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TWO HIGGS DOUBLET CONFIGURATION ADMITTING AN SO(2) GLOBAL SYMMETRY ANOMALOUSLY COUPLED TO MAGNETIC CHARGE

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We study the spherically symmetric configuration of two SO(2) doublets anomalously coupled to the electromagnetic field when the electromagnetic field has as its source magnetic charge. Though the configuration does not strictly represent a non-topological soliton, it mimics such a configuration if the potential is chosen to be of an appropriate form. The mass and the Q-charge of the configuration is calculated and a possible method of identifying such objects in the cosmos is pointed out.

1. Introduction

With the advent of gauge theories and the associated Higgs sector, the problem of finding solutions to gauge Higgs configurations has become an appealing field of endeavour for both the mathematician and the particle theorist[1]. The stunning success enjoyed by the standard model has also prodded theorists to ask what the significance would be of classical solutions to the gauge Higgs sector of the standard model SU(2) × U(1). Monopoles and dyons represent such solutions when the symmetry is broken to U(1) and the energy functional has a minimum[2]. Such monopole solutions have distinct homotophy characteristics and could be of significance in the early universe in providing a catalyst for proton decay[3]. With regard to alternative mechanisms to G. U. T.'s to generate the baryon asymmetry of the early universe the sphaleron has been suggested as a possible source to anomalous baryon number non-conservation[4]. The sphaleron represents a saddle point of the energy functional of the gauge Higgs lagrangian in the Weinberg Salam model. The above solution refers to the full gauge Higgs system with the characteristic properties of the solution having topological significance related to the group theoretic structure of a gauge group. If gauge symmetries are not present but global symmetries are, Coleman has asked what characteristics of field equations exist that have properties related to the global charge[5]. In this spirit he has invented the name Q-ball which represents the configuration of fields admitting a global symmetry stabilized against decay into the free pions of the theory by virtue of the fact that a Q-ball solution has a lower mass than that of the free pions of the theory with number specified by the Q-charge. Subsequent investigations along the same avenue of thought demonstrated that L-balls which were configurations of fermi fields and scalar fields inspired by the Galmini Roncadelli model of neutrino mass generation were also stable except for a slow decay into fermions at the surface, here the global symmetry is lepton number[6]. When gauge symmetries are studied simultaneously in Q-ball solutions, it is found at least for a U(1) gauge symmetry that the gauge charge prevents the Q-ball from growing too large in much the same way that the Coulomb force prevents the formation of a large Z nucleus[7]. In curved space we have studied the stability of a Q-ball held together by gravitation with the result that the Q-ball cannot get too large if it is to be stable[8]. These studies have prodded us to ask in general what other configurations of fields can possibly represent bound astrophysical objects and what are their characteristic properties on an astrophysical scale. In a previous note we have studied a configuration of two SO(2) doublets coupled to the electromagnetic field through a parity-violating anomalous term[9]. The configuration sat in the field of Abelian dyon. In this note we study the same configuration of fields only we couple it to the electromagnetic field through a distribution of magnetic charge. We immerse the configuration in the surrounding false vacuum of the two scalars and allow the center of the configuration to be in the true vacuum. We then calculate an approximate solution for the two scalar doublets, we also calculate the conserved Q-charge of the configuration and match the interior to the exterior solution to obtain the mass of configuration. Finally, we comment on possible probes for identifying such bound configuration of scalars coupled to magnetic charge on an astrophysical scale.

2. *Bound configuration of two scalar doublets coupled to magnetic charge*

We begin our analysis by writing the lagrangian of gravitation coupled to two SO(2) doublets with an anomalous scalar EM coupling:

$$L = \frac{C^4}{16\pi G} R\sqrt{-g} + \left\{ \frac{\partial^\mu \Phi^T \partial_\mu \Phi}{2} + \frac{\partial^\mu \psi^T \partial_\mu \psi}{2} - \frac{A_2}{4}\left(\Phi^T\Phi - \frac{A_1}{A_2}\right)^2 - \frac{\overline{A_2}}{4}\left(\psi^T\psi - \frac{\overline{A_1}}{\overline{A_2}}\right)^2 + \frac{\alpha \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\, \partial_\mu \Phi^T \partial_\nu \psi}{\sqrt{-g}} \right\} \sqrt{-g} \tag{2.1}$$

(α = constant, φ, ψ are two SO_2 doublets, A_1, A_2, $\overline{A_1}$, $\overline{A_2}$ are parameters in potential).

