YU ISSN 0015—3206
FZKAAA 21 (3) 267 (1989)

ALPHA PARTICLE CLUSTER STATES IN ^{40}Ca, ^{114}Pd, ^{188}Os AND ^{238}U

JOSEPH J. BEVELACQUA

GPU Nuclear Corporation, P. O. Box 480, Middletown, Pennsylvania 17057, U. S. A.

Received 22 August 1988

Revised manuscript received 18 April 1989

UDC 539.12

Original scientific paper

A single-channel alpha-particle plus core cluster model, based on the folding potential of Pal and Lovas (PL), is used to calculate the alpha-cluster states in ^{40}Ca, ^{188}Os and ^{238}U. The PL potential parameters (V_0 and R) were adjusted to give the best fit to the spectra of these nuclei. These potential parameters define an alpha-particle plus core (APPC) interaction which provides a theoretical representation of ^{40}Ca, ^{188}Os and ^{238}U in terms of rotational spectra. The radius parameter (R) for the APPC interaction is found to vary as a linear function of the mass number A. When the APPC interaction is applied to ^{114}Pd, the calculated spectrum is in reasonable agreement with the observed 0^+, 2^+, 4^+ and 6^+ spectrum. These results suggest the single-channel, alpha-particle plus core cluster model and the APPC interaction provide a reasonable description of the alpha clustering phenomena for nuclei with rotational or rotational-like spectra.

1. Introduction

The theoretical treatment of alpha-particle clustering in nuclei has received considerable attention[1-18]. It is well established that alpha particle cluster structure is present in many $A \leq 44$ nuclei[1-13]. It has also been suggested that alpha cluster states may be present in the rare earth[14], ^{212}Po[15] and actinide[16] nuclei. The study of the alpha clustering phenomena over a wide range of nuclear masses is of considerable interest. This study would permit an opportunity for a syste-

matic study of alpha clustering which could lead to a consistent theoretical model applicable to a wide range of nuclei.

Currently, a wide variety of models have been utilized to study alpha clustering in nuclei. Cluster models and their recent extensions[1-13] have provided a reasonable description of $16 \leq A \leq 44$ nuclei. The interacting boson model (IBM) has also led to a satisfactory description of alpha clustering with its greatest success lying within the rare earth[14] and actinide[16] regions. In addition, alpha particle clustering in heavy nuclei has been studied theoretically in connection with alpha decay[15]. Other models such as those based on the U(5) and O(5) symmetry groups have been applied to the ^{18}O and ^{20}Ne systems[17]. Additional discussion regarding the alpha clustering phenomena in nuclei has been provided by Bromley[18].

This paper will attempt to extend the cluster model of Buck et al.[1] to heavier nuclei ($A \geq 44$). This extension will permit a systematic study of alpha clustering and will permit the determination of the A-dependence of the cluster model potentials and associated parameters. A knowledge of the alpha clustering potential would permit a more thorough investigation of the alpha clustering phenomena and would assist in determining the importance of alpha clustering in the underlying spectroscopy of a wide range of nuclei. Approaches of this type have been utilized to determine nucleon-nucleus interactions with considerable success[19,20].

2. *Formalism*

The model used to describe the alpha plus core system represents an application of the standard method of Petrovich et al.[19] and Tanaka et al.[20]. Since the method is well established only salient features will be addressed.

The binding energy E_{NLSJ} of a cluster in the field of a nuclear core may be obtained by solving the Schrödinger equation

$$\left[\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) - E_{NLSJ} - V_{NLSJ}\right] U_{NLSJ}(r) = 0, \tag{1}$$

where r is the radial coordinate between the center of mass of the nuclear core (c) and the cluster; $V_{NLSJ}(r)$ is the model interaction; E_{NLSJ} is the core plus cluster binding energy; $U_{NLSJ}(r)$ is the radial wave function; and L, S and J are the orbital, spin and total angular momentum quantum numbers, respectively. The N quantum number is the harmonic oscillator radial quantum number, and μ is the reduced mass. For the present application, V_{NLSJ} is defined in terms of a geometry factor $g(r)$

