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BOSE-EINSTEIN CONDENSATION — AN AUTONOMOUS APPROACH

BIMAL KUMAR MAJUMDAR

Dept. of Physics, B. K. C. College, Calcutta 700035, India

and

RAJKUMAR ROYCHOUDHURY

Electronics Unit, Indian Statistical Institute, Calcutta 700035, India

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The symmetry breaking of B-E gas in 3 + 1 dimension has been investigated using autonomous normalization. Phase transition for different cases have also been investigated. Comparison has been made with previously published results.

1. Introduction

In recent years GEP (Gaussian Effective Potential) approach has been applied to various problems of quantum field theory[1-10]. It is argued that GEP, essentially a non perturbative method, has several advantages over loop expansion method and contains one-loop as well as $1/N$ expansion results in limiting cases for scalar fields.

However in GEP itself there are two different approaches of normalization in 3 + 1 dimension[1-3,5,6], one being the usual precarious version and other is autonomous version. In this paper we derive from first principle the finite temperature Gaussian effective potential for an ideal and interacting Bose gas with a net background charge. Of the two different approaches of normalisations mentioned above, we choose autonomous version[5,11,12]. As already discussed in Ref. 12 and references therein that the precarious version becomes trivial in 3 + 1 dimension and has some strange properties at finite temperatures. Autonomous theory on the other hand does not face triviality problem and exhibits SSB (Spontaneous Symmetry Breaking).

Here the autonomous $\lambda\varphi^4$ theory has been applied to a system having a net background charge. The phase transitional aspect of the system has also been studied.

For interacting Bose gas with $O(N)$ non-Abelian symmetry the chemical potential can be introduced only to mutually commutating generators. So for even N, the maximum number of commutating charges is $N/2$[13,14]. For the sake of simplicity we introduce a single μ in the present case. However, the generalisation to many μ case is straightforward. Since we have considered a single μ, the system will be invariant under $O(2) \times O(N-2)$ and not under the full $O(N)$ symmetry.

In Sect. 2 the derivation of FTGEP (Finite Temperature GEP) from basic principle is presented. While doing so we have followed the method syggested by Hajj and Stevenson[12]. The phase transitional aspect (B-E condensation) is analysed in Sect. 3. Section 4 is kept for discussion and remarks.

2. *Derivation of μ-dependent FTGEP for $0(N)$ symmetric φ^4*

The $O(N) \times O(N-2)$ symmetric Lagrangian for our case is given by

$$\mathscr{L} = \frac{1}{2}\,\partial_\nu\varphi_j\partial^\nu\varphi_j - \frac{1}{2}\,m_B\varphi_j\varphi_j - \lambda_B(\varphi_j\varphi_j)^2 - \mathrm{i}\mu(\dot{\varphi}_1\varphi_2 - \dot{\varphi}_2\varphi_1) + $$
$$+ \frac{1}{2}\,\mu^2(\varphi_1^2 + \varphi_2^2) \tag{1}$$

where j is summed from 1 to N.

The corresponding Hamiltonian is given by

$$H = \frac{1}{2}\sum_j \dot{\varphi}_j^2 + \frac{1}{2}\sum_j(\nabla\varphi_j)^2 + \frac{1}{2}\,m_B^2\sum_j\varphi_j^2 + \lambda_B\sum_j(\varphi_j^2)^2 +$$
$$+ \mathrm{i}\mu(\dot{\varphi}_1\,\varphi_2 - \dot{\varphi}_2\,\varphi_1) - \frac{1}{2}\,\mu^2(\varphi_1^2 + \varphi_2^2). \tag{2}$$

