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STUDY OF THE GATE CAPACITANCE OF METAL-OXIDE SEMICONDUCTOR STRUCTURES OF SILICON INVERSION LAYERS

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An attempt is made to investigate the gate capacitance of metal-oxide-semiconductor structures of silicon having both p and n-channel inversion layers under weak electric field limit of the basis of newly formulated $2D$ carrier energy spectra. It is found, that the same capacitance for both type of layers increases with increasing gate voltage and surface electric field, respectively. The theoretical formulation is in good agreement with the experimental observation as reported elsewhere and the corresponding results for inversion layers on parabolic semiconductors are also obtained from the expressions derived.

1. Introduction

In recent years, there has been considerable interest in studying the gate capacitance of inversion layers in silicion metal-oxide-semiconductor (MOS) structures[1-6] and the importance of such investigations has been stated in Ref. 6. Thus it would be of much interest to formulate the gate capacitance of MOS structures of Si inversion layers by considering the non-parabolicity of the conduction band and various types of anisotropies of the valence bands, respectively. In what follows, in Section 2.1 on theoretical background we shall derive a simplified expression of the gate capacitance in p-channel inversion layers of Si by formulating the appropriate modified energy spectra of heavy, light and split-off holes, respectively in p-channel inversion layers of Si as given elsewhere[6]. In Section 2.2 we shall derive an expression of the same capacitance for n-channel inversion layers of Si by formulating the basic $2D$ electron dispersion relation taking the band non-parabolicity into account. This will make our study a generalized one, since we can obtain the corresponding results of parabolic semiconductors. However, we shall consider the weak electric field limit since the condition for such limit is usually satisfied under the range of surface field normally applied in inversion layers. We shall study the surface electric field and the gate voltage dependence of both the capacitances, respectively.

2. Theoretical background

2.1: *Formulation of gate capacitance in p-channel inversion layers of Si*

The energy spectra of heavy, light, and split-off holes in p-channel inversion layers of Si under weak electric field limit can, respectively, be expressed[6]

$$G\varepsilon_1 = (\hbar^2 k_s^2/2m_0) + [\hbar G e F_s (2m_0)^{-1/2}]^{2/3} \cdot S(i_1) \tag{1}$$

$$\alpha\varepsilon^2 + \beta[P\varepsilon_2^2 + Q\varepsilon_2 + R]^{1/2} = (\hbar^2 k_s^2/2m_0) + \\ + [e\hbar F_s/\sqrt{2m_0}]^{2/3} \cdot \left[\alpha + \frac{Q + 2P\varepsilon_2}{\sqrt{P\varepsilon_2^2 + Q\varepsilon_2 + R}}\right]^{2/3} \cdot S(i_2) \tag{2}$$

$$\alpha\varepsilon_3 - \beta[P\varepsilon_3^2 + Q\varepsilon_3 + R]^{1/2} = (\hbar^2 k_s^2/2m_0) + \\ + \left(\frac{\hbar e F_s}{\sqrt{2m_0}}\right)^{2/3} \cdot \left[\alpha - \frac{Q + 2P\varepsilon_3}{\sqrt{P\varepsilon_3^2 + Q\varepsilon_3 + R}}\right]^{2/3} \cdot S(i_3) \tag{3}$$

where the notations are defined in Ref. 6.

The use of Eqs. (1) to (3) leads to the expression of the total hole concentration per unit area in p-channel inversion layers of Si as

$$p = \frac{m_0}{\pi\hbar^2}\left[\sum_{i_1=0}^{i_{1max}} \{\varrho(E_F) + \omega(E_F)\} + \sum_{i_2=0}^{i_{2max}} \{\delta(E_F) + \lambda(E_F)\} + \right. \\ \left. + \sum_{i_3=0}^{i_{3max}} \{\zeta(E_F) + \Theta(E_F)\}\right] \tag{4}$$

where $\varrho(E_F) \equiv [G E_F - \{\hbar G e F_s (2m_0)^{-1/2}\}^{2/3} S(i_1)]$, $E_F (\equiv e V_g - p e^2 d_{ins} \cdot \cdot \varepsilon_{ins}^{-1} - E_{fb})$, is the Fermi energy as measured from the edge of the valence band at the surface, e is the carrier charge, V_g is the gate voltage, d_{ins} and ε_{ins} are the permittivity and the thickness ofthe insulating layers, respectively, E_{fb} is the energy separation between the Fermi level and the valence band edge of the bulk of the substrate material,

