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A THEORETICAL INVESTIGATION OF A CIRCULAR APERTURE IN THE GROUND PLANE AND LAYERED DIELECTRICS

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A theoretical treatment of the dielectric effects on the polarizability of an circular aperture in a perfectly conducting plane has been performed. A solution is developed for computing the electric polarizability of an aperture in a ground plane with infinite small thickness surrounded by a dielectric half space on one side and a dielectric layer of finite thickness on the other side. An appropriate boundary-value problem is formulated and reduced to a set of dual integral equations. Later, we will formulate a Fredholm integral equation of the second kind with compact kernel and from the solution of this integral equation the electric polarizability of the aperture is calculated. The results of the calculations are reported in the form of graphs for the variations of the electric polarizability with the dielectric constant of the dielectric half space as well as the dielectric constant and thickness of the dielectric layer.

1. Introduction

The problem of the dielectric effects on the electric polarizability of an circular aperture in a perfectly conducting plane with infinite small thickness has been studied by several investigators[1,2]. For apertures whose linear dimensions are small compared to the wavelengths, the effect of the aperture can be thought of as due to electric and magnetic dipoles[1,2]. The electric and magnetic dipole moments are proportional to the normal electric field and the tangential magnetic field, respectively, of the incident electromagnetic wave. The currents and voltages induced inside a cable (apertures in a cable shield) by the fields in the aperture can be calculated from the dipole moments by invoking the Schelkunoff field equivalence theorem[3,4].

The dielectric effects on the electric dipole moment is readily accounted for when the aperture is located in a perfectly conducting plane with infinite small thickness and sandwiched between two homogeneous dielectric half space with different dielectric constants[2].

In this paper we will study the dielectric effects on the electric dipol moment when the circular aperture in the perfectly conducting plane is surrounded by a dielectric half space on one side and a dielectric layer of finite thickness on the other side. The problem under consideration might model an aperture in a cable shield with a dielectric insulation and a dielectric jacket. Apertures in a cable shield will cause leakage of electromagnetic energy from the region outside the cable shield into the region inside the cable shield and vice versa. To simplify the analysis the aperture shape's is chosen to be circular and located in an perfectly conducting plane.

Thus, an appropriate boundary-value problem is formulated and reduced to a set of dual integral equations. These equations are then solved, analytically in some limiting cases and numerically in the general case. Once the solution of these equations has been found, the equivalent dipole moment of the aperture is obtained by performing the simple integration, and the dielectric effects on the electric polarizability of the circular aperture can be determined. Results are shown in graphical form.

2. *Problem formulation*

The problem configuration considered here is shown in Fig. 1. It consists of a perfectly conducting plane with infinite small thickness separating two regions 1 and 2 which may have different electrical properties. Coupling between the two regions occurs through an aperture of the radii a. The perfectly conducting plane is infinite in the x and y directions. The problem consists of three regions separated by two boundaries.

The regions are defined as:

region 1 $z < 0$, all x

region 2 $0 < z < h$, all x

region 3 $z > h$, all x

and the dielectric constants are:

$$\varepsilon = \begin{cases} \varepsilon_1 = \varepsilon_{r1}\varepsilon_0, & z < 0 \\ \varepsilon_2 = \varepsilon_{r2}\varepsilon_0, & 0 < z < h \\ \varepsilon_3 = \varepsilon_0, & z > h \end{cases} \qquad \mu = \mu_0 \tag{1}$$

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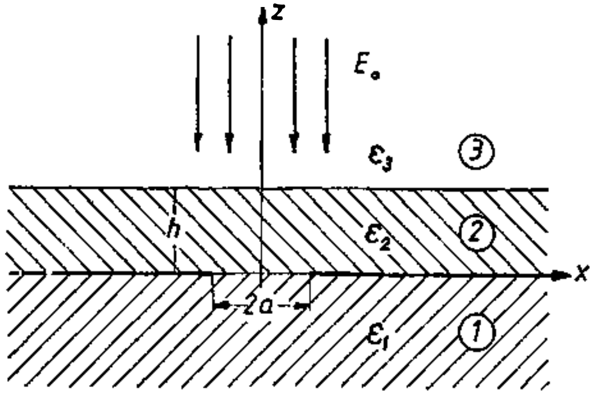

Fig. 1. A circular aperture in a plane screen sandwiched between a dielectric half space and a layer of dielectric.

where ε_{r_i} $(i = 1, 2, 3)$ is the relative dielectric constant, ε_0 is the dielectric constant of free-space and μ_0 is permeability of free-space.