Following $O(N)$ symmetric theory suggested by Stevenson et al.[11] we choose a coordinate system in which $(\varphi_0)_j$ points in the $j = 1$ direction, then writting $\varphi_j = (\varphi_0)_j + \hat{\varphi}_j$ and taking $(\varphi_0)_1 = \varphi_0$ we have,

$$\varphi^2 = \varphi_0^2 + 2\varphi_0\,\hat{\varphi}_1 + \sum\hat{\varphi}_j^2 \tag{3}$$

where

$$\varphi_j = (\varphi_0)_j + R_j^i(\Theta_1 \ldots \Theta_{N-1})\,\hat{\varphi}_i(\Omega_j) \quad \text{and} \quad \varphi_0 = |\varphi_0|$$

and

$$(\varphi^2)^2 = \varphi_0^4 + 4\varphi_0^2\,\hat{\varphi}_1^2 + 2\varphi_0^2 \sum_1^N \hat{\varphi}_j^2 + \Big|\sum_1^N \hat{\varphi}_j^2\Big|^2 + 4\varphi_0^3\,\hat{\varphi}_1 + 4\varphi_0\,\hat{\varphi}_1 \sum_1^N \hat{\varphi}_j^2. \tag{4}$$

As already mentioned, owing to the existence of a net charge, the system will be invariant under $O(2) \times O(N-2)$ and in general $(\varphi_0)_1$ and $(\varphi_0)_1$ will not be equal to zero and we choose $(\varphi_0)_i = 0$ for $i = 3, 4, \ldots, N$. However for the sake of simplicity we take $(\varphi_0)_2 = 0$ i. e., we take all $(\varphi_0)_j = 0$ excepting $j = 1$. We have attached trial mass Ω for radial field and trial mass w for transverse fields.

To compute ${}_{\Omega,\omega}\langle 0|\,H\,|0\rangle_{\Omega,w}$ and finite temperature μ — dependent GEP we follow quasiperturbative approach of Hajj and Stevenson[12]. The Hamiltonian H (Eq. (2)) is written as

$$H = H_0 + H_{int} \tag{5}$$

with

$$H_0 = \frac{1}{2}\,[\dot{\varphi}^2 + (\nabla\varphi)^2 + \Omega^2 \sum \hat{\varphi}_\alpha^2 + w^2 \sum \hat{\varphi}_{\alpha'}^2 + 2i\mu\,(\dot{\varphi}_1\,\varphi_2 - \dot{\varphi}_2\,\varphi_1) -$$

$$- \mu^2\,(\hat{\varphi}_1^2 + \hat{\varphi}_2^2)] \tag{5a}$$

and

$$H_{int} = -\frac{1}{2}\,[\Omega^2 \sum \hat{\varphi}_\alpha^2 + w^2 \sum \hat{\varphi}_{\alpha'}^2 + \mu^2\,\varphi_0^2] + \frac{1}{2}\,m_B^2\,(\varphi_0 + \hat{\varphi})^2 +$$

$$+ \lambda_B\,(\varphi_0 + \hat{\varphi})^4 \tag{5b}$$

where $\alpha = 1, 2,\ \alpha' = 3, 4, \ldots, N$.

In writing H_{int} we have used the fact that $\langle \hat{\varphi}_1 \rangle = 0$. Using standard thermodynamical relations we have,

$$V_G\,(\varphi_0, \Omega, w, \mu) = \frac{F}{V} = -\frac{1}{\beta V}\ln Z_0 + \langle H_{int}\rangle_T \tag{6}$$

where F is the free energy, $\beta = \frac{1}{k_B T}$, $Z_0 = \mathrm{Tr}\,e^{-\beta H_0}$ and $\langle H_{int}\rangle_T$ is the thermal average of H_{int}. Now minimisation of free energy F under the condition $\langle \varphi \rangle = \varphi_0$ gives the effective potential, viz.,

$$\overline{V}_G = \min_{\Omega,w} V_G\,(\varphi_0, \Omega, w, \mu). \tag{7}$$

Now

$$H_0\,|n_1, n_2, \ldots, n_1', n_2' \ldots\rangle = I_1\,V + n_1\,w_1 + n_2\,w_2 + \ldots$$

$$+ (N-2)\,[I_1'\,V + n_1'\,w_1' + n_2'\,w_2' + \ldots] \tag{8}$$

where $|n_1, n_2, \ldots, n_1', n_2', \ldots\rangle$ represents the eigenstates of H_0 corresponding to n_i and n_i' quanta in the i-th mode and taking into account the contribution from each mode we get the vacuum energy $I_1 V$ and $I_1' V$ as follows

$$I_1 V = V \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \frac{w_k}{2} \quad \text{and} \quad I_1' V = V \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \frac{w_k'}{2}. \tag{9}$$