We have omitted the free lagrangian of EM at this point but will include its effects in the Einstein equation through its energy momentum tensor. For the metric we have the sphereically symmetric ansatz

$$(\mathrm{d}s)^2 = \mathrm{e}^{\nu}(\mathrm{d}x^4)^2 - \mathrm{e}^{\lambda}(\mathrm{d}r)^2 - r^2(\mathrm{d}\Theta)^2 - r^2 \sin^2\Theta\,(\mathrm{d}\varphi)^2. \tag{2.2}$$

For the scalar doublets we have

$$\Phi = \begin{pmatrix} \Phi(r)\cos\omega t \\ \Phi(r)\sin\omega t \end{pmatrix}, \qquad \psi = \begin{pmatrix} -\psi(r)\sin\omega t \\ \psi(r)\cos\omega t \end{pmatrix} \tag{2.3}$$

which represent the SO(2) symmetric scalar doublet solutions with rotation frequency ω. The above lagrangian is invariant under the global SO_2 transformation

$$\Phi \to \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right] \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \tag{2.4}$$

$$\delta\Phi_1 = -\varepsilon\Phi_2, \qquad \delta\Phi_2 = \varepsilon\Phi_1$$

$$\psi \to \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2.5}$$

$$\delta\psi_1 = -\varepsilon\psi_2, \qquad \delta\psi_2 = \varepsilon\psi_1.$$

For the electromagnetic field we have a distribution of magnetic charge with local constant charge density ϱ_0, for the equation to determine $F_{\mu\nu}$ we have

$$\frac{\partial}{\partial x^{\nu}}\left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}}{2}\right) = 4\pi\sqrt{-g}\, J^{\mu}_{M} \tag{2.6}$$

which represents the equation for the dual to the EM field. We choose a constant radial magnetic field ($B_r = B_0$),

$$F_{23} = r^2 \sin\Theta B_r = r^2 \sin\Theta B_0,$$

giving

$$\frac{\partial}{\partial r}(\varepsilon^{4123} r^2 B_0) = 4\pi r^2 \mathrm{e}^{(\lambda+\nu)/2} \varrho_0 \mathrm{e}^{-\nu/2} = 4\pi\varrho_0 r^2 \mathrm{e}^{\lambda/2} \, (J^4_M = \varrho_0 \mathrm{e}^{-\nu/2}) \tag{2.7}$$

or

$$\varrho_0 = \frac{B_0 \mathrm{e}^{-\lambda/2}}{2\pi r} \tag{2.8}$$

after cancelling $\sin\Theta$ from Eq. (2.7).

Thus the magnetic charge density varies as $1/r$ to generate a constant radial component magnetic field. To calculate the radial component of the doublets we first evaluate the scalar part of the lagrangian including the anomalous EM scalar coupling from Eq. (2.1) after integrating over Θ, φ

$$L = \left[\begin{array}{l} \frac{e^{-\nu}\omega^2(\Phi(r))^2}{2c^2} - \frac{e^{-\lambda}(\Phi_{,r})^2}{2} + \frac{e^{-\nu}\omega^2(\psi(r))^2}{2c^2} - \frac{e^{-\lambda}(\psi_{,r})^2}{2} \\ -\frac{A_2}{4}\left((\Phi(r))^2 - \frac{A_1}{A_2}\right)^2 - \frac{\overline{A_2}}{4}\left((\psi(r))^2 - \frac{\overline{A_1}}{\overline{A_2}}\right)^2 \\ + a2r^2 B_0 \left[\frac{\omega\Phi\psi_{,r}}{c} + \frac{\omega\Phi_{,r}\psi}{c}\right] 4\pi. \end{array} \right] 4\pi r^2 e^{(\lambda+\nu)/2}. \tag{2.9}$$

