

THE GLUON SELF-ENERGY IN THE LIGHT-CONE GAUGE AND THE FIGUET-SIBOLD IDENTITY

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We discuss the dependence of the complete gluon self-energy tensor on the second light-like vector in the light-cone gauge. The Piguet-Sibold identities are verified.

1. Introduction

The light-cone gauge has become increasingly popular with the revival and intensive research of string theories. The gauge is not only ghost-free, but can also be used to remove from the theory all unphysical degrees of freedom associated with gauge transformations¹⁾. It is also the only gauge in which a quantum formulation of superstring theories is known. However, there are problems about defining axial gauges carefully. The most convenient way of defining the light-cone gauge seems to be by introducing two light-like vectors n and n^* , satisfying

$$n^2 = n^{*2} = 0, \quad n \cdot A \equiv 0. \quad (1)$$

The prescription for the gluon propagator after Mandelstam²⁾ is

$$G_{\mu\nu} = \frac{1}{k^2 + i\varepsilon} \left(-\delta_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k + i\eta n^* k} \right) \quad (2)$$

and after Leibbrandt is³⁾

$$G_{\mu\nu} = \frac{1}{k^2 + i\varepsilon} \left[-\delta_{\mu\nu} + \frac{(k_\mu n_\nu + k_\nu n_\mu) n^* \cdot k}{n \cdot k n^* \cdot k + i\eta} \right]. \quad (3)$$

Gauge-invariant quantities should be independent of the choice of n^* as well as n . This invariance is guaranteed by the Piguet-Sibold identity⁴⁾

$$\Delta\Gamma = \Gamma \times \Gamma'. \quad (4)$$

The infinitesimal change in the generating functional of one-particle-irreducible vertices $\Delta\Gamma$ due to an infinitesimal change Δn^* in n^* is given by the BRST convolution⁵⁾ of the generating functional with the functional containing the extra vertex

$$\bar{c} \Delta N_\mu(\partial) A_\mu, \quad (5)$$

where $N(A)$ is the functional in the gauge-fixing term and $\bar{c}$ the antighost field

$$\lim_{\alpha \rightarrow 0} \frac{1}{2\alpha} [N \cdot A]^2. \quad (6)$$

Mandelstam's prescription corresponds to the choice for $N^{(6)}$ in (6):

$$N_\mu = n_\mu + i\eta n_\mu^* \quad (7)$$

and so the ghost propagator is

$$(n \cdot k + i\eta n^* k)^{-1} \quad (8)$$

and the ghost-gluon vertex is

$$n_\mu + i\eta n_\mu^*. \quad (9)$$

Leibbrandt's prescription corresponds to the gauge-fixing term⁶⁾

$$N \cdot A = n \cdot A + i\eta (n^* \partial)^{-1} \square^{-1} \partial A, \quad (10)$$

with the ghost propagator

$$(n \cdot k n^* \cdot k + i\eta)^{-1} \quad (11)$$

and the vertex

$$n^* \cdot k n_\mu + i\eta (k^2)^{-1} k_\mu. \quad (12)$$

According to Piguet and Sibold, to ensure the gauge independence, the gluon self-energy tensor to one-loop order has to satisfy the identity

$$\Delta\pi(x, y) = \frac{\delta^2 \Delta\Gamma}{\delta A(x) \delta A(y)} = \frac{\delta^2 \Gamma}{\delta A(x) \delta A(y)} \cdot \frac{\delta \Gamma'}{\delta u(x) \delta A(y)} + (x \leftrightarrow y), \quad (13)$$

where $u(z)$ is the source in Γ'

$$u_\lambda D_\lambda c. \quad (14)$$

When we Fourier transform this, we get

$$\Delta\pi_{\mu\nu}(p) = (p_\nu p_\lambda - \delta_{\lambda\nu} p^2) \Gamma'_{\lambda\nu} + (p_\nu p_\lambda - \delta_{\lambda\nu} p^2) \Gamma'_{\mu\lambda}. \quad (15)$$