The introduction of chemical potential results in two values of frequencies w_i of the i-th mode $(K^2 + \Omega^2)^{1/2} \pm \mu$ (as can be verified by solving coupled Klein-Gordon equation) unlike single value $(K^2 + \Omega^2)^{1/2}$ in the case with $\mu = 0$. The upper sign corresponds to a particle with charge $+1$ and lower sign corresponds to a particle with charge -1. The factor $(N - 2)$ in the bracketted term on R. H. S. of Eq. (8) arises from the $(N - 2)$, μ-less transverse quantum fields with the same mass parameter w. Writing explicitly the trace appearing in Z_0 where summation is extended over all modes and corresponding occupation number corresponding to each mode n_i and n_i' it can be shown that

$$Z_0 = Z_0' (Z_0'')^{N-2} \tag{10}$$

with

$$Z_0' = \mathrm{e}^{-\beta I_1 V} \left[\sum_{n_1=0}^{\infty} \mathrm{e}^{-\beta n_1 w_1} \right] \left[\sum_{n_2=0}^{\infty} \mathrm{e}^{-\beta n_2 w_2} \right] = \mathrm{e}^{-\beta I_1 V} \prod_i (1 - \mathrm{e}^{-\beta w_i})^{-1} \tag{11}$$

and

$$Z_0'' = \mathrm{e}^{-\beta I_1' V} \left[\sum_{n_1'=0}^{\infty} \mathrm{e}^{-\beta n_1' w_1'} \right] \sum_{n_2'=0}^{\infty} \mathrm{e}^{-\beta n_2' w_2'}] = \mathrm{e}^{-\beta I_1' V} \prod_i (1 - \mathrm{e}^{-\beta w_i'})^{-1}. \tag{12}$$

Again since two values of w_k (as mentioned earlier) are independent of each other we have

$$Z_0' = \mathrm{e}^{-\beta I_1 V} \prod (1 - \mathrm{e}^{-\beta (w_k)_1})^{-1} \prod (1 - \mathrm{e}^{-\beta (w_k)_2})^{-1}. \tag{13}$$

Now defining

$$I_1^{FT} = I_1(\Omega) + I_1^\beta = -\frac{1}{\beta V} \ln Z_0' \tag{14}$$

and replacing $\sum_i$ by $V \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu}$ we get from (13) and (14)

$$I_1^\beta = \frac{1}{\beta} \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} [\ln(1 - \mathrm{e}^{-\beta (w_k)_1}) + \ln(1 - \mathrm{e}^{-\beta (w_k)_2}] =$$

$$= \frac{1}{\beta} \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} [\ln(1 - \mathrm{e}^{-\beta(E+\mu)}) + \ln(1 - \mathrm{e}^{-\beta(E+\mu)})] \tag{15}$$

where $E = (K^2 + \Omega^2)^{1/2}$ and $I_1(\Omega)$ is given by

$$I_1(\Omega) = \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \frac{1}{2} (w_{k_1} + w_{k_2}) = \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} (K^2 + \Omega^2)^{1/2}. \tag{16}$$

Following the same process we get

$$I_1'^{\beta} = \frac{1}{\beta} \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu_\varepsilon} \ln [1 - \mathrm{e}^{-\beta(K^2+w^2)^{1/2}}] \tag{17}$$

and

$$I_1'(w) = \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \frac{w_k'}{2}, \tag{18}$$

where $w_k' = (K^2 + w^2)^{1/2}$ and $I_1'^{\beta}$ and $I'(w)$ are same as that of Hajj and Stevenson[12] and Roditi[2]. Starting from the definition of thermal average one obtains easily

$$\langle \hat{\varphi}_\alpha^2 \rangle_T = I_0^{FT} = I_0(\Omega) + I_0^{\beta} \tag{19}$$

with

$$I_0(\Omega) = \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \frac{1}{(K^2 + \Omega^2)^{1/2}} \tag{20}$$

and

$$I_0^{\beta} = 2 \int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \frac{1}{(K^2 + \Omega^2)^{1/2}} \left[\frac{1}{\mathrm{e}^{\beta(E+\mu)} - 1} + \frac{1}{\mathrm{e}^{\beta(E-\mu)} - 1} \right]. \tag{21}$$