The incident electric field, which is perpendicular to the perfectly conducting plane at $z = 0$, is denoted by $\vec{E}_0$.

The electric field in the region 1 (see Fig. 1) and far away from the circular aperture can be expressed in terms of the dipole moment $\vec{p}$ of the aperture[2],

$$\vec{p} = -\frac{\varepsilon_{r1}\,\varepsilon_0}{2}\int_S \vec{r}_S \times (\vec{1}_z \times \vec{E})\,\mathrm{d}S = \frac{\varepsilon_{r1}\,\varepsilon_0}{2}\int_S \vec{r}_S \times (\vec{1}_z \times \vec{\nabla}\varphi)\,\mathrm{d}S =$$

$$= \frac{\varepsilon_{r1}\,\varepsilon_0}{2}\vec{1}_z\int_S \vec{r}_S \cdot \vec{\nabla}\varphi\,\mathrm{d}S = -\,\varepsilon_{r1}\,\varepsilon_0\,\vec{1}_z\int_S \varphi\,\mathrm{d}S \tag{2}$$

where $\vec{r_S}$ is a position vector between two arbitrary points in the aperture, that φ and $\vec{E}$ are the function of potential distribution and vector of electric field intensity, respectively.

The fields quantity, $\vec{E}$ and φ, satisfy the following equations:

$$\vec{\nabla} \times \vec{E} = 0 \tag{3}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{4}$$

$$\vec{E} = -\vec{\nabla}\varphi. \tag{5}$$

If we substitute Eq. (5) into Eq. (4), and using the cylindrical coordinate system, the Laplace equation for the electrostatic potential seemed:

$$\frac{1}{\varrho}\frac{\partial}{\partial \varrho}\left(\varrho\frac{\partial \varphi}{\partial \varrho}\right) + \frac{\partial^2 \varphi}{\partial z^2} = 0. \tag{6}$$

To find the electric dipole moment we will solve the Laplace equation for the electrostatic potential in regions 1, 2, 3 i. e.,

$$\varphi(\varrho, z) = \begin{cases} \varphi_1(\varrho, z), & z < 0 \\ \varphi_2(\varrho, z), & 0 < z < h \\ \varphi_3(\varrho, z), & z > h \end{cases} \tag{7}$$

together with the proper boundary conditions at the junctions of the media and the correct behaviour of the field at infinity. By performing a Hankel transform[5-7] on the Laplace equation (6), we can derive the following expressions for the electrostatic potential:

$$\varphi_1(\varrho, z) = \int_0^\infty A(\lambda)\exp(\lambda z)\, J_0(\lambda\varrho)\, \mathrm{d}\lambda, \qquad z < 0,$$

$$\varphi_2(\varrho, z) = \int_0^\infty [B(\lambda)\exp(\lambda z) + C(\lambda)\exp(-\lambda z)]\, J_0(\lambda\varrho)\, \mathrm{d}\lambda + $$
$$+ E_0 z/\varepsilon_{r2}, \qquad 0 < z < h, \tag{8}$$

$$\varphi_3(\varrho, z) = \int_0^\infty D(\lambda)\exp(-\lambda z)\, J_0(\lambda\varrho)\, \mathrm{d}\lambda + E_0 z - E_0 h(\varepsilon_{r2} - 1)/\varepsilon_{r2},$$
$$z > h,$$

where $J_0(\lambda\varrho)$ is the Bessel function of the first kind and order zero and the functions $A(\lambda)$, $B(\lambda)$, $C(\lambda)$ and $D(\lambda)$ are to be determined from the boundary conditions at the $z = 0$ and $z = h$. The integrals in Eqs. (8) can be viewed as the »scattered« potential, i. e., the influence on the field due to the aperture, and this part of the potential approaches zero as the radius of the aperture tends to zero.