2. Complete Γ' to one-loop order

Here we calculate the ultraviolet divergent part (in dimensional regularization) as well as the finite part of the two-point function under the change in $\Delta\pi^*$,

$$\frac{\delta \Gamma'}{\delta u_\lambda(x) \delta A_\mu(y)}.$$

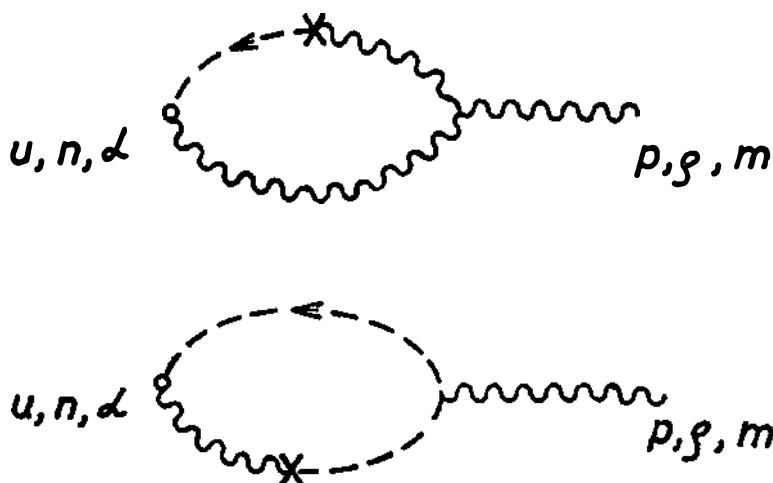


Fig. 1. The two graphs contributing to Eq. (16). The cross signifies the action of the term (5), and the circle indicates the action of the source (14). Wavy lines are gluons and dotted lines represent ghosts.

There are two graphs, shown in Fig. 1, which contribute, respectively,

$$\begin{aligned} \Gamma'(1) = & \eta g^2 C_{YM} \delta_{mn} \int \frac{d^n k}{(2\pi)^n} \frac{\Delta n^* \cdot k}{[n \cdot k \, n^* \cdot k + i\eta]^2} \cdot \frac{1}{k^2} \times \\ & \times [n_\beta (p - 2k)_\alpha + \delta_{\alpha\beta} (n \cdot k - 2p \cdot n) + n_\alpha (k + p)_\beta] \times \\ & \times \frac{1}{(p - k)^2} \left\{ \delta_{\alpha\beta} - \frac{n^* (p - k)}{n^* \cdot (p - k) n (p - k) + i\eta} \times \right. \\ & \left. \times [n_\alpha (p - k)_\beta + n_\beta (p - k)_\alpha] \right\} \end{aligned}$$

and

$$\begin{aligned} \Gamma'(2) = & \eta g^2 C_{YM} \delta_{mn} \int \frac{d^n k}{(2\pi)^n} \frac{\Delta n^* \cdot k \, n_\alpha}{[n \cdot k \, n^* \cdot k + i\eta]^2} \cdot \frac{1}{k^2} \times \\ & \times \frac{n^* (p - k) n_\alpha}{n^* (p - k) n (p - k) + i\eta}. \end{aligned} \quad (16)$$

Let us choose a frame in which

$$n = (1, 0, 0, 1), \quad n^* = (1, 0, 0, -1) \quad (17)$$

and make a Wick rotation $k_0 \rightarrow i k_4$. Then we write

$$k_\mu = (k_4; 0, 0, k_3) + K_\mu,$$

where $K_3 = K_4 = 0$ and rescale

$$k_3 \rightarrow \eta^{1/2} k_3, \quad k_4 \rightarrow \eta^{1/2} k_4.$$

Now, we perform the integration over k_3 and k_4

$$\int d k_3 d k_4 (k_4^2 + k_3^2 - i\eta)^{-2} = \frac{i\pi}{\eta} \quad (18)$$

and after a simple algebra we get

$$\begin{aligned} \Gamma' = & -g^2 C_{YM} \delta_{mn} \pi \int \frac{d^{n-2} K}{(2\pi)^n K^2} \frac{\Delta n^* \cdot K}{(p - K)^2} \times \\ & \times \left\{ -2p \cdot n [\delta_{\alpha\alpha} - \frac{1}{n \cdot p} (n_\alpha p_\alpha + n_\alpha p_\alpha - n_\alpha K_\alpha - n_\alpha K_\alpha)] + \right. \\ & \left. + n_\alpha [2K_\alpha + \frac{1}{n \cdot p} n_\alpha (K^2 - p^2)] \right\}. \end{aligned} \quad (19)$$

However, here $1/K^2$ and $1/(p-K)^2$ are still in n -dimensional space. First, we have to set them into $(n-2)$ -dimensional space, which means that

$$\begin{aligned}\frac{1}{K^2} &= -\frac{1}{\vec{K}^2}, \\ \frac{1}{(p-K)^2} &= \frac{1}{p_+p_- - (\vec{P} - \vec{K})^2}, \\ \Delta n^* \cdot \vec{P} &= -\Delta n^* \cdot p, \\ P^2 &= -\vec{P}^2.\end{aligned}\quad (20)$$