Again I_1^{FT} and I_0^{FT} are related in the following way

$$\frac{\mathrm{d}I_1^{FT}}{\mathrm{d}\Omega} = \Omega I_0^{FT}. \tag{22}$$

Starting from basic premises, straightforward calculation confirms the result $\langle \hat{\varphi}_\alpha^4 \rangle_T = 3 \langle \hat{\varphi}^2 \rangle_T^2$ (up to volume suppressed term) of Stevenson et al.[12]. After obtaining contributions of all terms of $\langle H_{int} \rangle_T$ and using Eqs. (3), (4) and (6) and keeping in mind that the system is invariant under $O(2) \times O(N-2)$ symmetry we get $\overline{V}_G$. Henceforth for convenience we write I_1^{FT} as I_1 and I_0^{FT} as I_0 and similarly $I_1'^{FT}$ and $I_0'^{FT}$ as I_1' and I_0', respectively. Finally we have

$$\overline{V}_G = \left[I_1 + \frac{1}{2} (m_B^2 - \Omega^2) I_0 \right] + (N-2) \left[I_1' + \frac{1}{2} (m_B^2 - w^2) I_0' \right] +$$

$$+ \frac{1}{2} m_B^2 \varphi_0^2 + \lambda_B \varphi_0^4 + \lambda_B [3 I_0^2 + (N^2 - 2N) I_0'^2 + 2(N-2) I_0 I_0' +$$

$$+ 6 I_0 \varphi_0^2 + 2(N-2) I_0' \varphi_0^2] - \frac{1}{2} \mu^2 \varphi_0^2. \tag{23}$$

Now minimisation of $\overline{V}_G(\varphi_0, \Omega, w, \mu)$ with respect to Ω and w and use of results $\mathrm{d}I_N/\mathrm{d}\Omega = (2N-1)\,\Omega I_{N-1}$ and similar results for I'_N yield the coupled equation for $\overline{\Omega}$ and $\overline{w}$. Denoting the optimum value of Ω and w by $\overline{\Omega}, \overline{w}$ we have

$$\overline{\Omega}^2 = m_B^2 + 4\lambda_B\,[3I_0 + (N-2)\,I'_0 + 3\varphi_0^2] \tag{24a}$$

$$\overline{w}^2 = m_B^2 + 4\lambda_B\,[I_0 + NI'_0 + \varphi_0^2]. \tag{24b}$$

Now to apply autonomous theory we first consider renormalisation of coupling constant λ_B. For this we define renormalised coupling constant

$$\lambda_R = \frac{1}{2}\,\frac{\mathrm{d}^2\,\overline{V}_G}{\mathrm{d}(\varphi_0^2)^2}\bigg|_{\varphi_0=0}.$$

From Eq. (24) the following equations are obtained

$$\frac{\mathrm{d}\overline{\Omega}^2}{\mathrm{d}\varphi_0^2} = 4\lambda_B\,[3 + 4\,(N+1)\,\lambda_B I'_{-1}]/A \tag{25a}$$

$$\frac{\mathrm{d}\overline{w}^2}{\mathrm{d}\varphi_0^2} = 4\lambda_B/A \tag{25b}$$

with $A = 1 + 2\lambda_B N I'_{-1} + 6\lambda_B I_{-1} + 8\lambda_B^2\,(N+1)\,I_{-1}I'_{-1}$.

From (23) and (25) we get,

$$\lambda_R = \lambda_B\,\frac{1 - 8\lambda_B I_{-1} - 16\,(N+1)\,\lambda_B^2 I_{-1}^2}{(1 + 2\,(N+1)\,\lambda_B I_{-1})(1 + 4\lambda_B I_{-1})} \tag{26}$$

where $I_{-1} = I_{-1}(\chi)$, with χ = finite mass.

Now we use the following renormalisation relation for coupling constant (similar to that of Stevenson et al.[11]).

$\lambda_B = a/I_{-1}(\chi)$. where a is N-dependent number so that the numerator in (26) vanishes i. e.,

$$1 - 8a - 16\,(N+1)\,a^2 = 0 \tag{27}$$

only positive root of Eq. (27) is acceptable (reasons discussed in Ref. 11).