$$V_{NLSJ}(r) = -V_0 g_0(r) - V_{s0} g_{s0}(r) F(LSJ) + Z_\alpha Z_c e^2 C(r) \tag{2}$$

where

$$F(LSJ) = J(J+1) - L(L+1) - S(S+1), \tag{3}$$

and

$$C(r) = \frac{1}{2R_c}\left[3 - (r/R_c)^2\right] \text{ for } r < R_c \tag{4}$$

$$C(r) = \frac{1}{r} \text{ for } r \geq R_c.$$

Since the potential is not a function of the Θ and φ spherical coordinates, the solution of the angular equation is most easily expressed in terms of spherical coordinates

$$Y_{LM}(\Theta, \varphi).$$

The total wave function for the relative motion of the core plus cluster interacting through a spherically symmetric potential is given by a product of space and spin wave functions:

$$\psi_{NLSJM}(\vec{r}) = \frac{1}{r} U_{NLSJ}(r) \sum_{M_L M_S} C(L, M_L, S, M_S; JM) \times Y_{LM_L}(\Theta, \varphi) X_{SM_S} \tag{5}$$

where M and M_S are the projections of angular momentum and spin, and X is the spin wave function. For the alpha cluster problem, $S = 0$ which simplifies the angular momentum algebra. In addition, $F(LSJ) = 0$ and the potential, Eq. (2), reduces to the form

$$V_{NLSJ}(r) = -V_0 g(r) + Z_\alpha Z_c e^2 C(r). \tag{6}$$

The strength of the potential required to obtain the experimental binding energy is obtained by rewriting the radial Schrödinger equation in the form

$$\left[\frac{d^2}{dr^2} - k(p, r)\right] U(p, r) = 0 \tag{7}$$

where

$$U(p, r) = U_{NLSJ}, \tag{8}$$

and

$$k(p, r) = \frac{L(L+1)}{r^2} + \frac{2\mu}{\hbar^2}\left[E_{NLSJ} + V_{NLSJ}(r)\right]. \tag{9}$$

The model searches for values of p in order to obtain the potential strength V_0 which yields the binding energy E_{NLSJ}. Equations (7) and (9) may be used to obtain E_{NLSJ} if V_0 is available. The method of searching for p is provided by Brown, Gunn and Gould[21]. Interior ($r < R_0$) and exterior ($r \geq R_0$) are matched, and the Fox-Godwin method is used to obtain a converged solution. Additional calculational details for bound states may be found in Refs. 19, 23 and 24. Resonance energies are obtained by the methods outlined in Ref. 25.

3. *Model potential*

The model interaction is based upon the potential of Pal and Lovas (PL)[8]. The PL potential describes the central part of the interaction between an alpha particle cluster and core nucleons. The local PL potential is obtained by folding the densities of the two constituents with an effective nuclear interaction. This interaction has been accurately parametrized in the form

$$V(r) = -V_0 g(r) \tag{10}$$

where the geometry factor $g(r)$ is given by

$$g(r) = \frac{1 + \cosh(R/a)}{\cosh(r/a) + \cosh(R/a)}. \tag{11}$$

The radius parameter R and diffuseness parameter a define the geometry of the potential. The depth V_0 can be chosen to reproduce the experimental value of the alpha cluster plus core separation energy. This interaction has been successful in the description of $A \leq 44$ systems[1-13].