Here p_+ and p_- are Mandelstam's variables

$$\begin{aligned}p_+ &= n \cdot p, \\ p_- &= n^* \cdot p.\end{aligned}$$

It means that in $(n-2)$ -dimensional space

$$\begin{aligned}\Gamma' &= -g^2 C_{YM} \delta_{mn} \frac{\pi}{(2\pi)^n} \Delta n_\mu^* \int d^{n-2}K \left\{ \frac{K_\mu}{p_+p_- - (\vec{P} - \vec{K})^2} \cdot \frac{n_e p_\alpha}{n \cdot p} - \right. \\ &\quad - \frac{K_\mu}{\vec{K}^2} \frac{1}{p_+p_- - (\vec{P} - \vec{K})^2} \left[-2p \cdot n \delta_{e\alpha} - \frac{p^2}{n \cdot p} n_e n_\alpha + 2n_\alpha p_e + \right. \\ &\quad \left. \left. + 2n_e p_\alpha - 2n_\alpha K_e \right] \right\}.\end{aligned}\quad (21)$$

To evaluate Γ' , we need only three different integrals:

$$\begin{aligned}(1) \quad &\int d^{n-2}K \frac{K_\mu}{\vec{K}^2 - 2\vec{P} \cdot \vec{K} + \vec{P}^2 - p_+p_-} = P_\mu \left\{ \Gamma\left(\frac{4-n}{2}\right) - \ln(-p_+p_-) \right\}, \\ (2) \quad &\int d^{n-2}K \frac{K_\mu}{\vec{K}^2 - 2\vec{P} \cdot \vec{K} + \vec{P}^2 - p_+p_-} \cdot \frac{1}{K^2} = -\frac{P_\mu}{\vec{P}^2} \ln \frac{p_+p_-}{p^2}, \\ (3) \quad &\int d^{n-2}K \frac{K_\mu K_e}{\vec{K}^2} \cdot \frac{1}{\vec{K}^2 - 2\vec{P} \cdot \vec{K} + \vec{P}^2 - p_+p_-} = \frac{1}{2} \delta_{\mu e} \Gamma\left(\frac{4-n}{2}\right) +\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \delta_{\mu e} [2 - \ln(-p_+ p_-)] - \frac{1}{2} \delta_{\mu e} \frac{p^2}{\vec{p}^2} \ln \frac{p_+ p_-}{p^2} + P_\mu P_e \left\{ -\frac{1}{\vec{p}^2} + \right. \\
& \quad \left. + \frac{p^2}{\vec{p}^4} \ln \frac{p_+ p_-}{p^2} \right\}.
\end{aligned} \quad (22)$$

Owing to $\Delta n_\mu^* P_\mu = -\Delta n^* p$, Γ' becomes

$$\begin{aligned}
\Gamma' \cdot C^{-1} = & -\Delta n^* p \left\{ \frac{n_e n_\alpha}{n \cdot p} \left[\Gamma \left(\frac{4-n}{2} \right) - \ln(-p_+ p_-) \right] \right\} - \\
& - \frac{\Delta n^* p}{\vec{p}^2} \ln \frac{p_+ p_-}{p^2} \left[-2p \cdot n \delta_{e\alpha} - \frac{p^2}{n \cdot p} n_e n_\alpha + 2n_\alpha p_e + 2n_e p_\alpha \right] + \\
& + n_\alpha \Delta n_e^* \left[\Gamma \left(\frac{4-n}{2} \right) + 2 - \ln(-p_+ p_-) - \frac{p^2}{\vec{p}^2} \ln \frac{p_+ p_-}{p^2} \right] - \\
& - 2n_\alpha \frac{\Delta n^* p}{\vec{p}^2} \left(-1 + \frac{p^2}{\vec{p}^2} \ln \frac{p_+ p_-}{p^2} \right) \left(p_e - \frac{1}{2} n_e^* p_+ - \frac{1}{2} n_e p_- \right),
\end{aligned}$$