Further we use the following renormalisation relations

$$m_B^2 + 4\,(N+1)\,\lambda_B I_0(0) = m_0^2/8I_{-1}(\chi) \tag{28a}$$

$$\varphi_0^2 = I_{-1}(\chi)\,\Phi_0^2 \quad \text{and} \quad \mu^2 = \mu_0^2/I_{-1}(\chi). \tag{28b}$$

The renormalisation relations for m_B and φ_0 have been taken from Ref. 5. χ, m_0 used in the above equations are finite mass parameters and both have dimension of mass. $I_{-1}(\chi)$ is logarithmically divergent integral. Now taking help of the following formulas[1)]

$$I_0(0) - I_0(\overline{\Omega}) = \frac{1}{2}\overline{\Omega}^2\left[I_{-1}(\overline{\Omega}) + \frac{1}{8\pi^2}\right]$$

$$I_{-1}(\overline{\Omega}) - I_{-1}(\chi) = -\frac{1}{8\pi^2}\ln(\overline{\Omega}^2/\chi^2) \tag{29}$$

and similar relations for $[I'_0(0) - I'_0(\overline{w})]$ and $[I'_{-1}(\overline{w}) - I'_{-1}(\chi)]$. Using (28), (29) we get,

$$\overline{\Omega}^2 = \frac{m_0^2}{8I_{-1}(\chi)} + \frac{4a}{I_{-1}(\chi)}\left[\frac{3}{16\pi^2}\overline{\Omega}^2(\ln\overline{\Omega}^2/\chi^2 - 1) + \right.$$
$$\left. + \frac{(N-2)}{16\pi^2}\overline{w}^2(\ln\overline{w}^2/\chi^2 - 1) + 3I_0^\beta + (N-2)I_0'^\beta\right] - \tag{30a}$$
$$- 6a\overline{\Omega}^2 - 2(N-2)a\overline{w}^2 + 12a\Phi_0^2$$

and

$$\overline{w}^2 = \frac{m_0^2}{8I_{-1}(\chi)} + \frac{4a}{I_{-1}(\chi)}\left[\frac{1}{16\pi^2}\overline{\Omega}^2(\ln\overline{\Omega}^2/\chi^2 - 1) + \right.$$
$$\left. + \frac{N}{16\pi^2}\overline{w}^2(\ln\overline{w}^2/\chi^2 - 1) + I_0^\beta + NI_0'^\beta\right] - 2a\overline{\Omega}^2 - 2Na\overline{w}^2 + 4a\Phi_0^2. \tag{30b}$$

Eqs. (30a) and (30b) are rewritten as

$$\overline{\Omega}^2(1 + 6a) + 2(N-2)a\overline{w}^2 = 12a\Phi_0^2 + \varepsilon_{\overline{\Omega}} \tag{31a}$$

$$2a\overline{\Omega}^2 + (1 + 2Na)\overline{w}^2 = 4a\Phi^2 + \varepsilon_{\overline{w}} \tag{31b}$$

where

$$\varepsilon_{\overline{\Omega}} = \frac{1}{8\pi^2 I_{-1}(\chi)}[\pi^2 m_0^2 + 2a(3\overline{\Omega}^2(\ln\Omega^2/\chi^2 - 1) +$$
$$+ (N-2)\overline{w}^2(\ln\overline{w}^2/\chi^2 - 1) + 48\pi^2 I_0^\beta + 16\pi^2(N-2)I_0'^\beta)] \tag{32a}$$

and

$$\varepsilon_{\overline{w}} = \frac{1}{8\pi^2 I_{-1}(\chi)}[\pi^2 m_0^2 + 2a(\overline{\Omega}^2(\ln\overline{\Omega}^2/\chi^2 - 1) +$$
$$+ N\overline{w}^2(\ln\overline{w}^2/\chi^2 - 1) + 16\pi^2 I_0^\beta + 16\pi^2 N I_0'^\beta)]. \tag{32b}$$