The boundary conditions at $z = h$ imply that

$$\begin{aligned} &\varphi_2(\varrho, h) = \varphi_3(\varrho, h) \\ &\varepsilon_{r2}\frac{\partial \varphi_2}{\partial z}(\varrho, h) = \frac{\partial \varphi_3}{\partial z}(\varrho, h), \qquad \varrho \geqslant 0. \end{aligned} \tag{9}$$

Similarly, the boundary conditions at $z = 0$ can be expressed mathematically in the following way,

$$\varphi_1(\varrho, 0) = \varphi_2(\varrho, 0), \qquad \varrho \geqslant 0$$

$$\varphi_1(\varrho, 0) = \varphi_2(\varrho, 0) = 0, \qquad \varrho > a \tag{10}$$

$$\varepsilon_{r1} \frac{\partial \varphi_1}{\partial z}(\varrho, 0) = \varepsilon_{r2} \frac{\partial \varphi_2}{\partial z}(\varrho, 0), \qquad 0 < \varrho < a.$$

The first of these equations expresses the continuity of the potential at $z = 0$, the second one the fact that the potential is zero on the perfectly conducting plate, and the third one the continuity of the normal component of the electric induction vector across the aperture. Substituting the expressions (8) into the boundary conditions (9) and (10) we arrive at the following set of dual integral equations for $A(\lambda)$,

$$\int_0^\infty A(\lambda) J_0(\lambda \varrho) \, \mathrm{d}\lambda = 0, \qquad \varrho > a,$$

$$\int_0^\infty F(\lambda) A(\lambda) J_0(\lambda \varrho) \, \mathrm{d}\lambda = E_0, \qquad \varrho < a \tag{11}$$

where

$$F(\lambda) = (\varepsilon_{r1} + \varepsilon_{r2}) \lambda [1 + k_1(\lambda)]$$

and

$$k_1(\lambda) = \frac{\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}} \left[\frac{\cosh \lambda h + \varepsilon_{r2} \sinh \lambda h}{\sinh \lambda h + \varepsilon_{r2} \cos h \lambda \mathrm{h}} - 1 \right].$$

It should be pointed out that $k_1(\lambda)$ tends to zero when $\varepsilon_{r2} \to 1$ or when $h \to \infty$. In these special cases the set of dual integral equations (11) has an explicit solution[8].

To find a solution of Eq. (11) in the general case we introduce the normalized quantities

$$u = \varrho / a, \qquad \xi = a\lambda, \qquad k(\xi) = k_1(\lambda)$$

$$X(\xi) = \frac{\varepsilon_{r1} + \varepsilon_{r2}}{E_0 a^2} A(\lambda) \tag{12}$$

and we arrive at the following set of equations for $X(\xi)$,

$$\int_0^\infty \xi [1 + k(\xi)] X(\xi) J_0(u \xi) \, \mathrm{d}\xi = 1, \qquad 0 < u < 1,$$

$$\int_0^\infty X(\xi) J_0(u \xi) \, \mathrm{d}\xi = 0, \qquad u > 1. \tag{13}$$

The electric dipole moment of the circular aperture can be expressed in terms of the solution of the integral equation (13) in the following way,

$$\vec{p} = -2\pi \varepsilon_{r1} \varepsilon_0 E_0 a^3 (\varepsilon_{r1} + \varepsilon_{r2})^{-1} \cdot \vec{1}_z \int_0^\infty \xi^{-1} X(\xi) J_1(\xi) \, \mathrm{d}\xi. \tag{14}$$

Thus, we have reduced the problem of finding the electrostatic potential in the aperture with a dielectric coating to the solution of the set of dual integral equations (13). Once the solution of these equations has been found, the equivalent electric dipole moment of the aperture is obtained by performing the simple integration (14), and the dielectric effects on the electric polarizability of the circular aperture can be determined.

3. *Solution of the integral equations*

In this section we will solve the set of dual integral equations (13). In the most general case it is not possible to find a closed form solution of Eqs. (13). Numerical methods must therefore be used to »solve« Eqs. (13). Later in this section we will formulate a Fredholm integral equation of the second kind with compact kernel and from the solution of this integral equation the electric dipole moment of the aperture is calculated. However, when $h \gg a$ perturbation procedures are used to obtain an approximate solution of Eqs. (13). Also, some limiting forms of the solutio n of Eqs. (13) are derived when $h \ll a$.

