YU ISSN 0015—3206
FZKAAA 21 (3) 227 (1989)

ON THE MAGNETIC SUSCEPTIBILITIES OF THE ELECTRONS IN SEMICONDUCTOR SUPERLATTICES IN THE PRESENCE OF A QUANTIZING MAGNETIC FIELD

KAMAKHYA PRASAD GHATAK and BASWATI MITRA

Department of Electronics and Telecommunication Engineering, Faculty of Engineering and Technology, University of Jadavpur, Calcutta-700032, India

Received 1 September 1988

UDC 538.955

Original scientific paper

An attempt is made to study the doping dependence of the dia- and paramagnetic susceptibilities of the electrons in semiconductor superlattices under strong magnetic quantization, taking $Ga_{1-x}Al_xAs$-AlAs superlattice as an example. It is found, that the values of both the susceptibilities are higher than their bulk values of GaAs and for relatively higher carrier degeneracy, they approximately obey the Landau relation.

1. *Introduction*

In recent years with the advent of MBE, MOCVD and other techniques semiconductors with superlattice structures (SL's), in which alternate layers of two different degenerate materials set up a periodic potential with a periodicity many times the crystal dimensions resulting in energy minibands have been experimentally realized[1-4]. The SL has found wide applications in many new device structures, such as avalanche photodiodes[5], photo detectors[6], transistors[7], light emitters[8], electro-optical modulators[9], etc. Though considerable work has already been done on the various physical aspects of SL's, there still remain scopes in the investigations made while the interest for further researches of the different other parameters of such heterostructures is becoming increasingly important. One such significant physical feature is the magnetic susceptibility of the electrons in degenerate materials which has been studied in the literature under various physical conditions[10-13]. Nevertheless, such susceptibility in SL's has yet to be worked out for the more interesting case which occurs from the presence of magnetic quantization. In the present communication an attempt is made to study theoretically the doping dependence of the dia- and paramagnetic susceptibilities of electrons in semiconductor superlattices in the presence of a quantizing magnetic field along superlattice direction, taking $Ga_{1-x}Al_xAs$-AlAs superlattice as an example.

2. Theoretical background

The $E - \vec{k}$ dispersion relation of the electrons in SL's can be expressed[14] in the absence of any quantization, under tight-binding approximations, as

$$E = \frac{\hbar^2}{2m^*}[k_x^2 + k_y^2] + E_{0s} - E_{1s} \cos(2\pi k_z/k_0) \tag{1}$$

where the notations are the same as in Ref. 14.

Incidentally, the presence of a quantizing magnetic field B in the z-direction, the energy eigenvalue equation takes the form

$$E\psi = H\psi \tag{2}$$

where ψ is the wave-function under magnetic quantization and the Hamiltonian H can be written as

$$H = \frac{p_x^2}{2m^*} + \frac{1}{2m^*}(p_y - eBx)^2 + E_{0s} - E_{1s} \cos(2\pi k_z/k_0) \tag{3}$$

in which $\vec{p} = \hbar\vec{K}$ and e is the electron charge.

Thus the modified electron energy spectrum in SL's under magnetic quantization can be expressed as

$$E = \left(n + \frac{1}{2}\right)\hbar\omega_0 + E_{0s} - (E_{1s} \cos(2\pi k_z/k_0) \tag{4}$$

where $n\,(= 0, 1, 2, \ldots)$ is the Landau quantum number and $\omega_0 = eB/m^*$.

The use of equation (4) leads to the expression of the density-of-states function as

$$N(E) = \frac{eB}{\pi^2\hbar} \sum_{n=0}^{n_{max}} \sum_{n_{min}^{(s)}}^{n_{max}^{(s)}} \frac{\partial k_z}{\partial E} =$$

$$= \frac{eB}{\pi^2\hbar d_0} \sum_{n=0}^{n_{max}} \sum_{n_{min}^{(s)}}^{n_{max}^{(s)}} (E_{1s})^{-1}\left[1 - \left\{\left(\left(n + \frac{1}{2}\right)\hbar\omega_0 + E_{0s} - E\right)\Big/ E_{1s}\right\}^2\right]^{-1/2} \tag{5}$$

where d_0 is the superlattice period.

