

GAJA ALAGA MEMORIAL VOLUME\*

THE SPECTROSCOPY OF THE ODD-*A* Ra ISOTOPES IN THE OCTUPOLE  
DEFORMED REGION

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Odd-*A* Ra spectroscopy is surveyed from  $^{217}\text{Ra}$  to  $^{227}\text{Ra}$  with a view toward understanding the effect of octupole deformation on the transition from spheric to quadrupole deformed Ra nuclei.  $^{221}\text{Ra}$ ,  $^{223}\text{Ra}$  and  $^{225}\text{Ra}$  are found to be the best examples of normal octupole deformation although definite differences exist in their low lying parity doublet bands — particularly the Coriolis coupling of their  $K = 3/2^\pm$  and  $1/2^\pm$  bands. The spectroscopy of  $^{217}\text{Ra}$  and  $^{219}\text{Ra}$  represents a transition from shell model spectroscopy with octupole correlations into an octupole deformed system which can describe the apparent weak coupling. The spectroscopy of  $^{227}\text{Ra}$  on the other side of the octupole deformed nuclei suggests the coexistence of parity doublet bands expected of an octupole deformed system and octupole vibrational bands of the normal type without octupole deformation.

### 1. Introduction

This article is written to honor Professor Gaja Alaga. He not only was a brilliant theoretical physicist, but also a very warm and caring human being.

In this article we will concern ourselves with a «cut» through the region of the highest octupole deformation along the Ra isotopes line. We will use only the odd-*A* Ra isotopes because the last odd neutron in these odd-*A* Ra isotopes acts

\* Owing to a grave technical error this paper was inadvertently left out of the Gaja Alaga Memorial Volume (*Fizika* 22, 1, 1990).

as a probe to the nuclear structure and thereby gives us information that would not be available from the even- $A$  Ra isotopes.

Our purpose in this study is to gain as much insight as possible from the new physical situations which occur from the addition symmetry breaking resulting from reflection asymmetry. With this additional insight we hope to better understand the transition from spherical to quadrupole deformation from the neutron region just beyond the 126 neutron shell closure to the neutron region of moderate quadrupole deformation and minimal octupole deformation.

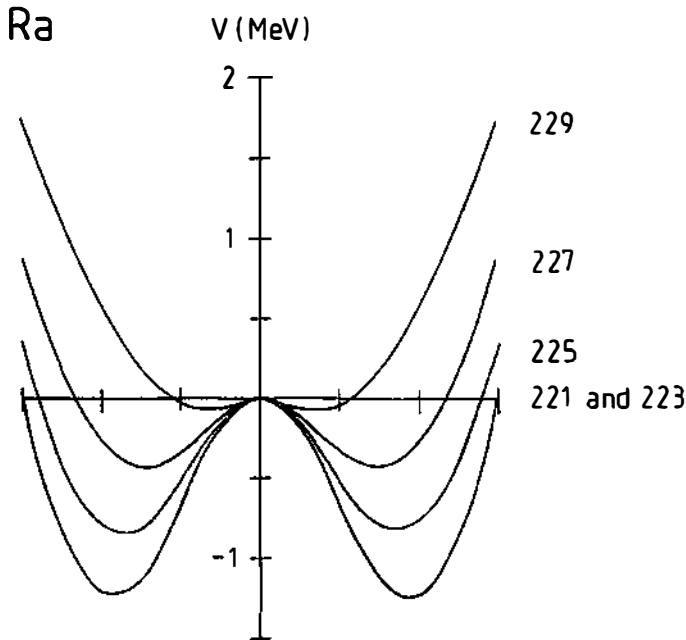


Fig. 1. The potential energy curves for the odd- $A$  Ra isotopes plotted against octupole deformation. Each curve follows the approximate<sup>2)</sup> minimal path in the  $\epsilon_3, \epsilon_4$  plane. Among the odd- $A$  Ra isotopes shown in this figure  $^{221}$  and  $^{223}$ Ra are the most octupole deformed, whereas  $^{229}$ Ra is the least.

Fig. 1 gives the potential energy for some of the odd- $A$  Ra isotopes as a function of the octupole deformation,  $\epsilon_3$ <sup>1)</sup>. This figure is an interpolation between neighboring even- $A$  potential energy curves obtained using the folded Yukawa single particle model. It should be noted that these one-dimensional potential diagrams have mirror minima with in most cases appreciable barriers between the minima. It is clear that  $^{221}$ Ra,  $^{223}$ Ra and  $^{225}$ Ra have the highest barriers between the mirror minima and should be the best examples of octupole deformation among the odd- $A$  Ra isotopes,  $^{227}$ Ra with a barrier  $< 0.5$  MeV between the mirror minima (Fig. 1) is the heaviest odd- $A$  Ra isotope where much experimental data are available, and is the heaviest Ra isotope considered in these studies. On the other hand, both  $^{217}$ Ra and  $^{219}$ Ra with 129 and 131 neutrons respectively, not shown in Fig. 1, are particularly interesting because they should represent the transition from a shell model spectroscopy into a modest quadrupole

spectroscopy both modified by octupole correlation. Although there is considerable ambiguity in the spectra of these nuclei, they will be considered in this paper.

The intrinsic single particle states with static reflection asymmetry are described in section 2. Section 3 presents in turn the spectra of each of the odd- $A$  nuclei from  $^{217}\text{Ra}$  through  $^{227}\text{Ra}$  with comparisons to the single neutron orbitals of Section 2. The various spectra of the odd- $A$  Ra nuclei are compared and contrasted in Section 4 with an attempt to obtain a coherent overall picture.

## 2. The intrinsic single particle neutron states with reflection asymmetry for $A \sim 219-229$

The calculation<sup>1)</sup> of the intrinsic single particle neutron states involves the coupling of a quasi particle to a rotor core with  $K = 0$ , but with an additional intrinsic reflection asymmetric degree of freedom. The coupling between the particle and the reflection asymmetric field is strong in this model, and therefore the basis states themselves are parity mixed. The coupling to octupole deformation is stronger in a potential with the more realistic flat bottomed radial shape<sup>2)</sup>. Therefore the folded Yukawa single particle potential<sup>3,4)</sup> is used to calculate the intrinsic single particle states instead of the modified oscillator potential of Nilsson. The single particle wave functions were calculated with the hexadecapole coordinate  $\varepsilon_4$  determined as a function of  $\varepsilon$  and  $\varepsilon_3$  by minimizing the potential energy for the appropriate Ra and Th isotope. This table is given as Table 1 in Ref. 1.