4. *Elimination of Pauli forbidden states*

The requirements of the Pauli principle are satisfied by treating the alpha cluster as a single particle whose relative motion about the core is characterized by the N and L quantum numbers. Moreover, we impose a restriction that N and L correspond to the microscopic situation in which the cluster nucleons all occupy a different major shell than the core nucleons. For example, if the alpha cluster model is applied to the ^{40}Ca system, the core nucleons form ^{36}Ar. Since the nucleons within the ^{36}Ar core reside within the 2S-1D shell, all cluster nucleons will be placed within the 3P-1F shell ($3\hbar\omega$). Therefore, the total excitation energy (E_t) available to the cluster will be

$$E_t = \sum_{i=1}^{4} (2N_i + L_i) = 2N + L \tag{12}$$

where i labels a nucleon within the alpha cluster. For the case of ^{40}Ca, $E_t = 12\hbar\omega$. Therefore, the alpha cluster quantum numbers satisfy the relationship

$$2N + L \geq 12. \tag{13}$$

For the ^{40}Ca system, the following alpha cluster quantum numbers (N, L) meet the criteria of Eq. (13): (7, 0), (6, 2), (5, 4), (4, 6), (3, 8), (2, 10) and (1, 12) which correspond to the $J^\pi = 0^+, 2^+, 4^+, 6^+, 8^+, 10^+$ and 12^+ states, respectively.

The reader should note that in the context of the present paper, we have assumed the model states have a core plus alpha particle character. The necessity for our discussion of Pauli forbidden states depends upon the existence of these core plus alpha particle states. A determination of the existence of these states would require that a calculation similar to that of Ref. 15 be performed. Such a calculation is beyond the scope of this paper.

5. *Comparison with other models*

In order to verify our model, we have performed a comparison with other models of the ^{40}Ca system. In order to describe ^{40}Ca, Merchant[13] choose a set of model parameters to fit the ^{36}Ar + alpha band head 0^+ level at 3.35 MeV excitation energy. Using these potential parameters, Merchant[13] used the following multi-channel model to describe the ^{36}Ar + alpha system:

$$\left[\frac{\hbar^2}{2\mu}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{L_i(L_i+1)}{r^2}\right) + e_i - E\right] U_i^J(\vec{r}) - \sum_k V_{ik}^J(\vec{r})\, U_k^J(\vec{r}) = 0 \qquad (14)$$

where $V_{ik}(r)$ is the potential matrix given by

$$V_{ik}^J(\vec{r}) = \langle Q_i^{JM}(\hat{r},\vec{y}) \mid V(\vec{r},\vec{y}) \mid Q_i^{JM}(\hat{r},\vec{y})\rangle. \qquad (15)$$

The wave function for a given total angular momentum J (with projection M) is written as a product wave function summed over channels (i)

$$\psi_{JM} = \sum_i \frac{1}{r}\, U_i^J(\vec{r})\, Q_i^{JM}(\hat{r},\vec{y}) \qquad (16)$$

where each channel i is defined by the core spin I_i, relative orbital angular momentum L_i and internal core energy e_i. In Eq. (15), r is the magnitude and $\hat{r}$ are the angular coordinates of the vector $\vec{r}$ which joins the centers-of mass of the alpha cluster and core, and $\vec{y}$ represents the internal coordinates of the core. In the case of ^{40}Ca, ^{36}Ar is the core which is joined to the alpha particle via the $\vec{r}$ coordinate.

The model of Merchant[13] reduces to our model if the following substitutions are made

$$y \to 0 \qquad (17)$$

$$e_i \to 0 \qquad (18)$$

$$\sum_k V_{ik}^J(\vec{r})\, U_k^J(\vec{r}) \to V_{NLSJ}(r)\, U_{NLSJ}(r) \qquad (19)$$

$$L_i \to L \tag{20}$$

$$U_i^J(\vec{r}) \to U_{NLSJ}(r). \tag{21}$$

Eq. (17) corresponds to our model limitation of not specifying any core internal structure. The limit on core excitation into states other than the ground state is provided by Eq. (18). Eqs. (19)—(21) represent the single channel nature of our model. With the restrictions of Eqs. (17)—(21), our model represents an extreme cluster model which does not permit core or cluster excitation or internal structure. However, the sacrifice of these model details are entirely justified in view of the nature of the phenomenological potentials utilized in cluster models. The extreme cluster model also permits a survey of the general alpha clustering phenomena without relying on numerous model corrections.