Therefore the free energy of the electrons in superlattices can be expressed by using equation (5) in the range of very low temperatures and considering only the first miniband, since in an actual superlattice only the lowermost miniband is significantly populated at low temperatures where the quantum effects become prominent, as

$$U(B) = n_0 E_F + \frac{eBc}{\pi^2 \hbar d_0} \sum_{n=0}^{n_{max}} \left[a \cos^{-1} a + \sqrt{1 - a^2}\right] \tag{6}$$

where n_0 is the electron concentration, E_F is the Fermi energy under magnetic quantization as measured from the edge of the conduction band in the absence of any quantization,

$$c = E_{||},\ a = (A - E_F)/c \text{ and } A = \left(n + \frac{1}{2}\right) \hbar\omega_0 + E_{01}.$$

Thus, the diamagnetic susceptibility is given by

$$\varkappa_d = -\mu_0 \frac{\partial^2 U(B)}{\partial B^2} = -\mu_0 \Bigg[2B^{-1} U_1(B) - 2B^{-2} U(B) + 2B^{-2} n_0 E_F -$$

$$- 2n_0 B^{-1} \frac{\partial E_F}{\partial B} + \frac{2eB}{\pi \hbar d_0} \left\{ \sum_{n=0}^{n_{max}} a (1 - a^2)^{-1/2} \right\} \frac{\partial^2 E_F}{\partial B^2} -$$

$$- \frac{eB}{\pi^2 \hbar d_0 c} \sum_{n=0}^{n_{max}} \left\{ \left(n + \frac{1}{2}\right) \hbar\omega_0 B^{-1} - \frac{\partial E_F}{\partial B} \right\}^2 \{3 (1 - a^2)^{-1/2} - a^2 (1 - a^2)^{-3/2}\} \Bigg] \tag{7}$$

where

$$U_1(B) = n_0 \frac{\partial E_F}{\partial B} + U(B) B^{-1} - n_0 E_F B^{-1} +$$

$$+ \frac{eB}{\pi^2 \hbar d_0} \sum_{n=0}^{n_{max}} \left[\left(n + \frac{1}{2}\right) \hbar\omega_0 B^{-1} - \frac{\partial E_F}{\partial B} \right] [\cos^{-1} a - 2a (1 - a^2)^{-1/2}].$$

Therefore, the determination of $\varkappa_d$ requires the expressions for $\frac{\partial E_F}{\partial B}$ and $\frac{\partial^2 E_F}{\partial B^2}$ which in turn require the corresponding electron statistics. Thus, using the same logic as used in obtaining equation (6), the electron concentration can be expressed as

$$n_0 = \frac{eB}{\pi^2 \hbar d_0} \sum_{n=0}^{n_{max}} \cos^{-1} a. \tag{8}$$

Since the electron concentration is not affected by magnetic quantization (neglecting magnetic freeze-out), we can write from equation (8), the expressions or $\frac{\partial E_F}{\partial B}$ and $\frac{\partial^2 E_F}{\partial B^2}$ respectively as

$$\frac{\partial E_F}{\partial B} = -(n_0 \pi^2 \hbar c d_0 / e B^2) \left[\sum_{n=0}^{n_{max}} (1 - a^2)^{-1/2} \right]^{-1} \tag{9}$$

and

$$\frac{\partial^2 E_F}{\partial B^2} = \left[\sum_{n=0}^{n_{max}} (1 - a^2)^{-1/2} \right]^{-1} \left[\frac{2\pi^2 n_0 \hbar c d_0}{e B^2} + \frac{1}{c} \left\{ \sum_{n=0}^{n_{max}} (1 - a^2)^{-3/2} \right\} \left(\frac{\partial E_F}{\partial B} \right)^2 \right]. \tag{10}$$

When spin-splitting is considered the magnetic moment can be expressed as

$$M = \frac{e B \beta g_0}{4\pi^2 d_0 \hbar} \sum_{n=0}^{n_{max}} [\cos^{-1} b_- - \cos^{-1} b_+] \tag{11}$$

where g_0 is the magnitude of the spectroscopic splitting factor at the edge of the conduction band, β is the Bohr magneton, $b_\pm = \frac{A - E_f \pm \frac{1}{2} g_0 B \beta}{c}$ and E_f i the Fermi energy when spin-splitting is considered.