The single neutron levels for the folded Yukawa potential are plotted in Fig. 2 as a function of the quadrupole deformation for  $\varepsilon_3 = 0.08$ . These asymmetric levels are labeled by  $\Omega$  which is the total angular momentum component along the symmetry axis and in parentheses by a set of single particle matrix elements:  $\langle \hat{s}_z \rangle$ ,  $\langle \hat{\pi} \rangle$  and if  $\Omega = 1/2$  also  $\langle \pi \text{ conj} | -\hat{j}_+ | R \text{ conj} \rangle$ . These reflection asymmetric Nilsson orbits are usually strongly parity mixed. Often they will contain approximately equal admixtures of two reflection symmetric orbitals with the same  $\Omega$  but opposite parities which lie close together in energy for  $\varepsilon_3 = 0$ . It should be noted that each of these levels gives rise to two bands of opposite parity which are known as parity doublets. Furthermore, the effect of intrinsic reflection asymmetry on the levels is great enough that the experimentally observed levels can only be explained by the Nilsson levels with reflection asymmetry (like Fig. 2) and not by the reflection symmetric Nilsson levels (See Refs. 5 and 6).

The quadrupole deformations ( $\varepsilon$ ) used with Fig. 2 in the interpretations are a monotonically increasing function of increasing Ra mass number. They are (with  $\varepsilon$  in parentheses)  $^{217}\text{Ra}$  (0.08);  $^{219}\text{Ra}$  (0.10);  $^{221}\text{Ra}$  (0.11);  $^{223}\text{Ra}$  (0.13);  $^{225}\text{Ra}$  (0.15);  $^{227}\text{Ra}$  (0.18).

## 3. The spectroscopy of the odd- $A$ nuclei $^{217}\text{Ra}$ , $^{219}\text{Ra}$ , $^{221}\text{Ra}$ , $^{223}\text{Ra}$ , $^{225}\text{Ra}$ and $^{227}\text{Ra}$

In this section we will discuss the spectroscopy of the odd- $A$  Ra nuclei from  $^{217}\text{Ra}$  through  $^{227}\text{Ra}$  in order of increasing mass number. We leave  $^{215}\text{Ra}$  out

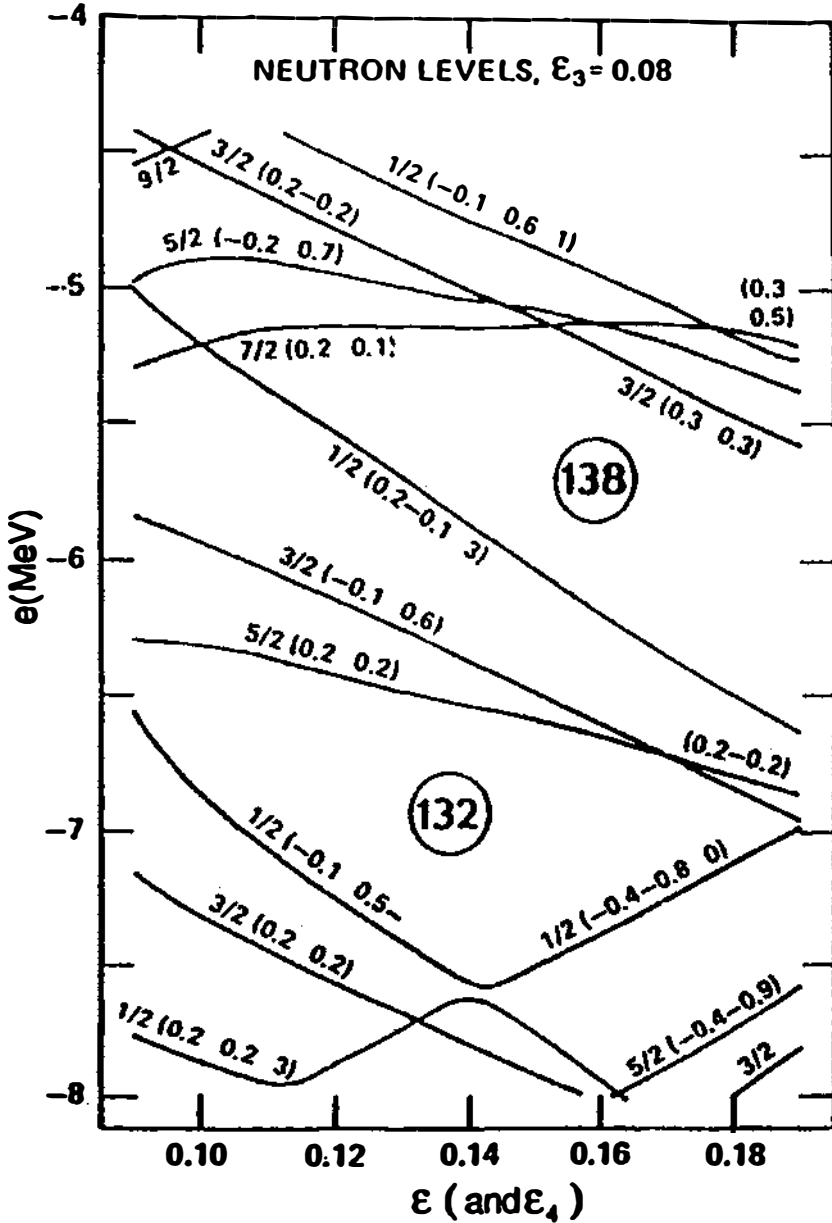


Fig. 2. The single neutron orbitals calculated with an axially symmetric but reflection asymmetric folded Yukawa potential ( $\epsilon_3 = 0.08$ ) plotted versus the quadrupole deformation coordinate  $\epsilon^{(1)}$ . The labeling of the orbitals is described in the text.

of our discussion because its spectroscopy is assumed to be explained on the basis of the coupling of the 127th neutron in shell model orbitals  $g_{9/2}$ ,  $i_{11/2}$ ,  $j_{15/2}$  ... coupled to the proton configuration  $(h_{9/2})^6 \equiv (h_{9/2})^{-4}$  and, as we shall see, a similar explanation is used for  $^{217}\text{Ra}$ . However, in  $^{217}\text{Ra}$  the presence of three neutrons beyond the closed shell enhances octupole correlation effects.