From (31a) and (31b) we get

$$\overline{\Omega}^2 = 8a\, \Phi_0^2 + \frac{2}{B}\left[(1 + 2Na)\, \varepsilon_{\overline{\Omega}} - 2\,(N - 2)\, \varepsilon_{\overline{\varpi}}\right] \tag{33a}$$

$$\overline{w}^2 = 8a\, \Phi_0^2/B + O\left(\frac{1}{I_{-1}(\chi)}\right), \tag{33b}$$

with

$$B = 3 + 4a\,(N + 1) = \frac{1 + 4a}{4a}. \tag{34}$$

Taking first derivative of $\overline{V}_G$ with respect to Φ_0^2 we have from (23) and using (24) and (28)

$$\frac{\mathrm{d}\overline{V}_G}{\mathrm{d}\Phi_0^2} = \frac{I_{-1}(\chi)}{2}\,[\overline{\Omega}^2 - 8a\,\Phi_0^2 - \mu^2]. \tag{35}$$

Now using (32) and (33a) the equation (35) may be rewritten as

$$\frac{\mathrm{d}\,\overline{V}_G}{\mathrm{d}\Phi_0^2} = \frac{m_0^2\, a}{2} + a\left[\frac{\overline{\Omega}^2}{4\pi^2}(\ln \overline{\Omega}^2/\chi^2 - 1) + \frac{(N-2)}{4\pi^2 B}\,\overline{w}^2\,(\ln \overline{w}/\chi^2 - 1) + \right.$$

$$\left. + 4I_0^\beta + \frac{4}{B}(N - 2)\, I_0'^\beta\right] - \frac{1}{2}\,\mu_0^2. \tag{36}$$

Finally GEP is obtained from (36)

$$\overline{V}_G - D = \frac{m_0^2\, a\, \Phi_0^2}{2} + \frac{a^2\, \Phi_0^4}{\pi^2 B}\left[2\,(B - 2)\left[\ln \frac{8a\, \Phi_0^2}{2} - \frac{3}{2}\right] - (B - 4)\ln B\right] +$$

$$+ I_1^\beta + (N - 2)\, I_1^\beta - \frac{1}{2}\,\mu_0^2\, \Phi_0^2 = (\overline{V}_G)_0 + I_1^\beta + (N - 2)\, I_1'^\beta - \frac{\mu_0^2\, \Phi_0^2}{2} \tag{37}$$

where D is vacuum energy constant and $(\overline{V}_G)_0$ gives the temperature independent part of GEP and is given by

$$(\overline{V}_G)_0 = \frac{m_0^2 a\, \Phi_0^2}{2} + a^2 \frac{\Phi_0^4}{\pi^2 B}\left[2\,(B - 2)\left[\ln 8a\Phi_0^2/\chi^2 - \frac{3}{2}\right] - (B - 4)\ln B\right]. \tag{38}$$

$(\overline{V}_G)_0$ has the form as that of GEP of Stevenson et al.[11]. It should be mentioned that in deriving (37) from (36) we have taken $\overline{\Omega}^2 \simeq 8a\Phi_0^2$ and $\overline{w}^2 = 8a\Phi_0^2/B$ since the term of order $\dfrac{1}{I_{-1}(\chi)}$ vanishes after the removal of regularisation.

3. B—E condensation and critical temperature

Apart from V_G another quality of interest in the study of phase transition is the effective mass $\bar{\Omega}$, defined by $\mathrm{d}^2\bar{V}_G/\mathrm{d}\Phi_0^2$, evaluated at the minimum of potential. In broken symmetry vacuum, $\Phi_0 = \pm v$, the effective mass

$$\bar{\Omega}_v = \left.\frac{\mathrm{d}^2 \bar{V}_G}{\mathrm{d}\Phi_0^2}\right|_{\Phi_0 = \pm v}.$$

From (36) we get $\bar{\Omega}_v^2 \simeq 8av^2$ hence the effective potential decreases with increase of temperature and minimum of GEP tends towards origin. The charge density ϱ is defined to be

$$\varrho = -\frac{\mathrm{d}\bar{V}_G}{d\mu_0} = -\left(\frac{\partial I_1^\beta}{\partial\mu_0}\right)_{(T,\,\bar{\Omega})_{fixed}} + \mu_0\Phi_0^2. \tag{39}$$

