40.2	1.748	1.161
60.1	1.009	.9
66.1	0.825	.818
69.6	0.767	.768
72.5	0.726	.728
72.9	0.723	.723
73	0.727	.725
73.1	0.734	.726
73.4	0.730	.730

Kinematic viscosity of CO_2 at 31.1 °C. Data taken from Ref. 9. ν_0 is taken as 0.723 × × 10^{-3} attained at a pressure 72.9 atm and γ takes the value 0.00065.

TABLE 6.

Pressure atm	ν Exp.	ν Calc. from Eq. (1)
38.3	0.001934	0.001281
41.1	0.001778	0.00125
61.4	0.001026	0.000980
71.8	0.000836	0.000822
75.4	0.000767	0.000765
78.7	0.000711	0.000713
79.6	0.000699	0.000699
79.8	0.000704	0.000702
79.9	0.000711	0.000704
81.0	0.000721	0.000721
81.7	0.000728	0.000732

Kinematic viscosity of CO_2 at 35 °C. Data taken from Ref. 9. ν_0 takes the value 0.000699 attained at a pressure 79.6 which is $P/2$ and γ is taken as 0.0008.

Terms of higher order are neglected. Referred to the surface of the earth density $\varrho_0 = 1.2250\ \mathrm{kg/m^3}$, mean molecular mass of air $m = 4.8 \times 10^{-23}$ g, $g = 981\ \mathrm{cms^{-2}}$, temperature $T = 273\mathrm{K}$, Boltzmann's constant $K_B = 1.38 \times 10^{-23}$ J/K. Third row of Table 7 gives the densities at those heights as calculated from (6). K_0 as calculated is 420.18×10^3, A_0 is 500 and $\lambda = \frac{\pi}{H} = \frac{\pi}{480\ \mathrm{m}}$, γ's are taken as 430 and 359.6 so as to get the calculated values of A and K at height 480 m equal to the observed values. 4th row gives the observed values of K obtained by dividing observed values of A by the respective densities given in the third row. Fifth row gives the kinematic eddy viscosity K as calculated with the formulae (4), when K_0, λ and γ's are known.

Table 7 is constructed in the manner and is published by the author[10]. Here it is given to show that it accounts for a similar phenomena.

TABLE 7.

z									
Height in meters	80	135	190	240	295	405	460	480	510
A (Observed) eddy viscosity g cm^{-1} s^{-1}	125	270	310	500	246	117	70	70	70
$\varrho \times 10^3$ density g cm^{-3}	1.217	1.205	1.117	1.19	1.182	1.166	1.158	1.155	1.152
$K(A/\varrho) \times 10^{-3}$ observed Kin. eddy vis. g cm^{-1} s^{-1}	102.71	224.02	258.91	420.18	208.12	100.35	60.45	60.61	60.76
$K \times 10^{-3}$ calculated from Eq (4)	108.79	192.07	304.61	420.18	293.52	103.13	63.70	60.61	

Kinematic eddy viscosity in the atmosphere. Observed data are taken from Mildner[3]) (1 g cm^{-1} s^{-1} = 0.1 Pa s).

3. Results and discussion

If the kinematic viscosity of the substance given in Tables 1—6 are plotted against pressure in a significant pressure range around the minimum i. e. in the case of nitrogen at 50 °C it is between 350 atm to 950 atm and so for others the resulting curve is typically as shown in Fig. 1, which shows that:

L is the line given by $p = P/2$; obviously it will pass through ν_0. Formula (1) can be written as

$$\begin{aligned} \nu &= \nu_0 + \gamma \cos \lambda p \quad p < P/2 \\ &= \nu_0 + \gamma \cos [\lambda p + (2n+1)\pi] \quad p > P/2 \\ &= \nu_0 + \gamma \cos \lambda p e^{i(2n+1)\pi} \end{aligned} \qquad \text{or} \qquad \begin{aligned} \nu &= \nu_0 + \gamma \sin \lambda p \quad p < 0 \\ &= \nu_0 - \gamma \sin [\lambda p + (2n+1)\pi] \quad p > 0 \\ &= \nu_0 - \gamma \sin \lambda p e^{i(2n+1)\pi} \end{aligned}$$