Varying Eq. (2.9) with respect to $\Phi(r)$, $\psi(r)$ we have

$$\frac{\omega^2}{c^2}\Phi r^2 e^{(\lambda-\nu)/2} - A_2\Phi\left(\Phi^2 - \frac{A_1}{A_2}\right) r^2 e^{(\lambda+\nu)/2} + \frac{2ar^2B_0\omega\psi_{,r}}{c} -$$

$$-\frac{d}{dr}\left(-e^{(\lambda-\nu)/2} r^2\Phi_{,r}\right) - \frac{d}{dr}\left(\frac{2ar^2B_0\omega\psi'_{,}}{c}\right) = 0 \tag{2.10}$$

$$\frac{\omega^2}{c^2}\psi r^2 e^{(\lambda-\nu)/2} - \overline{A_2}\psi\left(\psi^2 - \frac{\overline{A_1}}{\overline{A_2}}\right) r^2 e^{(\lambda+\nu)/2} + \frac{2ar^2B_0\omega\Phi_{,r}}{c}$$

$$\frac{d}{dr}\left(r^2 e^{(\lambda-\nu)/2}\psi_{,r}\right) - \frac{d}{dr}\left(\frac{2ar^2B_0\omega\Phi}{c}\right) = 0. \tag{2.11}$$

To obtain an approximate solution for $\Phi(r)$, $\psi(r)$ we approximate

$$e^{\nu} \cong e^{\lambda} \cong 1$$

in Eq. (2.10) and Eq. (2.11), giving

$$r^2\Phi_{,rr} + 2r\Phi_{,r} + \frac{\omega^2}{c^2}\Phi r^2 - A_2\Phi\left(\Phi^2 - \frac{A_1}{A_2}\right) r^2 + \frac{2ar^2B_0\omega\psi_{,r}}{c} -$$

$$-\frac{4ar}{c}B_0\omega\psi - \frac{2ar^2B_0\psi_{,r}\omega}{c} = 0 \tag{2.12}$$

$$r^2\psi_{,rr} + 2r\psi_{,r} + \frac{\omega^2}{c^2}\psi r^2 - \overline{A_2}\psi\left(\psi^2 - \frac{\overline{A_1}}{\overline{A_2}}\right) r^2 + \frac{2ar^2B_0\omega\Phi_{,r}}{c} -$$

$$-\frac{4arB_0\omega\Phi}{c} - \frac{2ar^2B_0\Phi_{,r}\omega}{c} = 0. \tag{2.13}$$

By studying the linearized version of Eq. (2.12) and Eq. (2.13) we may demonstrate that the linearized equations admit the solutions

$$\Phi = \sum_{i=0}^{\infty} a_i r^i \tag{2.14}$$

$$\psi = \sum_{i=0}^{\infty} b_i r^i \tag{2.15}$$

about $r = 0$. If the self couplings $A_2, \overline{A}_2$ can be treated as perturbations then Eq. (2.12), Eq. (2.13) possess the power series solution Eq. (2.14), Eq. (2.15) about $r = 0$. To develop an approximate solution for small r we write down the first four terms of the power series

$$\Phi = a_0 + a_1 r + a_2 r^2 + a_3 r^3 \tag{2.16}$$

$$\psi = b_0 + b_1 r + b_2 r^2 + b_3 r^3. \tag{2.17}$$

For the B. C. we have

$$\Phi(0) = \sqrt{\frac{A_1}{A_2}}, \qquad \psi(0) = \sqrt{\frac{\overline{A}_1}{\overline{A}_2}}$$

and

$$\Phi(R) = \psi(R) = 0$$

for the two Higgs doublets, or the true vacuum at $r = 0$, and the false vacuum at $r = R$ (boundary of configuration). Also for $r > 0$, we have the false vacuum

$$\Phi = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{2.18}$$

Substituting Eq. (2.16) and Eq. (2.17) into Eq. (2.12) and Eq. (2.13) and setting the coefficients of r^α up to $\alpha = 3$ equal to zero we obtain the following first four terms of the power series after realizing that