$$\omega(E_F) \equiv \sum_{r=1}^{s} t_r [\varrho(E_F)], \qquad t_r \equiv 2(k_B T)^{2r} \cdot (1 - 2^{1-2r}) \zeta_0(2r),$$

k_B is Boltzmann constant, T is the temperature, $\zeta_0(2r)$ is the zeta function of order $2r$[7], r is the set of real positive integers,

$$\delta(E_F) \equiv [\alpha E_F + \beta \{P E_F^2 + Q E_F + R\}^{1/2} -$$

$$- (\hbar e F_s/\sqrt{2m_0}^{2/3}) \cdot \left\{a + \frac{Q + 2P E_F}{\sqrt{P E_F^2 + Q E_F + R}}\right\}^{2/3} \cdot S(i_2)],$$

$$\lambda(E_F) \equiv \sum_{r=1}^{s} t_r [\delta^2(E_F)],$$

$$\zeta(E_F) \equiv \Bigg[\alpha E_F - \beta \{P E_F^2 + Q E_F + R\}^{1/2} - (\hbar e F_s/\sqrt{2m_0})^{2/3} \cdot S(i_3).$$

$$\left\{a - \frac{Q + 2P E_F}{\sqrt{P E_F^2 + Q E_F + R}}\right\}^{2/3} \Bigg] \quad \text{and} \quad \Theta(E_F) \equiv \sum_{r=1}^{s} t_r [\delta(E_F)].$$

The gate capacitance of MOS structures of inversion layers on semiconductors, can, in general, be expressed[4] as

$$C_g^{-1} = C_{ins}^{-1} + C_s^{-1} \tag{5}$$

where $C_{ins} (= \varepsilon_{ins}/d_{ins})$ is the fixed capacitance due to insulating layers, $C_s \left(= e \frac{dN}{dV_0}\right)$ is the surface capacitance due to space charge layer, N is the surface carrier concentration per unit area and V_0 is the surface potential.

Using equations (4) and (5) the gate capacitance of MOS structures of p-channel inversion layers of Si can be written as

$$C_{gp} = \tau_1/\tau_2 \tag{6}$$

where the functions are defined in the Appendix.

2.2: *Formulation of gate capacitance in MOS structures of n-channel inversion layers of Si*

The energy spectrum of the conduction electrons in bulk specimens of Si can be expressed[8], taking band non-parabolicity into account, as

$$\varepsilon = -\frac{E_g}{2} + \frac{\hbar^2 k_z^2}{2m_{\parallel}} + \left[\frac{E_g^2}{4} + E_g k_s^2 (\hbar^2/2m_{\perp})\right]^{1/2} \qquad (7)$$

where ε is the electron energy as measured from the edge of the conduction band, E_g is the band gap, $m_{\parallel}$ is the longitudinal effective mass at the edge of the conduction band, $k_s^2 = k_x^2 + k_y^2$ and $m_{\perp}$ is the transverse effective mass at the edge of the conduction band. Thus using the method as given elsewhere[5] the dispersion relation of the $2D$ electrons in n-channel inversion layers of Si can be written as

$$\frac{\hbar^2 k_x^2}{2m_{\perp}} + \frac{\hbar^2 k_y^2}{2m_{\parallel}} = \gamma(E, n) \qquad (8)$$

where $\gamma(E, n) \equiv [E(1 + a_0 E) + a_0 Z^2(n) - Z(n)(1 + 2a_0 E)]$, ε is the electron energy as measured from the edge of the conduction band at the surface, $Z(n) \equiv [\hbar e F_s / \sqrt{2m_{\perp}}]^{2/3} \cdot S(n)$, $n (= 0, 1, 2)$ is the electric subband index of the $2D$ electrons and $a_0 \equiv 1/E_g$. The use of equation (8) leads to the expression of electron concentration per unit area as

$$n_0 = l_0 k_B T \sum_{n=0}^{n_{max}} [(1 + 2a_0 \varepsilon_n - 2a_0 Z(n)) F_0(\eta_n) + 2a_0 k_B T F_1(\eta_n)] \qquad (9)$$