#### A The spectroscopy of $^{217}\text{Ra}$

$^{217}\text{Ra}$  has been studied by the heavy ion reaction  $^{208}\text{Pb} (^{12}\text{C}, 3n) ^{217}\text{Ra}^{\gamma}$  and by the alpha decay of  $^{221}\text{Th}^{8,9}$ . Unfortunately, the spectroscopies from these two different methods of study have not been related. The ground state of  $^{217}\text{Ra}$  is almost certainly  $9/2^+$  <sup>10</sup> because it alpha decays with a hindrance factor of 2.3 to the  $9/2^+$  ground state of the 127 neutron  $^{213}\text{Ra}$  daughter. Fig. 3 is a speculative attempt to synthesize the rather meager data from the alpha decay of  $^{221}\text{Th}$  into  $^{217}\text{Ra}$  with the heavy ion spectroscopy. The very close agreement between the energies of the first excited state observed in the heavy ion spectroscopy (330.8 keV;  $J^\pi = 11/2^+$ ) and in the alpha decay (331 keV) suggests that these states are identical. It is, however, somewhat surprising that the  $^{221}\text{Th}$  ground state which has been tentatively assigned <sup>6</sup> spin-parity  $7/2^+$  would populate the  $9/2^+$  ground state with hindrance factor 49 while populating the first excited state with spin  $11/2^+$  with hindrance factor 4.1 <sup>11</sup>. The most reasonable explanation is that the ground state of the  $^{221}\text{Th}$  alpha decaying parent is the  $7/2$  member of the  $(1/2 (-0.1; 0.5; -2))$  band (see Fig. 2) which decays with low hindrance factors to other members of this band in  $^{217}\text{Ra}$ . Thus, probably the 330.8 and 756 keV states can be assigned as the  $11/2^+$  and  $7/2^+$  members of the  $1/2 (-0.1; 0.5; -2)$  band in  $^{217}\text{Ra}$ . On the other hand, the  $9/2^+$  ground state in  $^{217}\text{Ra}$  probably arises from the  $1/2 (0.2; 0.2; 3)$  band (see Fig. 2) and therefore is populated with a much higher hindrance factor. From the shell model point of view this is equivalent to saying that the  $7/2^+$  ground state of  $^{221}\text{Th}$  arises from the  $i_{11/0}$  shell model orbital, as do the 330.8 and 756 keV states in  $^{217}\text{Ra}$ , while the  $9/2^+$  ground state in  $^{217}\text{Ra}$  arises from the  $g_{9/2}$  shell model orbital.

We can stretch this hypothesis even further in explaining the level scheme of Fig. 3. The ground state band with spin  $9/2^+$ , having arisen from the orbital  $1/2 (0.2; 0.2; 3)$  with a large decoupling parameter, will have the spins  $9/2^+$ ,  $13/2^+$ ,  $17/2^+$ ,  $21/2^+$  low, while the spins with the opposite signature will lie much higher. On the other hand, the parity doublet band with a large negative decoupling parameter will have the  $15/2^-$ ,  $19/2^-$ ,  $23/2^-$  ... low. One is forced to ask the question where are the  $11/2^-$ ,  $7/2^-$  states. Of course, if the negative decoupling parameter is large enough, these states would lie higher in the level scheme. In the same way, the  $11/2^+$ ,  $15/2^+$ ,  $19/2^+$  ... band arises from the  $1/2 (-0.1; 0.5; -2)$  orbital with a large negative decoupling parameter, which explains why this signature lies low. It would be of great interest to experimentally observe the negative parity doublet band which goes with the  $11/2^+$  band.

An alternative, and probably somewhat better, way to describe the bands in  $^{217}\text{Ra}$  is to use the shell model. In this case the appropriate shell model configurations are  $\nu g_{9/2}^3$ ,  $\nu g_{9/2}^2 i_{11/2}$ ,  $\nu g_{9/2}^2 j_{15/2}$  and  $\nu i_{11/2}^2 g_{9/2}$ . Comparison of the experimental bands with the theoretical bands in terms of these shell model configurations is shown in Fig. 4 <sup>7</sup>. The fit between experiment and theory is only qualitative, and suggests that important octupole correlations have not been success-

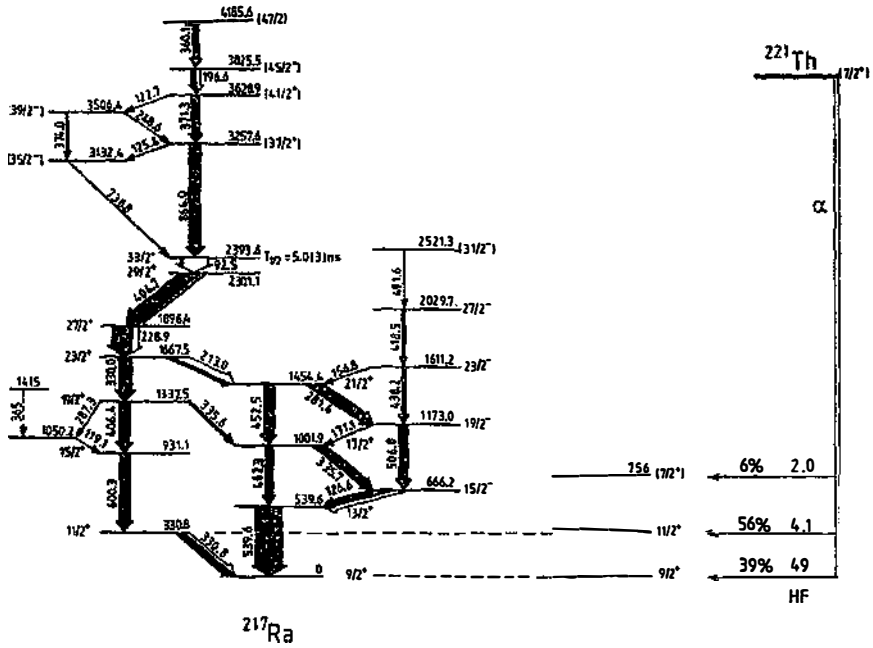


Fig. 3. The level structure of  $^{217}\text{Ra}$ . To the left, the level structure from heavy ion spectroscopy is presented<sup>7)</sup>; to the right, the level spectrum from the alpha decay of  $^{221}\text{Th}^{8,9)}$ .

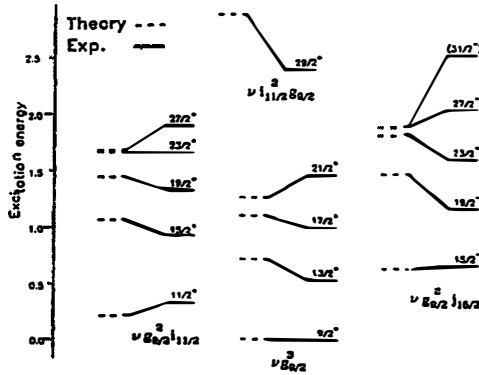


Fig. 4. A comparison of the experimental level energies of  $^{217}\text{Ra}$  with those calculated for pure shell model configurations<sup>7)</sup>.

fully taken into account. The real situation probably lies somewhere in between the simple shell model and the simple octupole deformed model descriptions. An additional indication of strong octupole correlations is the occurrence of strong *E1* transitions between the bands, much as are observed between parity doublet bands in the more usual octupole deformed region.