$$a_0 = \sqrt{\frac{A_1}{A_2}}; \qquad b_0 = \sqrt{\frac{\overline{A}_1}{\overline{A}_2}}$$

from

$$\Phi(0) = \sqrt{\frac{A_1}{A_2}}; \qquad \psi(0) = \sqrt{\frac{\overline{A}_1}{\overline{A}_2}} \tag{2.19}$$

$$a_0 = \sqrt{\frac{A_1}{A_2}}, \qquad a_1 = \left(\frac{2\alpha}{c}\right) B_0 \omega \sqrt{\frac{\overline{A}_1}{\overline{A}_2}} \tag{2.20}$$

$$a_2 = -\frac{\omega^2}{6c^2}\sqrt{\frac{A_1}{A_2}} + \frac{A_2}{6}\left(\frac{A_1}{A_2}\right)^{3/2} - \frac{1}{6}\frac{A_1^{3/2}}{A_2^{1/2}} + \left(\frac{4a}{6c}\right) B_0\omega \left(\frac{2a}{c}\right) B_0\omega \sqrt{\frac{A_1}{A_2}}$$

$$a_3 = -\frac{a_1\omega^2}{12c^2} + \frac{1}{6} A_2 a_0^2 a_1 + \frac{1}{12} A_2 a_1 a_0^2 - \frac{1}{12} A_1 a_1 + \frac{aB_0\omega}{3c} b_2$$

with b_0, b_1, b_2, b_3 being of the same form with the replacement $A_1 \leftrightarrow \overline{A}_1, A_2 \leftrightarrow \overline{A}_2$; $a_1, a_2, a_3, a_0 \to b_1, b_2, b_3, b_0$.

To determine a_4 and b_4 we match the solutions in Eq. (2.16), Eq. (2.17) with the quartic term added to the false vacuum

$$\Phi(R) = \psi(R) = 0$$

at $r = R$ (radius of configuration). Actually, a_4, b_4 from the power series solution inserted into Eq. (2.12), Eq. (2.13) would give different results than their determination from the matching to the false vacuum at $r = R$, but since we are only dealing with an approximate solution up to the quartic power, we will find the error small for small R. Thus at $r = R$

$$a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4 = 0 \quad \text{at} \quad r = R \tag{2.21}$$

$$b_0 + b_1 R + b_2 R^2 + b_3 R^3 + b_4 R^4 = 0 \quad \text{at} \quad r = R \tag{2.22}$$

or

$$a_4 = -\frac{1}{R^4}[a_0 + a_1 R + a_2 R^2 + a_3 R^3] \tag{2.23}$$

$$b_4 = -\frac{1}{R^4}[b_0 + b_1 R + b_2 R^2 + b_3 R^3]. \tag{2.24}$$

We next calculate the conserved Q-charge, by varying Eq. (2.1) using the field equation for $\Phi_1, \Phi_2, \psi_1, \psi_2$ we shave

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial \Phi_{1,\mu}}\delta\Phi_1 + \frac{\partial L}{\partial \Phi_{2,\mu}}\delta\Phi_2 + \frac{\partial L}{\partial \psi_{1,\mu}}\delta\psi_1 + \frac{\partial L}{\partial \psi_{2,\mu}}\delta\psi_2\right] = 0. \tag{2.25}$$

Integrating over r, Θ, φ, from 0 to R; $[0, \pi]$; $[0, 2\pi]$ we find after using Eq. (2.4), Eq. (2.5)

$$Q = \iiint \left[\frac{\partial L}{\partial \Phi_{1,4}}\delta\Phi_1 + \frac{\partial L}{\partial \Phi_{2,4}}\delta\Phi_2 + \frac{\partial L}{\partial \psi_{1,4}}\delta\psi_1 + \frac{\partial L}{\partial \psi_{2,4}}\delta\psi_2\right] \mathrm{d}r\mathrm{d}\Theta\mathrm{d}\varphi. \tag{2.26}$$