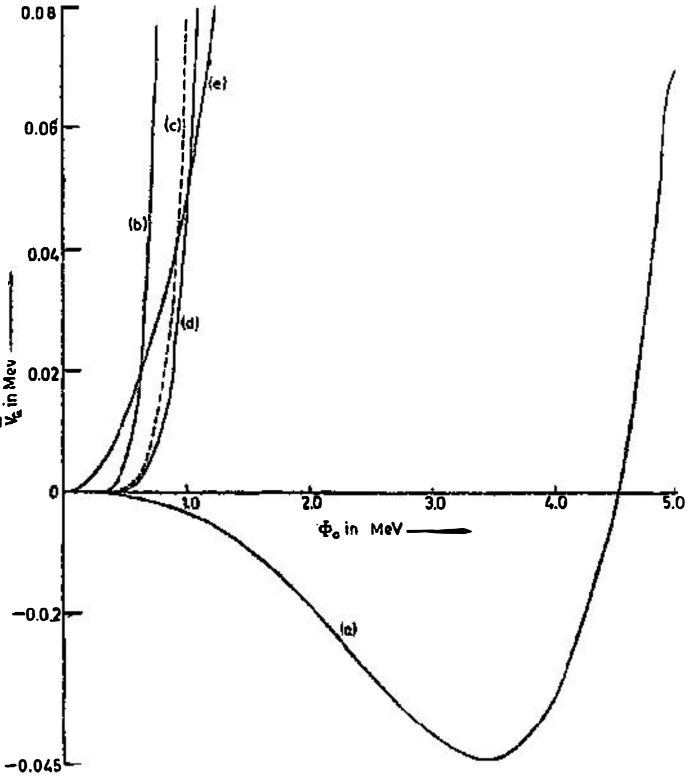

Fig. 1. Plot of the effective potential V_G against φ_0.

Since I_1^β depends on $\bar{\Omega}$ and hence on Φ_0, the value of chemical potential μ_0 may be obtained from Eq. (39) for a given charge density ϱ. The value of GEP $\bar{V}_G$ can be computed from (36) for given Φ_0 after obtaining μ_0 from 39) for a given ϱ.

In Fig. 1 we plot $\bar{V}_G$ against Φ_0 at different temperature, the values of the relevant parameters are as follows (expressed in MeV) $m_0^2 = -0.1$, $\varrho = 0.001$, $N = 10$, $\chi = 1$. The high temperature expansion of I_1^β and $I_1'^\beta$ are given by (see Refs. 14 and 15),

$$I_1'^\beta = -\frac{\pi^2 T^4}{90} + \frac{T^2 \bar{w}^2}{24} - \frac{T\bar{w}^3}{12\pi} - \frac{\bar{w}^4}{64\pi^2}\left[\ln\frac{\bar{w}^2}{T^2} - a\right] + O\left(\frac{\bar{w}^6}{T^2}\right) \quad (40)$$

with

$$a = \frac{3}{2} + 2(\ln 4\pi - \gamma) \simeq 5.41$$

and

$$I_1^\beta = -\frac{\pi^2 T^4}{45} + \frac{T^2(\bar{\Omega}^2 - 2\mu_0^2)}{12} - \frac{T(\bar{\Omega}^2 - \mu_0^2)^{3/2}}{6\pi} - \frac{\mu_0^2(3\bar{\Omega}^2 - \mu_0^2)}{24\pi^2} +$$
$$+ \frac{\bar{\Omega}^4}{16\pi^2}\left[\ln\left(\frac{4\pi T}{\bar{\Omega}}\right) - \gamma + \frac{3}{4}\right] + O\left(\frac{\bar{\Omega}^6}{T^2}, \frac{\bar{\Omega}^4\mu_0^2}{T^2}\right). \quad (41)$$

The result shows that for $m_0^2 = -0.1$ the phase transition (Bose condensation) occurs at $T = 0.236$ MeV. In order to make all the curves coincide at the origin we have added temperature dependent constant to each curve. Fig. 2 shows the variation of critical temperature with m_0^2. It is worth mentioning that for $m_0^2 \geq 0$ phase transition occurs at $T = 0$, which is not the case for $\mu = 0$, the case studied by Stevenson et al.[12]. In Fig. 3, T_c is plotted against ϱ to show the dependen-