6. *Determination of model potential parameters*

The PL model parameters for ^{40}Ca (^{36}Ar + alpha) were determined by Merchant[13]. In order to describe ^{40}Ca, Merchant[13] choose a radius $R = 2.9$ fm, a diffuseness of $a = 1.4$ fm and a depth $V_0 = 158.1$ MeV to fit the ^{36}Ar + alpha band head at 3.35 MeV excitation energy. The Coulomb potential was taken as that appropriate to a pointlike cluster interacting with a uniformly charged spherical core of radius $R_c = 4.0$ fm. In order to determine parameters for other systems, we follow the guidelines of Rost[26] which suggest the following relationships are appropriate

$$a(A) = a(^{40}\text{Ca}) = 1.4 \text{ fm} \tag{22}$$

$$R_c = R_{c0} A^{1/3} = 1.17\, A^{1/3}. \tag{23}$$

The numerical value of $R_{c0} = 1.17$ fm was determined from the ^{40}Ca PL value of R_c by using Eq. (23). Eqs. (22) and (23) were utilized to obtain a portion of the parameters of Table 1 for the nuclei considered herein. Values of R and V_0 are obtained from the best fit to the alpha cluster spectrum within the system of interest. The parameters a, R_c, R and V_0 define the model alpha-particle plus core (AAPC) interaction. In the next section of this paper, the APPC interaction will be used to investigate the ^{40}Ca, ^{188}Os and ^{238}U nuclei within the extreme cluster model framework.

7. *Results and discussion*

The calculated energies of low lying states with their experimental counterparts are given in Tables 2, 3 and 4 for the ^{40}Ca, ^{188}Os and ^{238}U systems, respectively. The model parameters utilized in these calculations are summarized in Table 1.

TABLE 1.

System	V_0[a] (MeV)	R (fm)	R_c (fm)	a (fm)	E (MeV)[b]
^{40}Ca	141.2	2.90	4.00	1.4	3.689[c]
^{188}Os	149.6	5.39	6.70	1.4	−2.137[d]
^{238}U	146.0	6.31	7.25	1.4	−4.270[d]

a) The potential strength is adjusted to fit the alpha + core band head energy.
b) E is the experimental band head energy of the 0^+ level. A negative value indicates the band head is unbound relative to the alpha-particle + core channel.
c) First excited 0^+ state, $E_x = 3.35$ MeV.
d) Ground state $J^\pi = 0^+$.

Alpha-particle plus core interaction parameters.

a. ^{40}Ca

Table 2 compares the multi-channel calculations of Merchant[13] with our cluster model and with experiment[27]. Our single channel calculations are found to be in good agreement with the multichannel calculations and with experiment. The ^{40}Ca results suggest the single-channel model is appropriate for describing alpha cluster states within this nucleus.

TABLE 2.

		THIS WORK[b]		
J^π	Merchant[a]	$B = 0.00$	$B = -0.06$	Experiment[c]
0^+	0.00	0.00	0.00	0.00
2^+	0.52	0.50	0.64	0.55
4^+	1.73	1.72	1.85	1.93
6^+	3.66	3.66	3.79	3.58
8^+	6.28	6.37	6.51	6.55
10^+	9.52	9.94	10.09	9.05
12^+	—	14.45	14.61	—
$E_x(0^+)$	3.34	3.35	3.35	3.35

a) Ref. 13.
b) All energies are in MeV and are relative to the 0^+ (3.35 MeV) level.
c) Ref. 27. The results are presented to two decimal places to facilitate comparison with Ref. 13.

Level energies for alpha clustering states in ^{40}Ca.

The agreement with experiment is best at the lower spin states. For the 2^+, 4^+, 6^+ and 8^+ levels the model and experiment differ by 0.05, 0.21, 0.08 and 0.18 MeV, respectively. However, the 10^+ model prediction is 0.89 MeV higher in excitation energy than the experimental level. Difficulty in reproducing the 10^+ level was also noted in multi-channel calculations[13].