3.1. *The case* $h \gg a$

To solve the set of dual integral equations (13) when $h \gg a$ it is first reduced to the following Fredholm integral equations of second kind[8],

$$X(\xi) + \frac{1}{\pi} \int_0^\infty \left[\frac{\sin(\xi - \eta)}{\xi - \eta} - \frac{\sin(\xi + \eta)}{\xi + \eta} \right] k(\eta)\, X(\eta)\, \mathrm{d}\eta = G(\xi), \quad \xi > 0, \tag{15}$$

where

$$G(\xi) = \frac{2(\sin \xi - \xi \cos \xi)}{\pi \xi}.$$

The functions $k(\xi)$ (see Eq. (12)) is then expanded in the series

$$k(\xi) = \frac{2\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}} \sum_{n=1}^{\infty} \gamma^n \exp(-n\,\beta\,\xi) \tag{16}$$

where

$$\beta = 2h/a, \qquad \gamma = (1 - \varepsilon_{r2})/(1 + \varepsilon_{r2}).$$

By inserting the expansion (16) into Eq. (15) one arrives at an approximate solution of Eqs. (13) when $h \gg a$, i. e., $\beta \gg 1$, as follows

$$X(\xi) = G(\xi) - r\,Q_3(\gamma)\,G(\xi)\,\beta^{-3} - r\,Q_5(\gamma)\,[2G^{\text{II}}(\xi) - $$

$$- 6G(\xi)/5]\,\beta^{-5} + r^2\,Q_3^2(\gamma)\,G(\xi)\,\beta^{-6} + O(\beta^{-7}) \tag{17}$$

where

$$r = \frac{8\varepsilon_{r2}}{3\pi(\varepsilon_{r1} + \varepsilon_{r2})}, \quad Q_m(\gamma) = \sum_{n=1}^{\infty} \gamma^n n^{-m}, \quad m = 3, 5.$$

The electric dipole moment of the aperture as defined by Eq. (2) can be expressed in the following form,

$$\vec{p} = \vec{p}_0 F(\varepsilon_{r1}, \varepsilon_{r2}, \beta), \qquad \vec{p}_0 = -\frac{4\varepsilon_{r1}\varepsilon_0 E_0 a^3}{3(\varepsilon_{r1} + \varepsilon_{r2})}\vec{1}_z \tag{18}$$

where $F(\varepsilon_{r1}, \varepsilon_{r2}, \beta)$ is given by (c. f. Eqs. (14) and (17))

$$F(\varepsilon_{r1}, \varepsilon_{r2}, \beta) = 1 - r\,Q_3(\gamma)\,\beta^{-3} + 2.4r\,Q_5(\gamma)\,\beta^{-5} + + r^2 Q_3^2(\gamma)\,\beta^{-6} + O(\beta^{-7}). \tag{19}$$

It is observed from Eq. (19) that the normalized factor $F(\varepsilon_{r1}, \varepsilon_{r2}, \beta)$ approaches unity rather fast for large values of β. From the numerical results that we will present in the next section it will be seen that the asymptotic expansion (19) is valid for $\beta \geqslant 2$, and that in this regime the electric dipole moment differs less than 2 percent from its value for $\beta \to \infty$.

3.2. *The case $h \ll a$*

To obtain an integral equation, which can be solved with perturbation procedures when $\beta \ll 1$, we first make the substitution

$$Y(\xi) = \frac{1 + \varepsilon_{r1}}{\varepsilon_{r1} + \varepsilon_{r2}} X(\xi) \tag{20}$$

and $Y(\xi)$ satisfies the set of dual integral equations

$$\int_0^\infty \xi\,[1 + l(\xi)]\,Y(\xi)\,J_0(u\,\xi)\,\mathrm{d}\xi = 1, \qquad 0 < u < 1,$$

$$\int_0^\infty Y(\xi)\,J_0(u\,\xi)\,\mathrm{d}\xi = 0, \qquad u > 1 \tag{21}$$

where

$$l(\xi) = \frac{\varepsilon_{r2} - 1}{\varepsilon_{r2} + 1} + \frac{\varepsilon_{r1} + \varepsilon_{r2}}{\varepsilon_{r1} + 1} k(\xi)$$

and

$$l(\xi) = (\varepsilon_{r2}^2 - 1)\,[2\varepsilon_{r2}(\varepsilon_{r1} + 1)]^{-1}\,\xi\,\beta^{-1} + O(\beta^{-3}), \qquad \beta \ll 1.$$