### B The spectroscopy of $^{219}\text{Ra}$

$^{219}\text{Ra}$  has been studied by heavy ion spectroscopy<sup>12,13)</sup> and also by the alpha decay of  $^{223}\text{Th}$ <sup>14)</sup>. It is of considerable interest that although a large number of transitions has been observed in each of these two types of studies, there is no overlap in the transition energies in these two studies. In part, this is because the heavy ion spectroscopy emphasizes high spin states, whereas the alpha spectroscopy from the  $^{223}\text{Th}$  ground state with presumed spin  $5/2^+$  emphasizes relatively low spin states. In Fig. 5 an attempt is made to synthesize the results from the alpha decay spectroscopy and the heavy ion spectroscopy. This combined level scheme must, however, be considered quite speculative in view of the fact that not a single transition in the two types of study overlaps. The ground state

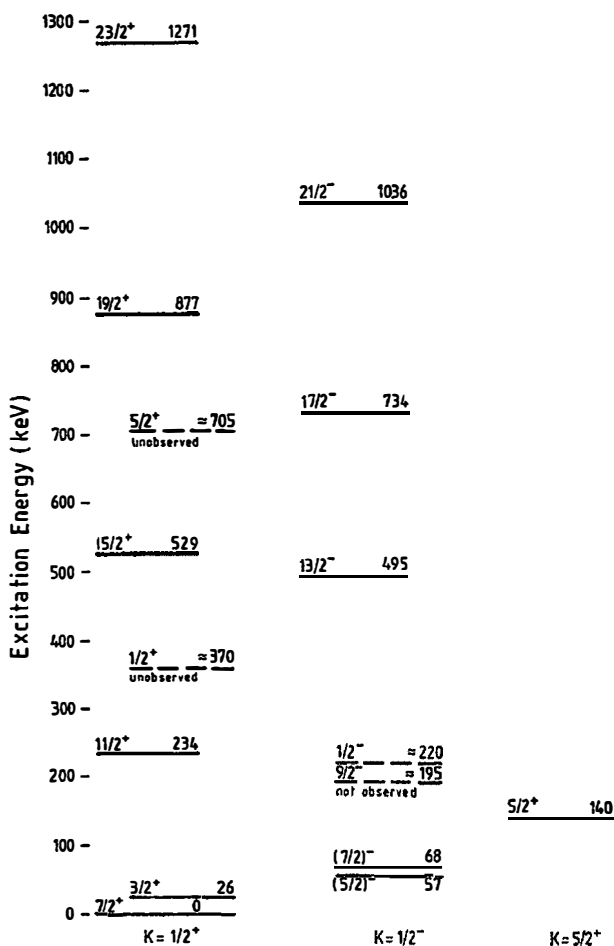


Fig. 5. A tentative composite of the low lying spectrum of  $^{219}\text{Ra}$  from the heavy ion spectroscopy<sup>12,13)</sup> and from alpha decay spectroscopy<sup>14)</sup>.

spin of  $^{219}\text{Ra}$  has been suggested as  $7/2^+$  both on the basis of the alpha decay study<sup>14)</sup> and on a theoretical basis<sup>6)</sup>. However, the most recent heavy ion study suggests spin  $9/2^+$  based on systematics<sup>13)</sup>. Since, however, the  $^{219}\text{Ra}$  ground state alpha decays to a  $9/2^+$  ground state in  $^{215}\text{Rn}$  with a hindrance factor of 110, it seems very unlikely that the ground state spin of  $^{219}\text{Ra}$  is  $9/2^+$ . A ground state spin of  $7/2^+$  in  $^{219}\text{Ra}$  may be understood as the lowest lying member of a  $1/2^+$  band from the octupole deformed orbital  $1/2 (-0.1; 0.5; -2)$ . This orbital has a large negative decoupling parameter which will become even more negative as the quadrupole deformation decreases. This could naturally, then give a  $7/2^+$  ground state with a fairly near lying  $3/2^+$  excited state, as suggested in Fig. 5. The large negative decoupling parameter would then give rise to excited states  $11/2^+$ ,  $15/2^+$ ,  $19/2^+$ , as suggested in Fig. 5. Thus we have decreased all spin assignments by  $1\hbar$  when compared with the spectroscopic assignments from the heavy ion reactions of Ref. 13. Because of the large negative decoupling parameter, the other members of the rotational band with opposite signature will lie much higher in energy and will not be observed. The states of negative parity are naturally explained, then, as the parity doublet band with large positive decoupling parameter, which will give rise to the states  $9/2^-$ ,  $13/2^-$ ,  $17/2^-$ ,  $21/2^-$  ... The states with spin  $11/2^-$ ,  $15/2^-$ ,  $19/2^-$  ... will lie much higher in energy and will not easily be populated in the heavy ion reactions. It would be of considerable interest to have very careful alpha spectroscopy of  $^{223}\text{Th}$  carried out in order to see the complex alpha spectra and the corresponding gamma transitions which must be present in the low energy spectrum of  $^{219}\text{Ra}$ .

### C. The spectroscopy of $^{221}\text{Ra}$

The ground state spin of  $^{221}\text{Ra}$  has been measured by co-linear laser spectroscopy<sup>15)</sup> to be  $5/2$ . The presence in the same measurements of large positive quadrupole moments implies that we are in the region of strong coupling where  $\Delta I = \pm 1$  sequences can be expected rather than the  $\Delta I = 2$  sequences observed in  $^{217}\text{Ra}$  and  $^{219}\text{Ra}$  above. Recently, mass separated sources of  $^{225}\text{Th}$  have been used to study the level structure of  $^{221}\text{Ra}$  following alpha decay<sup>16-18)</sup>. The final level scheme which evolved from these studies<sup>18)</sup> is shown in Fig. 6.

In contrast to the spectra of  $^{217}\text{Ra}$  and  $^{219}\text{Ra}$ , the level structure of  $^{221}\text{Ra}$  can be understood quite directly from the axially symmetric but reflection asymmetric orbital scheme shown in Fig. 2. Assuming a quadrupole deformation  $\varepsilon \cong \pm 0.1$ , the 133rd neutron in  $^{221}\text{Ra}$  is expected to occupy the  $5/2 (0.2; 0.2)$  orbital in its ground state according to Fig. 2. So we have a natural explanation for the  $K = 5/2^\pm$  parity doublet band in Fig. 6.