where $l_0 \equiv g_v \sqrt{m_{\parallel} m_{\perp}}/\pi \hbar^2$, g_v is the valley degeneracy, ε_n can be determined from the equation $\gamma(\varepsilon_n, n) = 0$, $F_j(\eta_n)$ is the one parameter Fermi-Dirac integral of order j as defined by Blakemore[9], $\eta_n \equiv (k_B T)^{-1} [E_{Fn} - \varepsilon_n]$, $E_{Fn} (\equiv e V_g - n_0 e^2 d_{ins} \varepsilon_{ins}^{-1} - E_{fbn})$ is the Fermi energy of the $2D$ electrons as measured from the edge of the conduction band at the surface and E_{fbn} is the energy separation between the Fermi level and the conduction-band edge of the bulk of the substrate material. Using equations (9) and (5), the gate capacitance in n-channel inversion layers of Si can be expressed as

$$C_{gn} = e^2 l_0 \tau_3 / \tau_4 \qquad (10)$$

where the notations are defined in Appendix.

Special cases:

Under the conditions $\Delta = 0$, $G = 1$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = E$, $i_1 = i_2 = i_3 = i$, $m_0 = m^*$ (the isotropic effective electron mass for parabolic energy bands), and $\frac{B}{2} + A \pm \frac{B\sqrt{5}}{2} = \frac{\hbar^2}{2m^*}$, equations (1) to (3) get simplified as[6]

$$E = \frac{\hbar^2 k_s^2}{2m^*} + \left[\frac{\hbar e F_s}{\sqrt{2m^*}}\right]^{2/3} \cdot S(i) \qquad (11)$$

which is the well-known dispersion relation of the $2D$ electrons in inversion layers on semiconductors[4]. Furthermore under the conditions, $E_g \to a$, $m_{||} = m_{\perp} = m^*$ and $n = i$, Eq. (7) reduces to Eq. (11). Thus the expressions for the carrier concentration and the gate capacitance for MOS structures of parabolic inversion layers assume the well-known forms as[4]

$$N = l_0 k_B T \sum_{i=0}^{i_{max}} F_0(\eta_i) \tag{12}$$

and

$$C_g = l_0 e^2 \left[\sum_{i=0}^{i_{max}} F_{-1}(\eta_i)\right] \left[1 + \sum_{i=0}^{i_{max}} F_{-1}(\eta_i)\left\{\frac{e^2 d_{ins}}{\varepsilon_{ins}} + \frac{2}{3}\frac{H(i)}{N}\right\}\right]^{-1} \tag{13}$$

where

$$\eta_i \equiv (k_B T)^{-1} [E_F - E_i], \qquad E_i \equiv \left[\frac{\hbar e F_s}{\sqrt{2m^*}}\right]^{2/3} S(i)$$

and

$$H(i) \equiv \left[\frac{3}{2} \pi e^2 N \hbar / \varepsilon_{sc} \sqrt{2m^*}\right]^{2/3}.$$

3. *Results and discussion*

Using the equations (4) and (6) and taking the parameters[6]

$$A = \frac{-4.28\,\hbar^2}{2m_0},\ B = \frac{-0.75\,\hbar^2}{m_0},\ \Delta = 0.03\,\text{eV},\ p = 10^{14}\,\text{m}^{-2},\ \varepsilon_{sc} = 11.8\,\varepsilon_0,$$

$$\varepsilon_{0x} = 3.8\,\varepsilon_0, \qquad T = 4.2\,\text{K}, \qquad d_{0x} = 15\,\mu\text{m}$$