An asymptotic solution of Eqs. (21), valid as $\beta \to 0$, is obtained by first reducing Eqs. (21) to a Fredholm integral equation of the second kind similar to Eq. (15) and then solving the resulting integral equation with perturbation procedures. With this method we derive the asymptotic representation of $Y(\xi)$:

$$Y(\xi) = \frac{2(\sin\xi - \xi\cos\xi)}{\pi\xi} + \frac{(\varepsilon_{r2}^2 - 1)\sin\xi}{\pi^2 \varepsilon_{r2}(\varepsilon_{r1} + 1)}\beta\ln\beta + O(\beta). \tag{22}$$

For small values of β the normalized factor $F(\varepsilon_{r1}, \varepsilon_{r2}, \beta)$ is asymptotically given by

$$F(\varepsilon_{r1}, \varepsilon_{r2}, \beta) = \frac{\varepsilon_{r1} + \varepsilon_{r2}}{\varepsilon_{r1} + 1} + \frac{3(\varepsilon_{r2}^2 - 1)(\varepsilon_{r1} + \varepsilon_{r2})}{2\pi\varepsilon_{r2}(\varepsilon_{r1} + 1)^2}\beta\ln\beta + O(\beta). \tag{23}$$

Comparing the two expressions (19) and (23) we observe that, as expected, the electric dipole moment of the aperture without the dielectric layer ($\beta = 0$) is larger than its value when the thickness of the dielectric layer is large compared to the aperture size ($\beta = \infty$). We also expect $F(\varepsilon_{r1}, \varepsilon_{r2}, \beta)$ to vary between the two limiting values given by Eqs. (23) and (19) as β varies between zero and infinity. The numerical calculations show that this is indeed the case.

3.3. *The general case*

To find an equation suitable for a numerical evaluation of the electric dipole moment in the general case we introduce the normalized potential $f(u)$ in the aperture, defined by

$$f(u) = (\varepsilon_{r1} + \varepsilon_{r2})\,\varphi(ua, 0)/(a E_0) \tag{24}$$

and $f(u)$ is related to the solution of the set of dual integral equations (13) by

$$f(u) = \int_0^\infty J_0(\xi u)\, X(\xi)\,\mathrm{d}\xi. \tag{25}$$

After some algebraic manipulations on Eqs. (13) and (25) we arrive at the following Fredholm integral equation for $f(u)$[8],

$$f(u) + \int_0^1 L(u, v) f(v)\,\mathrm{d}v = \frac{2}{\pi}\sqrt{1 - u^2}, \qquad 0 \leqslant u \leqslant 1 \tag{26}$$

where

$$L(u, v) = \frac{2v}{\pi}\int_0^\infty \int_u^1 \frac{\sin s\eta}{\sqrt{s^2 - u^2}}\,\eta\, k(\eta)\, J_0(v\eta)\,\mathrm{d}\eta\,\mathrm{d}s.$$

To find the solution of Eq. (25) we first extend the domain of definition of $L(u, v)$ to $(-1, 1) \times (-1, 1)$, so that $L(u, v)$ is an even function of both u and v. Similarly, we extend the domain of definition of $f(u)$ to $(-1, 1)$, so that $f(u)$ is an even function of u. Expanding $f(u)$ in terms of the Chebyshev polynomials of the second kind, $U_{2n}(u)$[9],

$$f(u) = \sqrt{1 - u^2} \sum_{n=0}^{\infty} f_n U_{2n}(u), \tag{27}$$

we can transform the integral equation (26) to the following set of algebraic equations by applying the Galerkin's method,

$$f_n + \sum_{m=0}^{\infty} L_{nm} f_m = \frac{2}{\pi} \delta_{n0} \tag{28}$$

where δ_{nm} is the Kronecker symbol and

$$L_{nm} = \frac{2}{\pi} \int_0^{\infty} \eta\, k(\eta)\, G_n(\eta)\, H_m(\eta)\, \mathrm{d}\eta. \tag{29}$$