According to Fig. 2, the first excited parity doublet band should result from the  $3/2 (-0.1; 0.6)$  orbital. Although, as we shall see, this gives rise to quite normal  $K = 3/2^\pm$  parity doublet bands in  $^{223}\text{Ra}$  and  $^{225}\text{Th}$ , the band structure in  $^{221}\text{Ra}$  is obviously anomalous. Fortunately, there is a natural explanation for these anomalies in terms of the Coriolis coupling of the  $K = 3/2^\pm$  bands with  $K = \pm 1/2^\pm$  bands expected in the near vicinity according to Fig. 2. One expects to observe the  $K = 1, 2^\pm$  parity doublet band arising from the hole orbital  $1/2 (-0.1; 0.5; -2)$  very near to the  $K = 3/2^\pm$  bands in  $^{221}\text{Ra}$ . This orbital, which is the ground state of  $^{219}\text{Ra}$ , has a negative decoupling parameter and the probable sequence  $7/2^+$ ,  $3/2^+$ ,  $11/2^+$ ,  $1/2^+$  ... Similarly, the negative parity  $K = 1/2$  band

should have a large positive decoupling parameter with a possible sequence  $5/2^-$ ,  $9/2^-$ ,  $1/2^-$  ... Furthermore, these  $K = 1/2^\pm$  bands have a large matrix element for coupling with the  $K = 3/2^\pm$  bands. Assuming that the  $K = 1/2^+$  band lies above the  $K = 3/2$  bands, the  $7/2^+$  member of the  $K = 3/2^+$  band and the  $5/2^-$  member of the  $K = 3/2^-$  band will be forced down through Coriolis mixing. One may ask why the  $K = 3/2^\pm$  bands are not assigned as  $K = 1/2^\pm$  bands. The hindrance factor of 1.9 for the population of the  $3/2^+$  state at 321 keV in  $^{221}\text{Ra}$

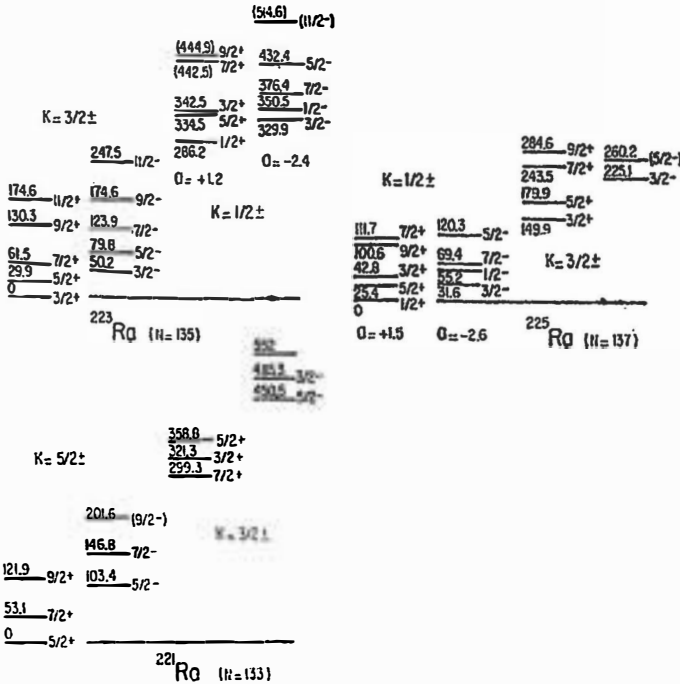


Fig. 6. The experimental level scheme of  $^{221}\text{Ra}$  as observed in the alpha decay of  $^{225}\text{Th}$ . Hindrance factors (HF) are shown to the right of the level scheme.

by the ground state  $3/2^+$  alpha decaying  $^{225}\text{Th}$  parent clearly implies that since the parent ground state results from the  $3/2$  ( $-0.1; 0.6$ ) orbital, then the  $^{221}\text{Ra}$  daughter state must also arise from this orbital. It, of course, would be of great interest to experimentally observe the  $K = 1/2^\pm$  states, not observed in these experiments, which must be postulated in order to explain the anomalous structure of the  $K = 3/2^\pm$  bands.

D. The spectroscopy of  $^{223}\text{Ra}$

The ground state spin of  $^{223}\text{Ra}$  has been measured by laser spectroscopy to be  $3/2^{15)}$ . In the same measurements a large positive quadrupole moment was determined for the ground state. This suggests that like  $^{221}\text{Ra}$ ,  $^{223}\text{Ra}$  is in a region of strong coupling and that the  $K^\pi = J^\pi$  in the ground state of this nucleus.

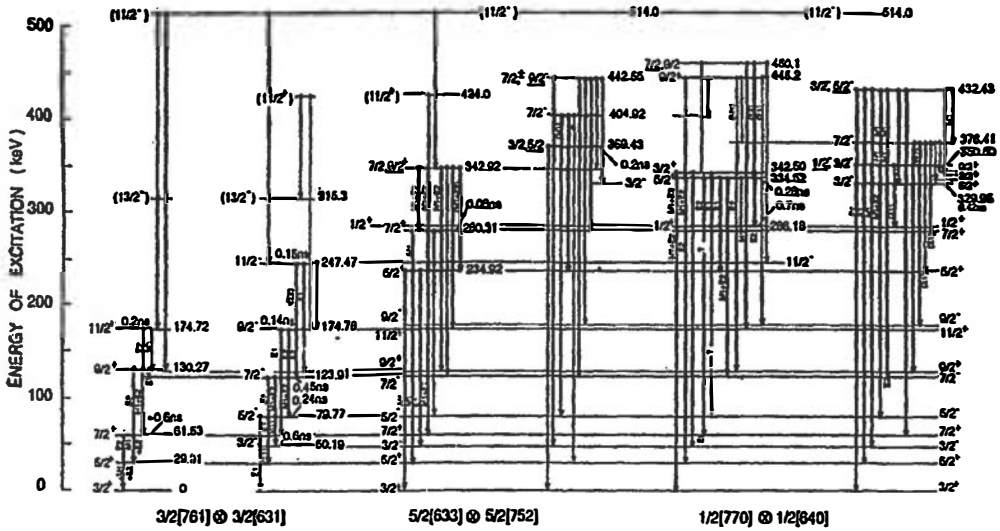


Fig. 7. The experimental level structure of  $^{223}\text{Ra}$ . The data are taken from Ref. 11. The states are divided into three sets of parity doublet bands. Tentative transitions are shown dotted. All observed states up to 514.0 keV have been included except the tentative state at 105.0 keV which is probably spurious.