TABLE 3.

	LEVEL ENERGY (MeV)	
J''	Model	Experiment[a)]
0^+	0.000	0.000
2^+	0.125	0.155
4^+	0.422	0.478
6^+	0.883	0.940
8^+	1.517	1.515
10^+	2.319	2.170
12^+	3.292	—
14^+	4.438	—

a) Ref. 27.
b) The 0^+ band head energy was fit to the experimental value (E) of Table 1.

Level energies for alpha clustering states in ^{188}Os.

TABLE 4.

	LEVEL ENERGY (MeV)	
J''	Model	Experiment[a)]
$0^{+b)}$	0.000	0.000
2^+	0.047	0.0449
4^+	0.152	0.1484
6^+	0.315	0.3072
8^+	0.532	0.5178
10^+	0.789	0.7757
12^+	1.085	1.0765
14^+	1.417	1.4157
16^+	1.775	1.7882

a) Ref. 27.
b) The 0^+ band head energy was fit to the experimental value (E) of Table 1.

Level energies for alpha clustering states in ^{238}U.

The difficulty of describing excited states, even in more detailed cluster models, is an additional motivation for retaining an extreme cluster model as the basis for our survey of the alpha clustering phenomena.

b. ^{188}Os

The ^{188}Os and heavier actinide systems would normally be investigated in terms of the interacting boson model[14,16] or a group theory model[17]. However, the results of Table 3 suggest the ^{188}Os level sheme is accurately described in terms of the extreme cluster model. As in the ^{40}Ca case, the model results are nearest to the experimental values at the lower spin states. For example, the 2^+, 4^+, 6^+, 8^+ and 10^+ model and experimental level energies differ by 0.030, 0.056, 0.057, 0.002 and 0.149 MeV, respectively.

c. ^{238}U

Table 4 summarizes model and experimental results for the ^{238}U system. The good agreement between model and experimental results noted in ^{40}Ca and ^{188}Os continues in the ^{238}U system. When the ^{238}U model and experiment are compared, the 2^+, 4^+, 6^+, 8^+, 10^+, 12^+, 14^+ and 16^+ states differ by 0.002, 0.004, 0.008, 0.014, 0.013, 0.008, 0.001 and 0.013 MeV, respectively. As noted in ^{40}Ca and ^{188}Os, the model results tend to degrade as the excitation energy increases. However, the results are encouraging and suggest the extreme cluster model and APPC interaction provide a reasonable description of alpha cluster states for a wide range of nuclei.

The poorer agreement between model and experiment at the higher spin states are an obvious model shortcoming. However, this result is not unexpected. As the alpha cluster + core excitation energy increases, the collective motion becomes more complex and its description will require more model flexibility than is available within the extreme cluster model utilized herein. Moreover, additional degrees of freedom may be required to properly describe a highly excited system. Even the incorporation of multiple channels, core excitation, or backbending phenomena may not be sufficient to accurately describe the higher spin states. This is an additional motivation for restraining our model to the extreme single-particle realm.

8. Variation of model parameters

Based upon the results of Tables 1—4, the parameters of our APPC interaction, described in terms of Eqs. (22) and (23), can be used to provide a reasonable description of alpha cluster states. The APPC radius parameter values of Table 1 appear to increase with A. In order to determine the functional dependence of this variation, the R values from ^{40}Ca, ^{188}Os and ^{238}U were fit to a variety of functional forms. The best fit was obtained for the linear relationship

$$R(A) = a + b\,A \tag{24}$$

where $a = 2.2056$ and $b = 0.0171$.

Eq. (24) completes the specification of the model APPC interaction and along with Eqs. (22) and (23) define its variation with A. As noted previously, values of V_0 are best chosen by fitting the band head energy of the 0^+ state. With the specification of these parameters, the APPC model interaction is completely defined.