The function $G_n(\eta)$ can be expressed in terms of the spherical Bessel functions,

$$G_n(\eta) = (-1)^n \eta\, [j_{n-1}(\eta/2)\, y_{n-1}(\eta/2) - j_{n+1}(\eta/2)\, y_{n+1}(\eta/2)]$$

while $H_m(\eta)$ is given by

$$H_m(\eta) = \sum_{l=0}^{m} \frac{(-1)^m (m+l)!\,(2l)!}{(l!)^2 (m-l)!\, \eta^{2l+1}} \times$$

$$\times \left[1 - \cos\eta \sum_{k=0}^{l} \frac{(-1)^k \eta^{2k}}{(2k)!} + \sin\eta \sum_{k=1}^{l} \frac{(-1)^k \eta^{2k-1}}{(2k-1)!}\right].$$

The normalized factor $F(\varepsilon_{r1}, \varepsilon_{r2}, \beta)$ is obtained from $f(u)$ and the solution of Eq. (28) as follows,

$$F(\varepsilon_{r1}, \varepsilon_{r2}, \beta) = \frac{3\pi}{2} \int_0^1 u f(u)\, \mathrm{d}u = \frac{3\pi}{2} \sum_{n=0}^{\infty} \frac{(-1)^{n+1} f_n}{(2n-1)(2n+3)}. \tag{30}$$

4. Results and discussion

Eq. (28) constitutes a form that is suitable for numerical solution. The integrand in Eq. (29) decays exponentially as $\exp(-4ha^{-1}\eta)$ (c. f. Eq. (12)), but for small values of h/a this decaying is very slow thus making it difficult to accurately evaluate Eq. (29) with numerical means. We therefore had to limit the numerical computations to $h/a > 0.1$. In the limiting case as h/a tends to infinity it is immediately seen from Eq. (29) that the solution of Eq. (28) is given by $f_n = 2/\pi\, \delta_{n0}$ and that the normalized factor $F(\varepsilon_{r1}, \varepsilon_{r2}, \infty) = 1$. In the numerical solution it was found that satisfactory accuracy ($< 1\%$) was obtained by truncating the infinite set of Eqs. (28) to a set of four equations and four unknowns. This fast convergence of the expansion (27) can partly attributed to the fact that each term in the expansion (27) satisfies the boundary condition at $\varrho = a$ ($u = 1$).

The parameters describing the electromagnetic properties of the aperture are the electric and magnetic polarizabilities. Only the electric polarizability a_e is affected by the presence of the dielectric layer and is related to the electric dipole moment p through

$$p = a_e\, q \tag{31}$$

where q is the charge density on the exterior surface of the conducting wall when the aperture is short circuited, i. e., (note that normal component of vector $\vec{D}$ is continuous) $q = \varepsilon_0 E_0$. We therefore get

$$a_e = \frac{p}{\varepsilon_0 E_0} = \frac{2\varepsilon_0 a^3}{3} \frac{2\varepsilon_{r1}}{\varepsilon_{r1}+\varepsilon_{r2}} F(\varepsilon_{r1}, \varepsilon_{r2}, \beta) \equiv a_e \overline{a}_e, \tag{32}$$

where a_e^f is the polarizability of a circular aperture in a perfectly conducting plane in vacuum,

$$a_e^f = 2\varepsilon_0 a^3/3 \tag{33}$$

and $\overline{a}_e$ is given by

$$\overline{a}_e = \frac{2\varepsilon_{r1}}{\varepsilon_{r1}+\varepsilon_{r2}} F(\varepsilon_{r1}, \varepsilon_{r2}, \beta). \tag{34}$$

In Fig. 2 we have graphed $\overline{a}_e(\varepsilon_{r1}, \varepsilon_{r2}, h/a)$ versus h/a for different values of ε_{r1} and ε_{r2}. It is seen from these figures that $\overline{a}_e$ is an increasing function of ε_{r1}, whereas it is a decreasing function of ε_{r2} and h/a, as expected.