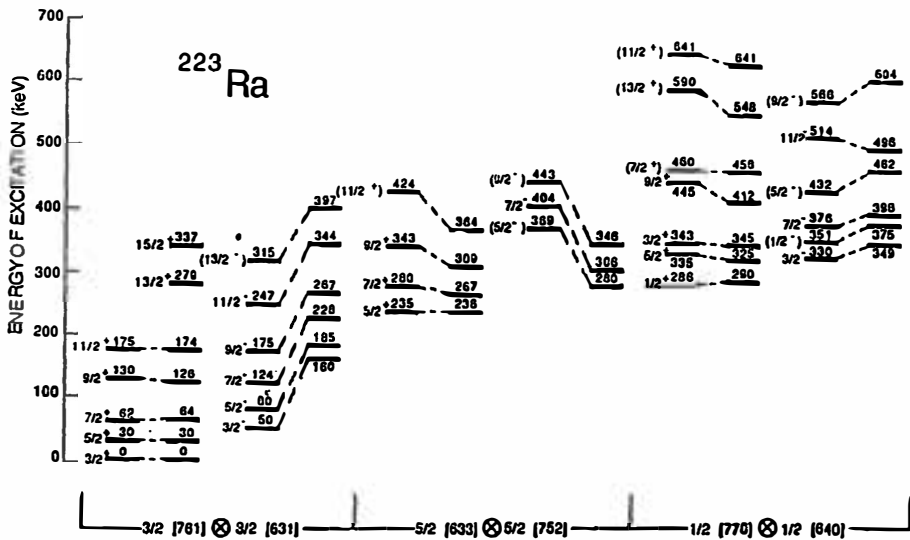


Fig. 8. Comparison of the experimental (left) and the theoretical (right) levels of  $^{223}\text{Ra}$ . It should be noted that a number of tentative experimental levels above the 514.0 keV limit of Fig. 7 have been included in this comparison.

The level structure for  $^{223}\text{Ra}$  is shown in Fig. 7<sup>19)</sup>. One sees quite clearly  $K = 3/2^\pm$ ,  $K = 5/2^\pm$  and  $K = 1/2^\pm$  parity doublet bands which are directly interpretable in terms of the  $3/2$  ( $-0.1; 0.6$ ), the  $5/2$  ( $0.2; 0.2$ ) and the  $1/2$  ( $0.2; -0.2; 3$ ) orbitals of Fig. 2. The 135th neutron is expected to occupy these orbitals, beginning with the ground state, then the hole state, and finally a particle state in this order. The rigid reflection asymmetric rotor model with the deformation parameters  $\beta_2 = 0.129$ ,  $\beta_3 = 0.10$ ,  $\beta_4 = 0.075$ ,  $\beta_5 = 0.01$  and  $\beta_6 = 0.004$  was used to calculate the energy levels for  $^{223}\text{Ra}$ . This calculation is compared with the experimental levels in Fig. 8. The experimental levels are given to the left, and the theoretical levels to the right. The approximate spacing of the different  $K$  values is well reproduced. The splitting between the bands of different parity is qualitatively reproduced — i. e., the negative parity states are always higher than the positive parity states, as observed experimentally. However, the absolute magnitude of the parity splitting is not very well reproduced.

Fortunately, the half lives of a number of the excited states in  $^{223}\text{Ra}$  have been measured. These half lives, together with the branching ratios and conversion coefficients, have allowed us to calculate the partial gamma ray half lives for

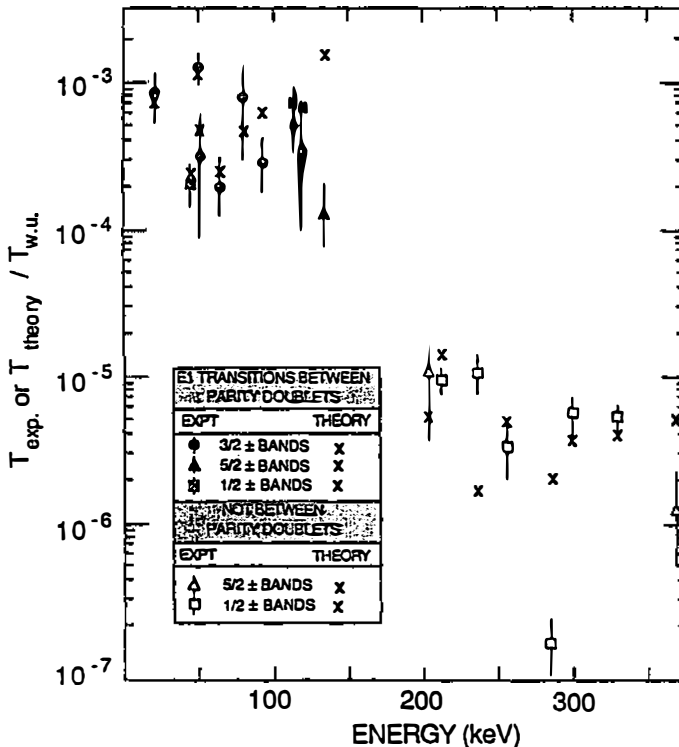


Fig. 9. The electric dipole transition probabilities in  $^{223}\text{Ra}$ . Experimental  $E1$  transition probabilities divided by the corresponding Weisskopf transition probabilities are plotted against the energy of the transition. Solid points correspond to transitions between parity doublet bands; open points to transitions not between parity doublet bands. The theoretical transition probabilities are shown as  $x$ 's.

a large number of transitions in  $^{223}\text{Ra}$ . The experimental transition probabilities divided by the Weisskopf transition probabilities are plotted against the energies of all the  $E1$  transitions in  $^{223}\text{Ra}$  in Fig. 9. Solid points correspond to  $E1$  transitions between parity doublet bands, whereas open points correspond to  $E1$  transitions which do not involve parity doublets. Two distinct groupings of points are observed in which the  $E1$  transitions between parity doublet bands are about two orders of magnitude more enhanced than the  $E1$  transitions which do not involve parity doublet bands. This, of course, is exactly what is expected theoretically since the  $E1$  transitions between parity doublet bands represent transitions within the same configuration, whereas the  $E1$  transitions not involving parity doublet bands do not involve transitions within the same configuration.

It is also possible to calculate this enhancement theoretically using the rigid reflection asymmetric rotor model with the same parameters as those used to calculate the energy levels in Fig. 8. The calculated points are shown in Fig. 9 as  $x$ 's. The value of the intrinsic electric dipole moment used for these calculations was 0.13 efm.