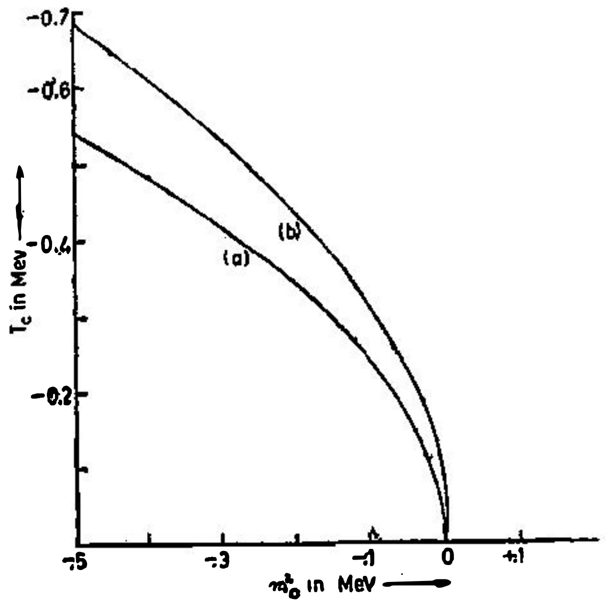

Fig. 2. Plot of the critical temperature T_c against m_0.

ce of critical temperature on chemical potential μ_0 (here $m_0^2 = -0.05$). It is interesting to note that the result of autonomous theory differs from that of previous results obtained by loop expansion method[14,16]. Specifically in our case there apears no precondition that for $T < T_c$, the effective mass of the system need to be

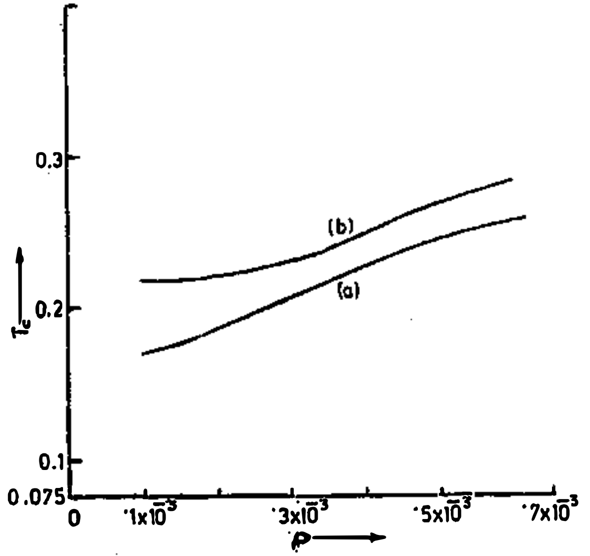

Fig. 3. Plot of the critical temperature T_c against ϱ.

equal to chemical potential as in the case of Refs. 13, 14 and 16. Another notable difference is that for autonomous theory (irrespective of $\mu = 0$ and $\mu \neq 0$) in its broken symmetry phase the particles are always massless. These are expected since the autonomous theory does not reproduce the one loop expansion result.

4. *Discussion and remarks*

The method adopted here has advantage over the one-loop expansion method in the sense that latter does not give the correct high temperature behaviour[12]. Also we observed here that in the autonomous φ^4 theory the finite temperature corrections appear in simple form even for a system with a net background charge and symmetry restoration occur at high temperature. So possibly one can claim that autonomous $\lambda\varphi^4$ theory in meaningfull and encouraging as far as finite temperature behaviour is concerned. Also there is scope of further investigation for massless, symmetric vacuum. Finally we like to add that although we have considered a single μ, the present study can be extended in a straightforward manner to a system with more than one μ.

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BOSE-EINSTEINOVA KONDENZACIJA — AUTONOMNI PRISTUP

BIMAL K. MAJUMDAR* i RAJKUMAR ROYCHOUDHURY**

**B. K. C. College, Calcuta 700 035, India*

***Electronics Unit, Indian Statistical Institute, Calcutta 700 035, India*

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Koristeći autonomnu normalizaciju istraženo je lomljenje simetrije u Bose-Einsteinovu plinu u 3 + 1 dimenziji. Također su razmotreni fazni prijelazi za razne slučajeve. Rezultati su uspoređeni s prije objavljenima.