Eq. (2.26) gives after using Eq. (2.1), Eq. (2.4), Eq. (2.5) and Eq. (2.7) for F_{23} (Ref. 9) and approximating $e^{\nu} \simeq e^{\lambda} \simeq 1$

$$Q = \frac{4\pi\omega}{c} \int_0^R (\Phi(r))^2 r^2 \mathrm{d}r + \frac{4\pi\omega}{c} \int_0^R (\psi(r))^2 r^2 \mathrm{d}r + 8\pi a B_0 \int_0^R r^2 (\Phi \psi_{1r} + \Phi_{1r} \psi) \, \mathrm{d}r. \tag{2.27}$$

If the solutions in Eq. (2.16), Eq. (2.17) including the quartic term are substituted into Eq. (2.27) we have an equation to determine ω (the angular frequency) in terms of the conserved Q-charge. Our final task is to evaluate the mass of the configuration of scalars coupled to the magnetic charge, for $r > R$ from Eq. (2.6) we have

$$\frac{\partial}{\partial x^{\nu}} \left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}}{2} \right) = 0$$

or

$$\frac{\partial}{\partial r} (r^2 B_r) = 0, \qquad B_r = \frac{q}{r^2} \tag{2.28}$$

where q = total magnetic charge of configuration. Matching the constant internal magnetic field at $r = R$ to Eq. (2.28) we have

$$B_0 = \frac{q}{R^2}, \qquad q = B_0 R^2. \tag{2.29}$$

This represents an expression for the magnetic charge of the sphere ($R \geq r$) in terms of the constant internal radial field B_0. The energy momentum tensor for the EM field has the same form in the presence of magnetic charge as it does in the case when magnetic charge is absent as emphasized by Semiz[10]. From Eq. (2.1) we have the energy momentum tensor for the scalar doublets coupled to electromagnetic field

$$\begin{aligned} (T_{\mu\nu})_1 = \frac{2}{\sqrt{-g}} \frac{\partial L}{\partial g^{\mu\nu}} &= \partial_\mu \Phi^T \partial_\nu \Phi + \partial_\mu \psi^T \partial_\nu \psi - \\ &- \frac{g_{\mu\nu}}{2} [\partial^\alpha \Phi^T \partial_\alpha \Phi + \partial^\alpha \psi^T \partial_\alpha \psi] + \\ &+ g_{\mu\nu} \left[\frac{A_2}{4} \left(\Phi^T \Phi - \frac{A_1}{A_2} \right)^2 + \frac{\overline{A}_2}{4} \left(\psi^T \psi - \frac{\overline{A}_1}{\overline{A}_2} \right)^2 \right]. \end{aligned} \tag{2.30}$$

For the electromagnetic field we have

$$(T_{\mu\nu})_2 = \frac{g_{\mu\nu}}{16\pi} (F_{\alpha\beta} F^{\alpha\beta}) - \frac{1}{4\pi} F_{\mu\alpha} F^{\alpha}_{\nu}. \tag{2.31}$$

Thus for $0 < R$ we have

$$T_4^4 = (T_4^4)_1 + (T_4^4)_2$$

$$T_4^4 = \frac{\omega^2 (\Phi(r))^2}{2c^2} + \frac{\omega^2 (\psi(r))^2}{2c^2} + \frac{1}{2}(\Phi_{1r})^2 + \frac{1}{2}(\psi_{1r})^2 + \tag{2.32}$$

$$+ \frac{A_2}{4}\left((\Phi(r))^2 - \frac{A_1}{A_2}\right)^2 + \frac{\overline{A}_2}{4}\left((\psi(r))^2 - \frac{\overline{A}_1}{\overline{A}_2}\right)^2 + \frac{B_0^2}{8\pi}$$

where we have approximated $e^{\nu} \cong e^{\lambda} \cong 1$ in the scalar terms of the energy momentum tensor.