We have gone somewhat further in describing  $^{223}\text{Ra}$  than for the other odd- $A$  Ra nuclei. In part this was done because the calculations were available, and in part to show that is possible

*E The spectroscopy of  $^{225}\text{Ra}$*

The ground state spin of  $^{225}\text{Ra}$  has been measured by laser spectroscopy to be  $1/2^+ (20)$ . Recent experimental data and concurrent interpretation<sup>21-24</sup> have

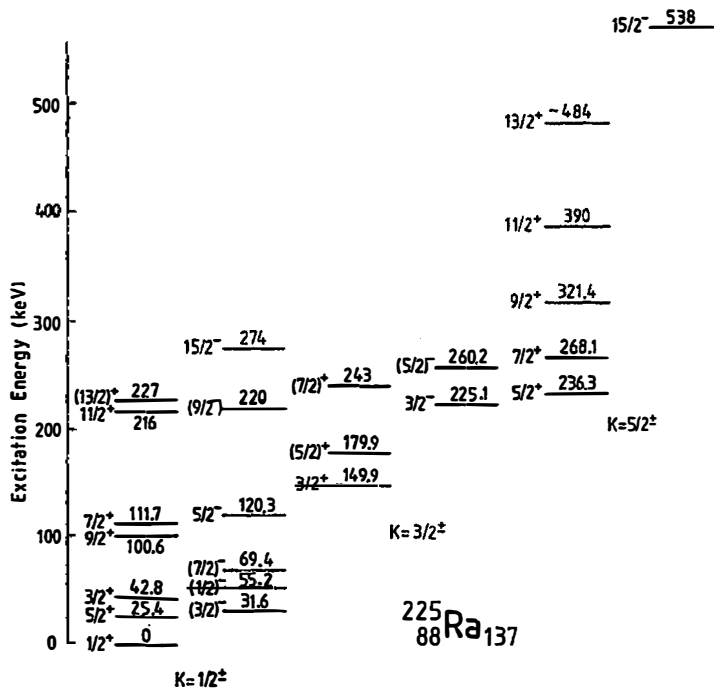


Fig. 10. The experimental level structure of  $^{225}\text{Ra}$ . The data and interpretation are taken from Refs. 21-24.

made possible the level scheme shown in Fig. 10. It is of considerable interest that the negative parity levels are not strongly populated in the alpha decay spectroscopy from the  $^{229}\text{Th}$  parent. Indeed, the  $K = 5/2^-$  band is not populated at all. This can be understood when it is realized that the  $^{229}\text{Th}$  alpha decaying parent has a reflection *symmetric*  $K = 5/2^+$  ground state. In fact, the only member of the  $K = 5/2^-$  band which is observed is populated in the  $^{226}\text{Ra} (^3\text{He}, \alpha) ^{225}\text{Ra}$  reaction.

The interpretation in terms of the  $1/2 (0.2; -0.1; 3)$ ,  $3/2 (-0.1; 0.6)$  and  $5/2 (-0.2; 0.7)$  orbitals of Fig. 2 would appear to be quite natural. However, there has always been difficulty with the  $1/2 (0.2; -0.1; 3)$  orbital, as noted in Refs. 6 and 19, both in  $^{223}\text{Ra}$  and  $^{225}\text{Ra}$ . This difficulty demands additional theoretical studies which are beyond the scope of this paper.

It should, however, be noted that experimentally one sees clear evidence for  $K = 1/2^\pm$  parity doublet bands for  $K = 3/2^\pm$  parity doublet bands, and for a  $K = 5/2^+$  band, and probably the  $15/2^-$  member of the  $K = 5/2^-$  parity doublet band. If the  $15/2^-$  member of the  $K = 5/2^-$  band is correctly assigned at 538 keV, then the  $5/2^-$  band can be expected in the vicinity of, but somewhat above, the  $K = 5/2^+$  band.

#### F. The spectroscopy of $^{227}\text{Ra}$

The ground state spin of  $^{227}\text{Ra}$  has been determined to be  $3/2$ , with a large positive quadrupole moment<sup>15</sup>. This nucleus has been extensively studied, using  $(n, \gamma)$ ,  $(d, p)$  and  $(d, t)$  reactions, and  $\beta$  decay<sup>25</sup>. The spectrum experimentally observed is shown in Fig. 11. One sees clear evidence for  $K = 3/2^\pm$  parity doublet bands at the ground state and 90.04 keV, and a second set of parity doublet bands with  $K = 1/2^\pm$  beginning at 120.71 and 284.28 keV. It is of considerable interest that a  $5/2^+$  state just 1.74 keV above the ground state with a  $9/2^+$  band member at 84 keV seems to have no  $K = 5/2^-$  parity doublet counterpart. This is very similar to the isotope  $^{229}\text{Th}$  where  $K = 3/2^\pm$  and  $K = 1/2^\pm$  parity doublet bands are observed, and a  $5/2^+$  band beginning less than a keV above the ground state with no parity doublet counterpart is also observed. In the case of  $^{229}\text{Th}$  a  $5/2^-$  octupole vibration is observed at 512 keV. We postulate, therefore, the coexistence of parity mixed reflection asymmetric and parity unmixed octupole-octupole vibrational spectroscopies in both  $^{229}\text{Th}$  and  $^{227}\text{Ra}$ .

If we look at the Nilsson levels with octupole deformation in Fig. 2 immediately above the 138 neutron gap (appropriate for 139 neutron  $^{229}\text{Th}$  and  $^{227}\text{Ra}$ ) for quadrupole deformation  $\varepsilon = 0.18$ , we see that we would expect the orbitals  $3/2 (0.3; 0.3)$ ,  $5/2 (-0.2; 0.7)$  and  $1/2 (-0.1; 0.6; 1)$  in this order. Therefore we would have expected  $3/2^\pm$ ,  $5/2^\pm$  and  $1/2^\pm$  parity doublet bands. Experimentally, however, we see that the  $5/2^-$  band lies 512 keV above the  $5/2^+$  band in  $^{229}\text{Th}$ , which stamps it as an octupole vibration. In a similar way no parity doublet band is found for  $^{227}\text{Ra}$ , and therefore it must be presumed that it also involves an octupole vibration. However, if instead of using the strong coupling model in which a quasi particle is coupled to a rotor core containing an intrinsic  $K = 0$  reflection asymmetric degree of freedom, we use the weak coupling model, it is able to account for the coexistence of these reflection symmetric and reflection asymmetric states in  $^{229}\text{Th}$  and  $^{227}\text{Ra}$ . The wave functions in order of decreasing importance for



With the addition of just two neutrons to give us  $^{219}\text{Ra}$ , we had already moved to a nucleus with ground state spin  $7/2^+$ , which is not easy to explain in the simplest shell model picture. However, the rather incomplete spectra of  $^{219}\text{Ra}$  can be explained with the octupole deformed Nilsson model. It is indeed quite interesting that the weak coupling structures observed in both  $^{217}\text{Ra}$  and  $^{219}\text{Ra}$  tend to arise quite naturally from  $K = 1/2^\pm$  parity doublet bands with large decoupling parameters of opposite sign. This gives rise to  $\Delta I = 2$  sequences with opposite signatures for the opposite parity bands, very much as observed in both  $^{217}\text{Ra}$  and  $^{219}\text{Ra}$ .