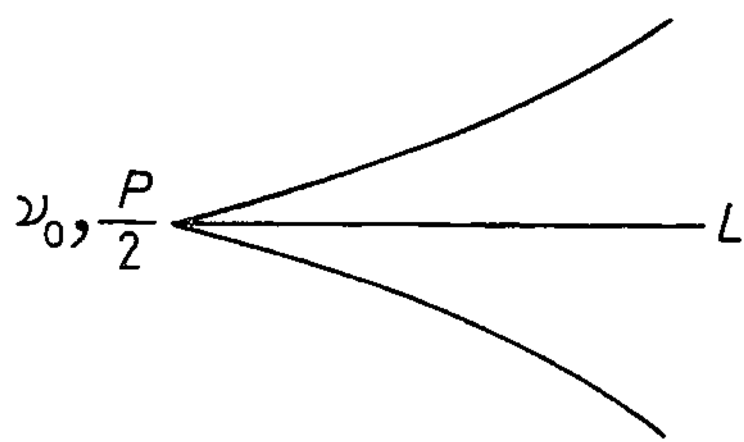

Fig. 1. Kinematic viscosity vs. pressure (see text).

when origin is at $P/2$. First part of this formulae is valid below the line L, the second part is valid above it, and on the line L both are valid as ν takes the value ν_0. Thus function ν as defined is a continuous function of p but it's first derivative is discontinuous. This can be interpreted that above and below L it is the same substance but they are separated by a well defined phase rule of kinematic viscosity. Kinematic viscosity is a molecular property. It may be thought of that in every situation the fluid has a minimum kinematic viscosity ν_0, could be defined as molar viscosity, and due to some particular rearrangement of molecules which occur at certain pressure the fluid attain this minimum (or molar) kinematic viscosity. For any other rearrangement of the molecules which would occur for any change of pressure, kinematic viscosity of the fluid would increase by an amount appearing due to this rearrangement and is given by the pressure dependent variable $\gamma \cos \lambda p$ as written in the second term of the kinematic viscosity formulae.

A rotation through an angle amounting an odd multiple of π is an operation which converts the phase of the variable part of the kinematic viscosity of the first phase to the variable part of the second phase and vice-versa. Thus it can be interpreted that in the first phase with increase of pressure kinematic viscosity decrease till it reaches it's minimum value and as it cannot go beyond it, so at this stage the molecules rotate in a manner so as to take the opposite direction performed by an odd multiple of π rotation giving the second phase of kinematic viscosity resulting an increase to it.

An analogy could be drawn with the mesophase of liquid crystal when the line L can be compared with the disclination line (Gray and Winsor[11)]). With the same analogy we can consider a surface σ drawn in the upper half bound by the line L and rotate each molecule on σ through an angle which is an odd multiple of π, supposing that this movement is transmitted on one side of σ leaving the other side in the vicinity of σ unaffected. This is a singularity in the molecular distribution but it is perfectly ordered phase. This could be that, nonequilibrium brings this order as defined by Prigogine[12)]. That the pattern begins well below the transition point and then continues above it with a transition (or a reversal), is in partial agreement with other evidence like nematic and cholesteric isotropic transition involve processes which occur over a significant temperature range. It may be mentioned that a raising of the pressure at constant temperature of the system has the same effect as a lowering of the temperature of the system at constant pressure, thus similar results are also obtained for the temperature dependence of kinematic viscosity of co-existing gases and liquid helium. Eq. (4) for kinematic eddy viscosity has been written and could be interpreted as, the eddice rotate giving a phase change resulting a change in the kinematic eddy viscosity from increase to decrease with height[10)]. Wind hodographs in the form of spiral observed at the turbulent boundary layer of the atmosphere gives evidence to it.

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RAZMATRANJE TRANSPORTNIH FENOMENA I FAZNIH PRIJELAZA KORIŠTENJEM KINEMATIČKE VISKOZNOSTI

DURGA RAY

Service de Chimie Physique II, Campus Plaine, Bruxelles, Belgium

UDK 532.13

Originalni znanstveni rad

Nađena je nova relacija za kinematičku viskoznost u kojoj ona, pri konstantnoj temperaturi, ovisi o tlaku. Usporedba pokazuje dosta dobro slaganje s eksperimentalnim rezultatima kod visokog tlaka. Na sličan način dobivena je ovisnost vrtložne viskoznosti o visini koja se dobro slaže s opaženim rezultatima u atmosferi, te također objašnjava obrnuće tečenja u konvekciji tekućine.