The reader will note that the functional dependence of $R(A)$ described by Eq. (24) was found by the least-squares fit from the three calculations presented in Table 1. Therefore, we limited the number of coefficients to two in obtaining a functional dependence for $R(A)$.

9. Application of the model to the ^{114}Pd system

In Section 8, a model interaction was derived from calculational results in the ^{40}Ca, ^{188}Os and ^{238}U systems. As a test of the validity of this interaction,

Eqs. (22)—(24) will be used to derive the interaction parameters for a system midway between $A = 40$ and 188 — i. e. an $A = 114$ nucleus. In choosing an $A = 114$ nucleus, we require that the following criteria be met: (1) sufficient data should be available to permit comparison with the model result; and (2) the $A = 114$ nucleus should have a 0^+, 2^+, 4^+, ... level structure which could be indicative of alpha cluster states. Based upon a review of these criteria, the ^{114}Pd system was selected for investigation with our model.

Utilizing Eqs. (22)—(24), the ^{114}Pd interaction was found to have the following parameters:

$$R = 4.155 \text{ fm} \tag{25}$$

$$a = 1.4 \text{ fm} \tag{26}$$

$$R_c = 5.673 \text{ fm}. \tag{27}$$

The ^{114}Pd value for V_0 is chosen to fit the binding energy of an alpha particle in the field of a ^{110}Ru core. This binding energy is 5.885 MeV which yields a V_0 value of 159.7 MeV. The ^{114}Pd interaction leads to 2^+, 4^+ and 6^+ levels which differ from their experimental counterparts by 0.077, 0.008 and 0.317 MeV, respectively. These results are sufficiently representative of the ^{114}Pd system to add credibility to the applicability of the cluster model and APPC interaction throughout the periodic table. Although the potential parameters have not been fully optimized, the ^{40}Ca, ^{114}Pd, ^{188}Os and ^{238}U results provide evidence for the applicability of our model and APPC interaction for describing alpha clustering phenomena.

TABLE 5.

LEVEL ENERGY (MeV)		
J^π	Model	Experiment[a]
0^{+b}	0.000	0.000
2^+	0.256	0.3329
4^+	0.862	0.8536
6^+	1.820	1.5030
8^+	3.150	—
10^+	4.877	—
12^+	7.026	—
14^+	9.636	—
16^+	12.735	—

a) Ref. 27.

b) The 0^+ band head energy was fit to the experimental value of the ground state binding energy (5.885 MeV) relative to the ^{110}Ru + alpha channel.

Level energies for alpha clustering states in ^{114}Pd.

The applicability of the alpha clustering model will not necessarily extend to every nucleus exhibiting a 0^+, 2^+, 4^+, ... or 1^-, 3^-, 5^-, ... level structure. However, the model should provide a convenient method to investigate the alpha clustering phenomena.

10. Model extensions

The model potential of Eq. (6) is limited in that a number of possible enhancements are not included. These enhancements could include the incorporation of deformation, core polarization, L-dependent potentials, multiple channels, backbending, etc. The most obvious extension to the model would be the incorporation of an explicit deformation term

$$V_0 g(r) \rightarrow V_0' g(r) + V_0' \left(-r \frac{dg(r)}{dr} \right) B\, Y_{20}(\hat{r}). \tag{28}$$

If the deformation extension is included, the strength V_0' will be chosen to fit the alpha + core band head energy with the constraint that the deformation parameter (B) is adjusted to provide the best fit to the level spectrum. However, B should be consistent with the quadrupole deformation of the core.

11. Inclusion of deformation into the model

Of the nuclei considered herein, the most appropriate place to test the correction due to the deformation of the core would be ^{238}U. Prior to performing the ^{238}U calculation, we will test the model deformation term by comparing its results to those obtained by Merchant[13] in the ^{40}Ca region.