as appropriate of p-Si we have plotted the gate capacitance of p-channel inversion layers on Si as a function of gate voltage as shown in plot A of Fig. 1. Using Eqs. (11) and (9) and taking the additional parameters[4] $g_v = 2$, $m_{\perp}^* = 0.916\,m_0$, $E_g = 1.1$ eV, $n_0 = 10^{14}\,\text{m}^{-2}$ and $m_{||} = 0.2\,m_0$ we have plotted the same dependence for n-channel inversion layers of Si as shown in (B) of Fig. 1 in which the same dependence for parabolic energy bands has also been plotted for the purpose of comparison. Using the same procedure as used in obtaining Fig. 1 we have plotted the above capacitances in accordance with various band models as a function of surface electric field as shown in Fig. 2. It appears from Fig. 1 that the gate capacitance increases with increasing gate voltage for both layers and the variations shown in the figure are in good agreement with the experimental observation as reported elsewhere. It appears from Fig. 2 that the gate capacitances increase with increasing surface electric field showing a tendency of saturation at relatively high values of the surface electric field. Since most of the carriers occupy the lowest electric subband at low temperatures, where the quantum effects become prominent, it is sufficiently accurate for such temperatures to consider the occupation of the lowest electric subband for the subsequent numerical computations[10]. Finally it may be noted that though in a more rigorous treatment the method of achieving the self-consistent solution, the many-body effects, effects of surface states and charges, the mixing of holes and the formation of band tails should properly be considered, this simplified triangular potential well approximation exhibits the basic features of gate capacitance of MOS structures.

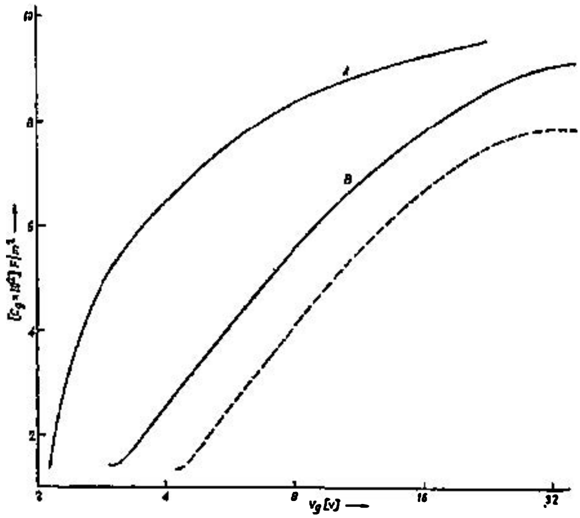

Fig. 1. The graph A is the plot of gate capacitance in p-channel inversion layers of Si in the weak electric field limit as a function of gate voltage. Plot B exhibits the same capacitance for n-channel inversion layers. The dotted plot corresponds to inversion layers on parabolic semiconductors.

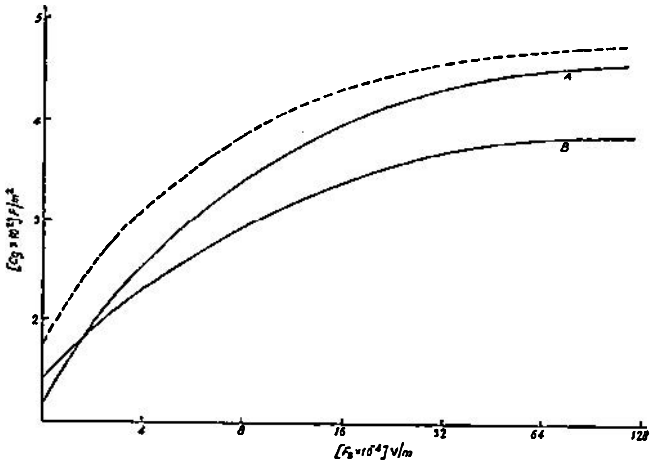

Fig. 2. The graph A is the plot of gate capacitance in p-channel inversion layers of Si in the weak electric field limit as a function of surface electric field. Plot B exhibits the same capacitance for n-channel inversion layers. The dotted plot corresponds to inversion layers on parabolic semiconductors.

Appendix

The functions τ_1, τ_2, τ_3 and τ_4 are defined as follows:

$$\tau_1 = \frac{m_0 e^2}{\pi \hbar^2} \left[\sum_{i_1=0}^{i_{1max}} \{G - t_{r+1} \varrho (E_F)\} + \sum_{i_2=0}^{i_{2max}} \left\{ a + \right.\right.$$

$$\left. + \frac{\beta}{2} g_{1,1} - L_2 (a + g_{1,1})^{-1/3} \cdot \left(2P g_{0,1} - \frac{1}{2} g_{2,3} \right) \right\} +$$