The asymptotic form (19) for $F(\varepsilon_{r1}, \varepsilon_{r2}, \beta)$ can be used to obtain an asymptotic form of $\overline{a}_e$ for large values of h/a. This asymptotic form for $\overline{a}_e$ deviates less than 10% from the exact form when $h/a > 0.8$. However, the difference between the asymptotic form and the exact form increases rapidly when $h/a < 0.8$. The limiting form (23) can be used to obtain the following expression for $\overline{a}_e$ in the special case of vanishing thickness of the dielectric layer,

$$\overline{a}_e(\varepsilon_{r1}, \varepsilon_{r2}, O) = \frac{2\varepsilon_{r1}}{1+\varepsilon_{r1}} \tag{35}$$

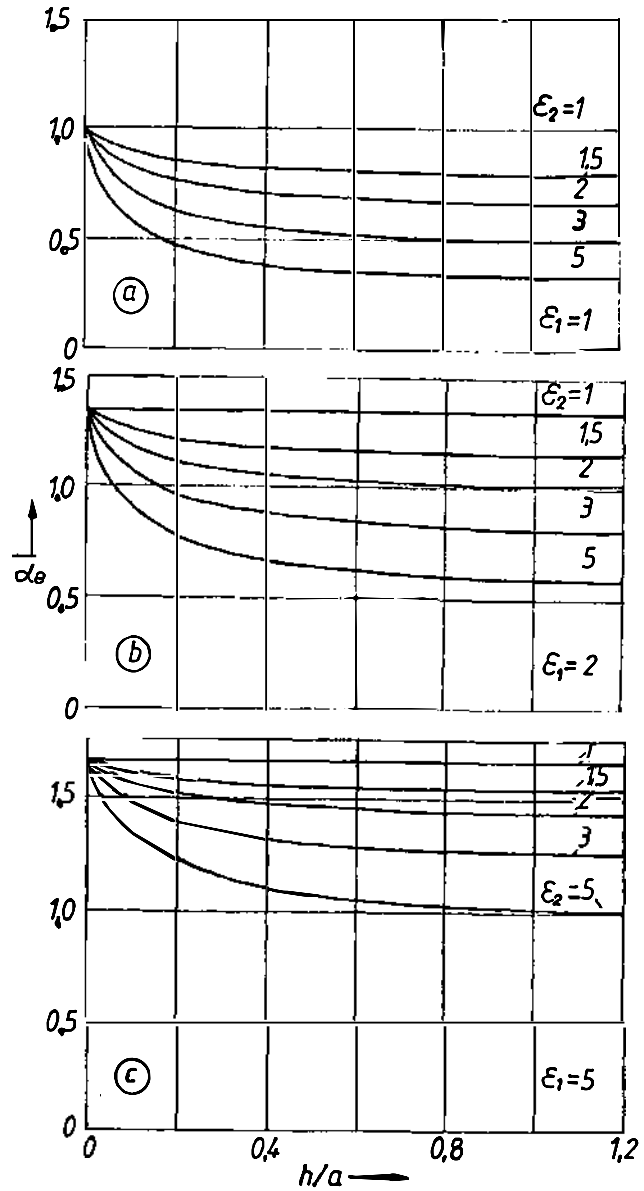

Fig. 2. The variation of the electric polarizability with the permittivity and the thickness of the dielectric layer.

5. *Conclusion*

The dielectric effects on the electric polarizability of an circular aperture in a perfectly conducting plane are studied. The circular aperture is surrounded by a dielectric half space on one side and a dielectric layer of finite thickness on the other side. The results of the calculations are presented in the form of graphs for the variations of the electric polarizability with the relative dielectric constant of the dielectric half space as well as the relative dielectric constant and the thickness of the dielectric layer. Asymptotic expressions for the electric polarizability are also found for thick and thin layers. The difference between the asymptotic form and the exact form increases rapidly when $h/a < 0.8$.

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TEORIJSKO IZUČAVANJE KRUŽNOG ZRAČEĆEG PROREZA NA IDEALNO VODLJIVOJ PLOHI I SLOJEVITOM DIELEKTRIKU

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U radu je teoretski izučavan uticaj dielektrika na električnu polarizabilnost kružnog zračećeg proreza na idealno vodljivoj plohi. Problem rubnog uvjeta je formulisan a zatim redukovan na sustav dualnih integralskih jednadžbi. Približnim rešavanjem Fredholm-ove integralske jednadžbe druge vrste sa kompaktnim jezgrom dobivena je električna polarizabilnost zračećeg proreza. Dobiveni numerički rezultati su prikazani grafički a za promjene polarizabilnosti u funkciji relativne dielektrične konstante i debljine dielektrika s konačnom debljinom.