^{40}Ca

The use of Eq. (28) to incorporate quadrupole deformation into the ^{40}Ca cluster model is summarized in Table 2. For the ^{40}Ca region, an oblate deformation is expected. Following Merchant[13], we use a value $B = -0.06$. This value of the deformation parameter generally results in an improved description for the levels in ^{40}Ca for the $J^\pi \leq 8^+$ levels. Specifically, the 2^+, 4^+, 6^+ and 8^+ model predictions with (and without) deformation differ from experiment by 0.09 (0.05), 0.08 (0.21), 0.210 (0.08) and 0.04 MeV (0.18 MeV), respectively. However, the improvement is not dramatic and the 10^+ level prediction with (without) the incorporation of quadrupole deformation lies 1.04 MeV (0.89 MeV) above its experimental counterpart[27]. The model results are sufficiently close to those of Ref. 13 to verify the model's formulation of the quadrupole deformation term.

^{238}U

The use of quadrupole deformation in the ^{238}U cluster model is summarized in this section. Ground state quadrupole moments (Q_0) or nuclei in the vicinity of the ^{234}Th core (^{233}U and ^{235}U) are in the range of +3.5 to +7.9 b[27]. By using standard a relationship[28,29],

$$Q_0 = \frac{3}{(5\pi)^{1/2}} \bar{R}^2 Z B (1 + (5/64\pi)^{1/2} B), \tag{29}$$

the deformation parameter (B) can be obtained in terms of the core charge (Z) and equilibrium radius ($\bar{R}$)[29] where

$$\bar{R} = 1.2\, A^{1/3}\ \text{fm} \tag{30}$$

with A being the core mass.

In order to illustrate the effect of the deformation term in ^{238}U, calculations were performed for $B = +0.15$ which corresponds to the average of the Q_0 values noted above. In utilizing the deformation term, we have not altered the strength (V_0) utilized in the $B = 0$ calculation. Therefore, the $B = +0.15$ calculations have not been optimized to yield the best possible results. However, these calculations do serve to illustrate the sensitivity of the ^{238}U model results to the deformation parameter.

The impact of including deformation in the ^{238}U system may be illustrated by a comparison with the ^{40}Ca results. As an example, the inclusion of deformation shifts the calculated 8^+ levels by 140 keV (2.2%) and 91 keV (17.1%) in ^{40}Ca and ^{238}U, respectively. The impact on ^{238}U is considerably more significant than in ^{40}Ca and choices of deformed potentials must properly account for this effect. Including deformation in the choice of our potential model would alter the formulation of Eqs. (22)—(24). Therefore, we have only discussed ^{238}U deformation in a general fashion. A more detailed discussion of the impacts of deformation upon global potential parameters will be presented in a subsequent paper.

12. Conclusions

The alpha cluster plus core model using a Pal-Lovas folding interaction geometry has been shown to provide a consistent description of alpha cluster levels in ^{40}Ca, ^{114}Pd, ^{188}Os and ^{238}U. An alpha particle plus core interaction, with parameters which scale only with the mass number A, has been derived from calculations in the $A = 40 - 238$ nuclei. Good agreement between the predicted and experimental levels was noted for the four nuclei considered herein. The model and alpha-particle plus core interaction presented herein can be applied to a wide variety of nuclei and offers a convenient tool for the investigation of alpha cluster levels.

References

1) B. Buck, C. B. Dover and J. P. Vary, Phys. Rev. **C11** (1975) 1803;
2. B. Buck and A. A. Pilt, Nucl. Phys. **A280** (1977) 133;
3) B. Buck, H. Friedrich and A. A. Pilt, Nucl. Phys. **A290** (1977) 205:
4) B. Buck and A. A. Pilt, Nucl. Phys. **A295** (1978) 1;
5) A. A. Pilt, in *Clustering Aspects of Nuclear Structure and Nuclear Reactions*, Winnipeg, Canada, edited by W. T. H Van Oers, J. P. Svenne, J. S. C. McKee, and W. R. Falk, American Institute of Physics Conference Proceedings No. 47, New York (1978) 173;
6) A. A. Pilt, Phys. Lett. **73B** (1978) 274;