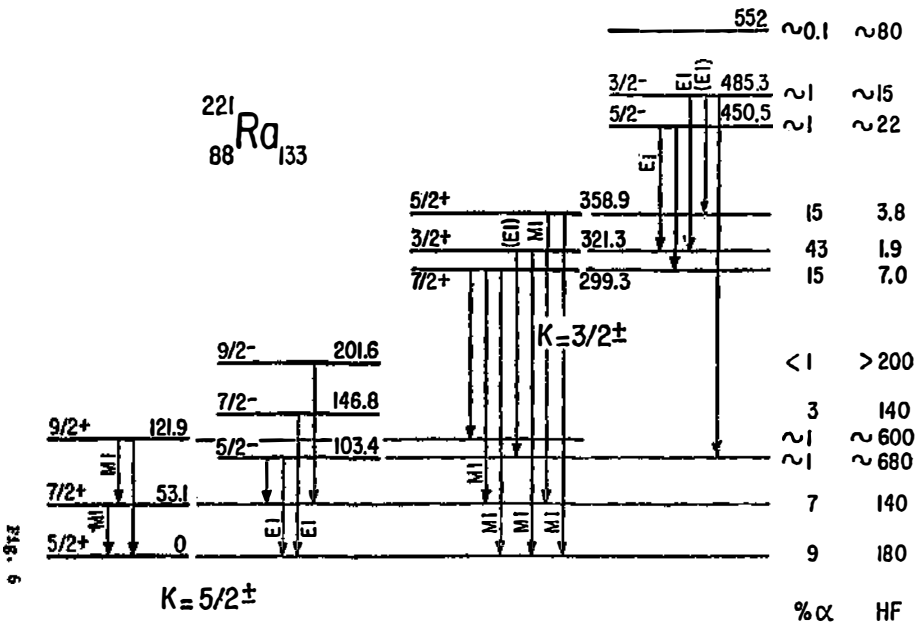


Fig. 12. Comparison of the low lying parity doublet bands in  $^{221}\text{Ra}$ ,  $^{223}\text{Ra}$  and  $^{225}\text{Ra}$ . Experimental data are from Refs. 16–24.

By the time we reach  $^{221}\text{Ra}$ ,  $^{223}\text{Ra}$  and  $^{225}\text{Ra}$ , the nuclei have developed large positive quadrupole moments which implies strong coupling and regular Nilsson-like rotational bands with, however, the additional proviso that one sees parity doublet bands with enhanced  $E1$  transitions between the parity doublets. Fig. 12 compares the parity doublets of  $^{221}\text{Ra}$ ,  $^{223}\text{Ra}$  and  $^{225}\text{Ra}$ . The startling differences between the  $K = 3/2^\pm$  bands are especially noticeable. Whereas a completely anomalous spin sequence is present in  $^{221}\text{Ra}$ , there is only a slight tendency in this direction in the ground state of  $^{223}\text{Ra}$ . Here the  $3/2^+$ ,  $7/2^+$  and  $11/2^+$  are slightly lowered with respect to the  $5/2^+$  and the  $9/2^+$  and the  $5/2^-$  and  $9/2^-$  are slightly lowered with respect to the  $3/2^-$ ,  $7/2^-$  and  $11/2^-$ . In  $^{221}\text{Ra}$  the  $7/2^+$  has not only been lowered below the  $5/2^+$  but also below the  $3/2^+$ . In a similar way the  $5/2^-$  has been lowered below the  $3/2^-$ . This is clearly the effect of the  $1/2$  ( $-0.1$ ;  $0.5$ ;  $-2$ ) orbital mixing with the  $3/2$  ( $-0.1$ ;  $0.6$ ) orbital. However, the mixing is much less than in  $^{221}\text{Ra}$  because the  $K = 3/2^\pm$  and  $K = 1/2^\pm$  pa-

rity doublet bands are much further apart in energy. By the time we reach  $^{225}\text{Ra}$  with 137 neutrons, the 131st neutron orbital is a fairly deep hole state, and the signature splitting in the  $K = 3/2^\pm$  band is reversed from that in  $^{223}\text{Ra}$  and  $^{221}\text{Ra}$ .

By the time we reach  $^{227}\text{Ra}$ , octupole correlations have begun to decrease and the degree of octupole deformation or correlation of the core is very sensitive to the ability of the last odd neutron to polarize the core toward reflection asymmetry or toward reflection symmetry. For this reason we see the coexistence of reflection asymmetric spectroscopies implied by the  $K = 3/2^\pm$  and  $1/2^\pm$  parity doublets, and the  $K = 5/2^+$  reflection symmetric band with superimposed octupole vibration.

As we move on to  $^{229}\text{Ra}$ , even though virtually nothing is known about this nucleus experimentally, we must presume that the nuclear structure has returned to the more usual quadrupole deformed reflection symmetric shape.

It should be noted that specific calculations have been carried out<sup>6)</sup> for a number of the Ra nuclei discussed in this paper.

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## SPEKTROKOPIJA NEPARNIH IZOTOPA RADIJA U OKTUPOLNO DEFORMIRANOM PODRUČJU

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Nadeno je da su  $^{221}\text{Ra}$ ,  $^{223}\text{Ra}$  i  $^{225}\text{Ra}$  najbolji primjeri normalnih oktopolnih vibracija. Spektroskopija  $^{217}\text{Ra}$  i  $^{219}\text{Ra}$  predstavlja prijelaz od ljuskastog modela sa oktopolnim korelacijama u oktopolno deformiran sistem sa slabim vezanjem. Spektroskopija  $^{227}\text{Ra}$  ukazuje na koegzistenciju paritetno dvojnih vrpca oktopolno deformiranog sistema i oktopolno vibracionih vrpca normalnog tipa bez oktopolnih vibracija.