Therefore, using equation (11) the diamagnetic part of the susceptibility can be written as

$$\varkappa_p = \mu_0 M B^{-1} + \frac{e g_0 \mu_0 B}{4\pi^2 \hbar d_0 c} \sum_{n=0}^{n_{max}} (S_+ - S_-) + \left(\sum_{n=0}^{n_{max}} L \right) \frac{e B g_0 \mu_0 \beta}{4\pi^2 \hbar d_0 c} \cdot \frac{\partial E_F}{\partial B} \tag{12}$$

where

$$S_\pm = (1 - b_\pm^2)^{-1/2} \left[\left(n + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} g_0 \beta B \right]$$

and

$$L = [(1 - b_-^2)^{-1/2} - (1 - b_+^2)^{-1/2}].$$

Incidentally, following the method as given above the expressions for n_0 and $\frac{\partial E_F}{\partial B}$ in the presence of spin-splitting can respectively be expressed as

$$n_0 = (eB/2\pi^2 \hbar d_0) \Sigma [\cos^{-1} b_- + \cos^{-1} b_+] \tag{13}$$

and

$$\frac{\partial E_F}{\partial B} = \left[\sum_{n=0}^{n_{max}} L \right]^{-1} \left[- \frac{2\pi^2 \hbar d_0 c n_0}{e B^2} + \sum_{n=0}^{n_{max}} \frac{S_-}{B} + \frac{S_+}{B} \right] \tag{14}$$

3. Results and discussion

Using the appropriate equations together with the parameters[15–16] $m^* = 0.069\,m_0, g_0 = 2, E_0 = 0.05$ eV, $E_1 = 0.01$ eV and $d_0 = 60$ nm for $Ga_{1-x}Al_xAs$-GaAs superlattice, the doping dependence of the dia- and paramagnetic susceptibilities of electrons has been plotted at low temperatures corresponding to two given values of the magnetic field as shown in Fig. 1. It appears from the figure

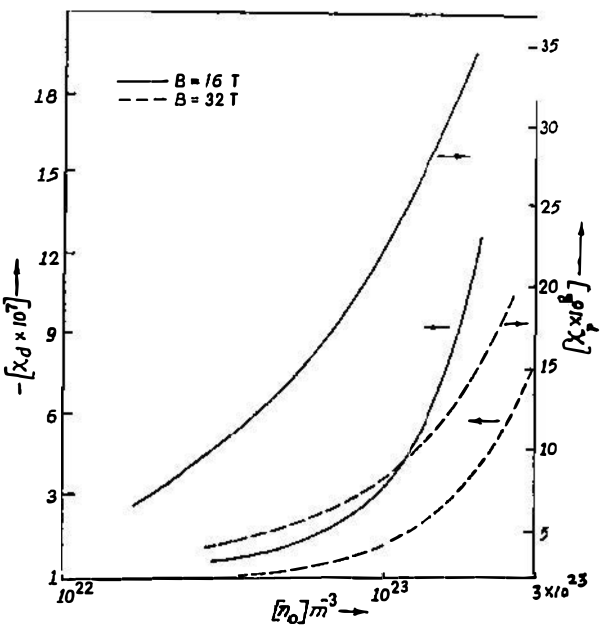

Fig. 1. Doping dependence of the diamagnetic and paramagnetic susceptibilities of the electrons in $Ga_{1-x}Al_xAs$-AlAs superlattice at low temperatures.