where

$$C = g^2 C_{YM} \delta_{mn} \cdot i\pi^{n/2} (2\pi)^{-n}. \quad (23)$$

We can write

$$\ln(-p_+ p_-) = -\ln(-p^2) + \ln \frac{p^2}{p_+ p_-} \quad (24)$$

and after a simple algebra we get

$$\begin{aligned}
\Gamma' C^{-1} = & - \left[\Gamma \left(\frac{4-n}{2} \right) + 2 - \ln(-p^2) + \frac{p_+ p_-}{\vec{p}^2} \ln \frac{p^2}{p_+ p_-} \right] \times \\
& \times (\Delta n^* p \frac{n_\alpha n_e}{n \cdot p} - n_\alpha \Delta n_e^*) + \frac{\Delta n^* p}{\vec{p}^2} \left[2n_\alpha p_e - n_\alpha n_e^* p_+ - n_\alpha n_e p_- + 2\vec{p}^2 \frac{n_\alpha n_e}{n \cdot p} \right] - \\
& - \frac{\Delta n^* p}{\vec{p}^2} \ln \frac{p_+ p_-}{p^2} \left[-2p \cdot n \delta_{e\alpha} - \frac{2p^2}{n \cdot p} n_e n_\alpha + 2n_\alpha p_e + 2n_e p_\alpha + \frac{p^2}{\vec{p}^2} (2n_\alpha p_e - \right. \\
& \quad \left. - n_\alpha n_e^* p_+ - n_\alpha n_e p_- + 2\vec{p}^2 \frac{n_\alpha n_e}{n \cdot p}) \right].
\end{aligned}$$

We want to check Eq. (15) which is the Piguet-Sibold equation for the gluon self-energy. With our Γ' , the right-hand side of Eq. (15) becomes

$$\begin{aligned}
 & (p_\mu p_\alpha - \delta_{\alpha\mu} p^2) \left\{ -2n_\alpha p_\varrho + n_\alpha n_\varrho^* p_+ + n_\alpha n_\varrho p_- - 2\vec{p}^2 \frac{n_\alpha n_\varrho}{p_+} \right\} + \\
 & + (\mu \leftrightarrow \varrho) = R_{u\varrho} - \frac{4T_{\mu\varrho}}{p_+}, \\
 & (p_\mu p_\alpha - \delta_{\alpha\mu} p^2) \left(\frac{n_\alpha n_\mu}{n \cdot p} \Delta n^* p - n_\alpha \Delta n_\mu^* \right) + (\mu \leftrightarrow \varrho) = -\frac{1}{p_+} \Delta R_{\mu\varrho}, \\
 & (p_\mu p_\alpha - \delta_{\alpha\mu} p^2) \left[2p \cdot n \delta_{\alpha\mu} + \frac{2p^2}{n \cdot p} n_\alpha n_\mu - 2n_\alpha p_\mu - 2n_\mu p_\alpha \right] + \\
 & + (\varrho \leftrightarrow \mu) = -\frac{4p^2}{p_+} S_{\mu\varrho}, \tag{25}
 \end{aligned}$$

where

$$\begin{aligned}
 R_{\mu\nu} &= 2p^2 p_- n_\mu n_\nu + p_+^2 (p_\mu n_\nu^* + p_\nu n_\mu^*) - p_+ p_- (p_\mu n_\nu + p_\nu n_\mu) - \\
 & - p^2 p_+ (n_\mu n_\nu^* + n_\nu n_\mu^*), \\
 S_{\mu\nu} &= p^2 n_\mu n_\nu + p_+^2 \delta_{\mu\nu} - p_+ (p_\mu n_\nu + p_\nu n_\mu), \\
 T_{\mu\nu} &= (p^2)^2 n_\mu n_\nu + p_+^2 p_\mu p_\nu - p^2 p_+ (p_\mu n_\nu + p_\nu n_\mu), \\
 Q_{\mu\nu} &= p^2 \delta_{\mu\nu} - p_\mu p_\nu. \tag{26}
 \end{aligned}$$

Finally, the r. h. s. of Eq. (15) is

$$\begin{aligned}
 & (p_\mu p_\alpha - \delta_{\alpha\mu} p^2) \Gamma'_{\alpha\nu} + (\mu \leftrightarrow \nu) = \frac{1}{p_+} \Delta R_{\mu\nu} \left[\Gamma' \left(\frac{4-n}{2} \right) + 2 - \ln(-p^2) + \right. \\
 & + \left. \frac{p_+ p_-}{p_+ p_- - p^2} \ln \frac{p^2}{p_+ p_-} \right] + \frac{\Delta n^* p}{p_+ p_- - p^2} \left[-R_{\mu\nu} + \frac{4}{p_+} T_{\mu\nu} \right] - \\
 & - \frac{\Delta n^* p}{(p_+ p_- - p^2)^2} \cdot p^2 \ln \frac{p_+ p_-}{p^2} \left[-R_{\mu\nu} + \frac{4}{p_+} T_{\mu\nu} \right] - \\
 & - \frac{\Delta n^* p}{p_+ p_- - p^2} \ln \frac{p_+ p_-}{p^2} \cdot \frac{4p^2}{p_+} S_{\mu\nu}. \tag{27}
 \end{aligned}$$