For $r > R$ we have,

$$(T_4^4)_1 + (T_4^4)_2 = \frac{A_1^2}{4A_2} + \frac{\overline{A}_1^2}{4\overline{A}_2} + \frac{q^2}{8\pi r^4}. \tag{2.33}$$

From the $\binom{4}{4}$ component of the Einstein equations we have

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\mathrm{e}^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 T_4^4. \tag{2.34}$$

Inserting Eq. (2.32) into Eq. (2.34) we have for $r = R$ after integration from 0 to R

$$(\mathrm{e}^{-\lambda})_R = 1 - \frac{8\pi G}{c^4 R}\int_0^R r^2 T_4^4 \mathrm{d}r. \tag{2.35}$$

For $r > R$ we have from Eq. (2.33)

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\mathrm{e}^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 \left(\frac{A_1^2}{4A_2} + \frac{\overline{A}_1^2}{4\overline{A}_2} + \frac{q^2}{8\pi r^4}\right). \tag{2.36}$$

Integrating gives for $r > R$

$$\mathrm{e}^{-\lambda} = 1 - \frac{2GM}{rc^2} - \frac{8\pi G}{3c^4} r^2 \left(\frac{A_1^2}{4A_2} + \frac{\overline{A}_1^2}{4\overline{A}_2}\right) + \frac{Gq^2}{r^2 c^4}. \tag{2.37}$$

Matching Eq. (2.35) to Eq. (2.37) at $r > R$ will give an expression for M. (Mass of configuration of the SO(2) scalar doublets coupled to the magnetic charge).

To look for such configurations of SO_2 scalar fields coupled to magnetic charge we note that ω in Eq. (2.32) depends on Q (conserved Q-charge) through Eq. (2.27), when this value of ω is substituted in Eq. (2.32) and the metric is evaluated from Eq. (2.35) at $r = R$ and matched to Eq. (2.37) we find that the mass depends on both the conserved Q-charge (SO_2 global charge) and the magnetic charge within the configuration for $R > r$. Any red shift of radiation emitted by such objects would have characteristics depending on both Q and the magnetic charge q which might serve to identify a class of these objects when the red shift is plotted versus the two parameters Q (the SO_2 charge), and q (the magnetic charge). In general magnetic charge is the inevitable consequence of a grand unified theory when a semi-simple gauge group is broken to a sub-group with a residual $U(1)_{EM}$ factor. The long range field of a G.U.T. monopole or dyon (a dyon being a monopole with added electric charge) appears as a abelian. Monopoles or dyons may affect the mass density of the universe and can also be the component of missing dark matter in galaxies, if this is the case, their density might be set to agree with the cosmological mass density to give a flat universe and on a galatic scale a population density to generate the fraction of missing dark matter required in galaxies. The »Parker bound« on magnetic charge or monopole density results from the requirement that the magnetic galatic field not be depleted[11], this bound gives a flux density of incident monopoles of less than 10^{-13} cm^{-2} sr^{-1} s^{-1} for monopoles of $m = 10^{19}$ GeV[12]. However there also exists an opposing viewpoint that monopoles or magnetic charge might generate the magnetic field of the galaxy through slow magnetic monopole plasma oscillations of a very long period[13]. It is also well known that inflation provided a resolution to the cosmological problem of the overproduction of monopoles that would form when the G. U. T. Higgs field was uncorrelated in the early universe, however even if monopoles were not produced in the early G. U. T. era they might be produced by energetic particle collisions around the time of helium synthesis and combined with antimonopoles to generate monopolonium with appreciable density that survives until the present epoch[14]. Such a bound state of a monopole and antimonopole could be observed through its decay into a whole spectrum of G. U. T. particles which have characteristic signatures finally manifested in the quark lepton signatures produced. The lifetime of monopolonium can be very large ($\tau \approx 10^{22}$ s) and its initial stages are characterized by the emission of everything from infrared to γ rays. In a laboratory superconducting monopole detection[15], and acoustic detection of monopoles[16] represent experimental probes to the upper limit of the monopole flux in the galaxy. In short, G. U. T. theory predicts monopoles and the possible localization of an ensemble of monopoles or magnetic charges produced around the period of helium synthesis might mimic the physical model in this paper wherein the effective electro-magnetic field of the magnetic charge distribution couples to the Higgs field through loops of G. U. T. particles or quarks circulating in the interior of a Feynman diagram that has external legs of two external scalars and the effective electromagnetic field as in Eq. (2.1). The model we use might be different than the actual G. U. T. model stemming from a G. U. T. group but the general features of the classical field structure is similar.