7) B. Buck, A. C. Merchant and N. Rowley, Nucl. Phys. **A327** (1979) 29;
8) K. F. Pal and R. G. Lovas, Phys. Lett. **96B** (1980) 19;
9) A. C. Merchant, Nucl. Phys. **A417** (1984) 109;
10) R. A. Baldock, B. Buck and J. A. Rubbio, Nucl. Phys. **A426** (1984) 222;
11) F. Michel, G. Reidemeister and S. Ohkubo, Phys. Rev. Lett. **57** (1986) 1215;
12) A. C. Merchant, Phys. Rev. **C36** (1987) 778;
13) A. C. Merchant, Phys. Rev. **C37** (1988) 414;
14) H. J. Daley and M. A. Nagarajan, Phys. Lett. **166B** (1986) 379;
15) G. Dodig-Crnković, F. A. Janouch, R. J. Liotta and Z. Xiaolin, Physica Scripta **37** (1988) 523;
17) Bo-jun Yang and Z. Hwang, Phys. Rev. **C35** (1987) 851;
18) D. A. Bromley, in *Proceedings of the Fourth International Conference on Clustering Aspects of Nuclear Structure and Nuclear Reactions,* Chester, United Kingdom, edited by J. S. Lilley and M. A. Nagarajan, Reidel, Dordrecht (1985) 1;
19) F. Petrovich, R. J. Philpott, D. Robson, J. J. Bevelacqua ,M. Golin and D. Stanley, Phys. Rev. Lett. **37** (1976) 558;
20) Y. Tanaka, Y. Oda, F. Petrovich and R. K. Sheline, Phys. Lett. **83B** (1979) 279;
21) G. E. Brown, J. H. Gunn and P. Gould, Nucl. Phys. **46** (1963) 598;
22) L. Fox and E. T. Godwin, Proc. Cambridge Philos. Soc. **45** (1949) 373;
23) J. J. Bevelacqua, Ind. J. Phys. **57A** (1983) 26; **58A** (1984) 79; **61A** (1987) 115;
24) J. J. Bevelacqua, Physics Essays **1** (1988) 47;
25) B. Buck, A. P. Stamp and P. E. Hodgson, Philos. Mag. **8** (1963) 1805;
26) E. Rost, Phys. Lett. **26B** (1968) 184;
27) C. M. Lederer and V. S. Shirley, *Table of Isotopes, Seventh Edition,* Wiley, New York (1978);
28) P. Marmier and E. Sheldon, *Physics of Nuclei and Particles-Volume II,* Academic Press, New York (1970);
29) R. C. Barrett and D. F. Jackson, *Nuclear Sizes and Structure,* Clarendon Press, Oxford (1977).

STANJA TIPA ALFA NAKUPINE U ^{40}Ca, ^{114}Pd, ^{188}Os I ^{238}U

JOSEPH J. BEVELACQUA

GPU Nuclear Corporation, P. O. B. 480, Middletown, Pennsylvania 17057, U. S. A.

UDK 539.12

Originalni znanstveni rad

Jednokanalni alfa čestica plus sredica model osnovan na preklopnom potencijalu Pala i Lovaša (PL) uzet je za račun alfa nakupina stanja u ^{40}Ca, ^{114}Pd, ^{188}Os i ^{238}U. Parametri potencijala PL (V_0 i R) odabrani su za najbolji opis spektara tih jezgara. Oni definiraju alfa čestica plus sredica (APPC) interakciju koja vodi na teorijsku interpretaciju ^{40}Ca, ^{188}Os i ^{238}U kao rotacionih spektara. Parametar radijusa (R) za APPC varira linearno sa A. Primijenjena na ^{114}Pd APPC interakcija daje spektar u razumnom slaganju s opaženim 0^+, 2^+, 4^+ i 6^+ spektrom. Ti rezultati ukazuju da je APPC model s APPC interakcijom u jednokanalnoj verziji razuman opis dijela alfa nakupine fenomena za jezgre s rotacionim ili kvazirotacionim spektrom.