3. Complete $\pi_{\mu\nu}$ to one-loop order

In Eq. (34) of Ref. 7 we evaluated the complete self-energy tensor to one-loop order. The graph is shown in Fig. 2.

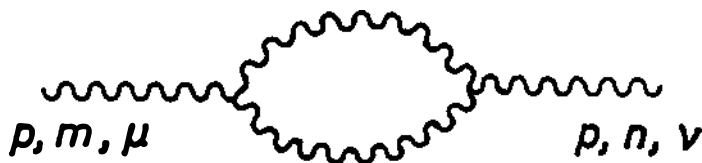


Fig. 2. The gluon self-energy.

It is

$$\begin{aligned} \pi_{\mu\nu} G^{-1} = & \frac{11}{3} \left[\Gamma \left(\frac{4-n}{2} \right) + \frac{67}{33} - \ln(-p^2) \right] Q_{\mu\nu} + \frac{1}{p_+} R_{\mu\nu} \left[\Gamma \left(\frac{4-n}{2} \right) + \right. \\ & + 2 - \ln(-p^2) + \left. \frac{p_+ p_-}{p_+ p_- - p^2} \ln \frac{p^2}{p_+ p_-} \right] - \frac{4p_-}{p_+ (p_+ p_- - p^2)} \ln \frac{p^2}{p_+ p_-} T_{\mu\nu} + \\ & + \frac{4p_-}{p_+} p^2 F(p^2, p_+ p_-) S_{\mu\nu}, \end{aligned} \quad (28)$$

where

$$\begin{aligned} F = & \int_0^1 dx \int_0^1 dy (1-y+xy)^{-1} [-p_+ p_- + y(p_+ p_- - p^2)]^{-1} \doteq \\ & \doteq \int_0^1 dy y^{-1} \ln(1-y) [p_+ p_- - y(p_+ p_- - p^2)]^{-1}. \end{aligned} \quad (29)$$

The change in Δn^* in F is

$$\Delta F = \frac{\partial F}{\partial p_+ p_-} \cdot p_+ \Delta p_- = \frac{\partial F}{\partial p_+ p_-} p \cdot n p \cdot \Delta n^*. \quad (30)$$

We introduce the variable

$$\lambda = \frac{p_+ p_-}{p_+ p_- - p^2}. \quad (31)$$

In fact, we want

$$\begin{aligned} \frac{\partial}{\partial p_+ p_-} [p_+ p_- F(p^2, p_+ p_-)] &= \frac{\partial}{\partial p_+ p_-} \left\{ \int_0^1 dy \ln(1-y) \left[\frac{1}{y} + \frac{1}{\lambda-y} \right] \right\} = \\ &= p^2 (p_+ p_-)^{-2} \lambda^{-2} \int_0^1 dy \ln(1-y) (\lambda-y)^{-2}. \end{aligned} \quad (32)$$

This we integrate by parts to get

$$(p_+ p_- - p^2)^{-1} \ln \frac{p^2}{p_+ p_-}.$$

Now it is trivial to check that the change in Δn^* of $\pi_{\mu\nu}$ in Eq. (28) is identical to Eq. (27).

In a similar way we can verify the Piguet-Sibold equation for a change in Δn .

4. Conclusion

We have verified that the dependence of the gluon self-energy on n^* is consistent with the Piguet-Sibold identity to one-loop order. We have studied ultra-violet divergent parts as well as the finite parts contributing to $\pi_{\mu\nu}$ and Γ' .

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VLASTITA ENERGIJA GLUONA U BAŽDARNOM UVJETU
SVJETLOSNOG KONUSA I PIGUET-SIBOLDOV IDENTITET

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Razmatrana je ovisnost potpunog tenzora vlastite energije gluona o drugom svjetlosnom vektoru u baždarnom uvjetu svjetlosnog konusa. Verificirani su Piguet-Siboldovi identiteti.