3. *Conclusion*

The above expressions in Eq. (2.14), Eq. (2.15) and Eq. (2.37) represent a solution for the scalar doublets coupled to magnetic charge when the magnetic field for $R > r$ is constant and the magnetic charge density is given by Eq. (2.8) for $R > r$. For the metric we have the $\binom{1}{1}$ component given by Eq. (2.35) and Eq. (2.37) where matching the two expressions at $r = R$ gives an expression for the mass of the Q configuration of scalar fields coupled to magnetic charge. The angular frequency can be expressed in terms of the Q-charge using Eq. (2.27). The anomalous scalar E. M. coupling in Eq. (2.1) can be motivated by considering a photon coupled to two external scalars through a quark loop in much the same way that the standard axion coupling[17] to the E. M. field results when an axion is coupled to a quark loop with the internal quarks coupled to two external photons, in our case the scalars replace the photon of the axion E. M. coupling and the axion is replaced by a photon through $F_{\mu\nu}$ as

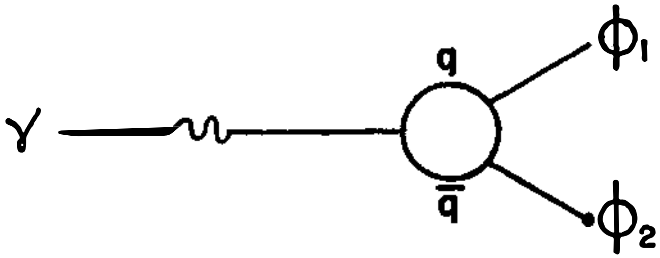

Fig. 1.

In a true G. U. T. theory or super-symmetric G. U. T. theory containing two SU(2) scalar doublets at low energy, a lagrangian analogous to Eq. (2.1) would result when the scalars couple to opposite ends of a quark loop with an external photon connected to the loop. Though upper limits have been given for the monopole flux or the magnetic charge flux in the galaxy which discourages any accumulation of magnetic charge in the galaxy, it is not out of the question that accumulation of magnetic charges outside the galaxy might result around the period of helium synthesis that have not been observed yet or have been mistaken for other objects. If such magnetic charge configurations are coupled to Higgs fields in the manner represented by Eq. (2.1) then they would generate red shifts which would depend on both the Q-charge and the magnetic charge of the configuration. A study of the red shift versus these two parameters (Q, q) could then be used to identify a series of these condensed objects with similar properties. Since magnetic charge (through magnetic monopoles) and scalar doublet Higgs fields have received widespread attention in G. U. T. theories and symmetry breaking mechanisms it is hoped that the model studied in this paper suggests a fusion of these two aspects of particle theory that suggest the presence of astro-physical objects carrying the properties discussed above. Magnetically charged Q-balls, discussed in this paper, normal Q-balls, black holes, Kerr black holes[18] and neutrino balls[19] all represent condensed objects predicted by particle theory that still await to be discovered by the experimentalist in the cosmos.

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KONFIGURACIJA DUBLETA DVA HIGGSA KOJA OMOGUĆUJE SO(2) GLOBALNU SIMETRIJU ANOMALNO VEZANU S MAGNETSKIM NABOJEM

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Proučavana je sferično simetrična konfiguracija dvaju SO(2) dubleta anomalno vezanih s elektromagnetskim poljem kada elektromagnetsko polje ima magnetski naboj kao svoj izvor. Premda ova konfiguracija striktno ne predstavlja netopološki soliton, ona je slična takvoj konfiguraciji ako se izabere potencijal odgovarajućeg oblika. Masa i *Q*-naboj konfiguracije su izračunati te je istaknuta moguća metoda identificiranja takvih objekata u kozmosu.