that the values of $\varkappa_d$ and $\varkappa_p$ are both higher than their bulk values of GaAs and increase with increasing electron concentration. It has also been observed that $\varkappa_d = -\lambda\varkappa_p$, where the factor λ decreases with increasing electron concentration in the presence of a constant magnetic field (the value of λ being typically varying between 1 to 1/3). Thus at relatively high values of carrier degeneracy, we have approximately the relation $\varkappa_d = -\left(\frac{1}{3}\right)\varkappa_p$ which is in fact the relation first derived by Landau[10] for $3D$ electron gases by considering the magnetic quantum number to be fairly large. It appears then that a strong quantizing magnetic field would not affect the validity of the Landau's basic relation for a superlattice structure. This follows from the fact that the electron concentration increases very much with the consequent rise of the Fermi level with respect to the bottom of the lowest Landau level resulting in the occupation of a large number of Landau levels even under strong magnetic quantization. Thus the magnetic quantum number becomes large for large values of the electron concentration leading to the picture assumed in the Landau theory. Incidentally, the effect of collision broadening has not been considered here. Though the effects of collision broadening are usually small at low temperatures, the sharpness of the amplitude in the figure would be reduced by its influence. It may also be stated that the electron-electron interactions which become increasingly important with increasing electron concentration have not been considered in the present treatment. Besides, if the orientation of the quantizing magnetic field be taken as an arbitrary one instead of the superlattice direction as assumed in the present work, the expressions of the dia- and paramagnetic susceptibilities would be different analytically. Finally it may be remarked that though in a more rigorous treatment, the above modifications should be considered along with a self consistent procedure, this simplified analysis exhibits the major features of the magnetic susceptibility of the electrons in semiconductor heterostructures.

References

1) L. L. Chang and L. Esaki, Progr. Crystal Growth Charact. **2** (1979) 3;
2) R. Dingle, H. L. Stormer, A. C. Gossard and W. Weigman, Appl. Phys. Letts. **33** (1978) 665;
3) G. A. Sai-Halasz, L. L. Chang, J. M. Welter, C. A. Chang and L. Esaki, Solid State Communn. **27** (1978) 935;
4) G. A. Sai-Halasz, L. Esaki and W. Harrison, Phys. Rev. **B18** (1978) 2812;
5) F. Capasso in *Semicond. Semimetal*, Ed. R. W. Willardson and A. C. Beer, Academic Press, N. Y., **22** (1985) 2;
6) J. S. Smith, L. C. Chiu, S. Magralit and A. Yariv, J. Vac. Sci. Tech. **31** (1983) 376;
7) F. Capasso and R. A. Keihl, J. Appl. Phys. **58** (1985) 1366;
8) K. Ploog and G. H. Dohler, Adv. Phys. **32** (1983) 285;
9) D. R. Suifres, C. Kindstrom, R. D. Burnhaum, W. Streifer and T. L. Paoli, Electronics Letts. **19** (1983) 170;
10) L. D. Landau, Z. Phys. **64** (1930) 629;
11) R. E. Peierls, *Quantum Theory of Solids*, Oxford, 1955;
12) E. M. Gashimsade, Sov. Phys. Semicond. **6** (1972);
13) D. Roychowdhury and P. K. Basu, Int. J. Electronics **44** (1978) 397;
14) D. Mukherjee, A. N. Chakravarti and B. R. Nag, Phys. Stat. Sol. (a) **26** (1974) K27;
15) L. Esaki, L. L. Chang, W. E. Howard and V. I. Rideout, Proc. 11th Int. Conf. Phys. Semicond. Warsaw 1972, p. 431;
16) L. Esaki and R. Tsu, IBM J. Res. Develop. **14** (1970) 61.

ELEKTRONSKA MAGNETSKA SUSCEPTIBILNOST U SUPERREŠETKI POLUVODIČA U PRISUSTVU KVANTIZIRAJUĆEG MAGNETSKOG POLJA

KAMAKHYA PRASAD GHATAK i BASWATI MITRA

Faculty of Engineering and Technology, University of Jadavpur, Calcutta-700 032, India

UDK 538.955

Originalni znanstveni rad

Proučavana je ovisnost o dopiranju dia- i paramagnetske susceptibilnosti elektrona u poluvodičima u režimu jake magnetske kvantizacije. Kao primjer uzeta je superrešetka $Ga_{1-x}Al_xAs$-AlAs. Nađeno je da su vrijednosti obje susceptibilnosti veće od vrijednosti u unutrašnjosti GaAs, te da za relativno visoke degeneracije nosilaca naboja zadovoljavaju Landauovu relaciju.