$$\left. + \sum_{i_3=0}^{i_{3max}} \left\{ a - \frac{\beta}{2} g_{1,1} - L_3 (1 - g_{1,1})^{-1/3} \cdot \left(\frac{1}{2} g_{2,3} - 2P g_{0,1} \right) \right\} \right] \quad (1.1)$$

$$\tau_2 = \left[1 + \frac{m_0}{\pi \hbar^2} \sum_{i_1=0}^{i_{1max}} \left\{ \frac{G\, d_{ins}\, e^2}{\varepsilon_{sc}} + \frac{2}{3} G^{2/3} L_1 p^{-1} + \right.\right.$$

$$\left. + \frac{d_{ins}\, e^2}{\varepsilon_{ins}} \left\{ \sum_{r=1}^{s} t_{r+1}\, \varrho (E_F) \right\} \right\} + \frac{e^2 d_{ins} m_0}{\varepsilon_{ins} \pi \hbar} \sum_{i_2=0}^{i_{2max}} \left\{ a + \right.$$

$$\left. + \frac{\beta}{2} g_{1,1} - L_2 (a + g_{1,1})^{-1/3} \left(2P g_{0,1} - \frac{1}{2} g_{2,3} \right) \right\} + \frac{2}{3} \frac{m_0}{\pi \hbar^2 p} \cdot$$

$$\cdot \left\{ \sum_{i_2=0}^{i_{2max}} L_2 (1 + g_{1,1})^{2/3} \right\} + \frac{e^2 d_{ins} m_0}{\varepsilon_{sc} \pi \hbar^2} \left\{ \sum_{i_2=0}^{i_{2max}} \sum_{r=1}^{s} t_{r+1}\, \delta E(_F) \right\} +$$

$$+ \frac{m_0 e^2}{\pi \hbar^2} \frac{d_{ins}}{\varepsilon_{ins}} \sum_{i_3=0}^{i_{3max}} \left\{ a - \frac{\beta}{2} g_{1,1} - L_3 (1 - g_{1,1})^{-1/3} \left(-2P g_{0,1} + \right.\right.$$

$$\left.\left. + \frac{1}{2} g_{2,3} \right) \right\} + \frac{2m_0}{3\pi \hbar^2 \varphi} \sum_{i_3=0}^{i_{3max}} L_3 (a - g_{0,1})^{2/3} +$$

$$\left. + \frac{e^2 d_{ins} m_0}{\varepsilon_{sc} \pi \hbar^2} \left\{ \sum_{i_3=0}^{i_{3max}} \sum_{r=0}^{s} t_{r+1}\, \zeta (E_F) \right\} \right] \quad (1.2)$$

$$\tau_3 = \sum_{n=0}^{n_{max}} \left[[(1 + 2a_0 \varepsilon_n - 2a_0 Z(n))] F_{-1} (\eta_n) + 2a_0 k_B T F_0 (\eta_n) \right] \quad (1.3)$$

$$\tau_4 = 1 + l_0 \sum_{n=0}^{n_{max}} [\{1 + 2a_0 \varepsilon_n - 2a_0 Z(n)\} F_{-1} (\eta_n) +$$

$$+ 2a_0 k_B T F_0 (\eta_n)] \left[\frac{e^2 d_{ins}}{\varepsilon_{ins}} + \frac{2}{3} \frac{Z(n)}{n_0} \right] \quad (1.4)$$

where

$$g_{i,j} = \frac{(2PE_F + Q)^i}{(PE_F^2 + QE_F + R)^{j/2}}, \quad L_r \equiv \left(\frac{\hbar e F_s}{\sqrt{2m_\perp}}\right)^{2/3} S(i_r)$$

i, j and r are real numbers.

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ISTRAŽIVANJE KAPACITETA UPRAVLJAČKE ELEKTRODE MOS STRUKTURE Si S INVERZIONIM SLOJEVIMA

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Razmatran je kapacitet upravljačke elektrode MOS strukture Si sa p i n inverzionim slojevima u granici slabih polja. Pri tome je korištena nova formulacija za energetski spektar nosilaca naboja u dvije dimenzije. Nađeno je da kapacitet za oba inverziona sloja raste s naponom na elektrodi i porastom površinskog električnog polja. Teorijski opis u dobrom je slaganju s eksperimentalnim opažanjima što su objavljena u drugim radovima. Dobiveni su i rezultati za paraboličnu vrpcu.