

## STABILITY OF SYMMETRY BREAKING IN AN SU (5) MODEL WITH THE 75 HIGGS FIELD

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We consider the symmetry breaking of the SU (5) symmetry to the physical  $SU(3) \times SU(2) \times U(1)$  symmetry induced by an effective 75-dimensional Higgs field. We present a one-loop renormalization-group analysis of the stability of the  $SU(3) \times SU(2) \times U(1)$  symmetry of the absolute minimum. We have found regions in the full parameter space where all stability requirements, between the Planck scale  $10^{19}$  GeV and the typical GUT scale  $10^{16}$  GeV, are satisfied. The usefulness of the sum-of-square potential in the stability analysis is explicitly demonstrated.

### 1. Introduction

During the last two decades there has been a permanent search for a theory beyond the standard model. Although such a theory has not been found, there is hope to approach it through the construction of some effective theories, most of them based on spontaneously broken gauge symmetries. It is true that such a strategy does not lead directly to a more fundamental theory; however, it might shed some light on specific problems. For example, one of the latest attempts to construct an effective theory has been motivated by the intention to understand the origin of mass in the standard model<sup>1)</sup>.

The true origin of symmetry breaking in the standard model, namely the breaking of the  $SU(2) \times U(1)$  to the  $U(1)$  group, is still not understood. Never-

theless, it is very well described effectively by replacing the true mechanism by the Higgs mechanism. In the same way, the Higgs mechanism is usually used in effective theories beyond the standard model, and is generally very complicated from the technical point of view. In fact, there are two main technical problems. First, it is necessary to find the absolute minimum of the corresponding Higgs potential, which generally depends on many variables, i. e. to find the vacuum of the theory. Second, the stability of the physically desired vacuum should be ensured under renormalization-group equations (RGE's) between physically relevant momentum scales, i. e. the stability under radiative (one-loop) corrections. The methods for solving these problems were extensively discussed and applied to some grand-unified models (GUT's)<sup>2-4)</sup>.

In this paper we extend the stability analysis of the symmetry breaking of the SU(5) GUT model with the 75 Higgs field<sup>4,5)</sup>, to the SU(3) × SU(2) × U(1) standard model. It was shown<sup>4)</sup> that in the original SU(5) × Z<sub>2</sub> model, with the additional discrete Z<sub>2</sub> symmetry, the breaking to the physical SU(3) × SU(2) × U(1) phase was highly unstable under RGE's. It was suggested that the extended RGE stability analysis applied to the most general SU(5)-invariant Higgs potential might lead to the stable SU(3) × SU(2) × U(1) phase, which is an essential natural requirement.

Our additional motivation is to demonstrate the usefulness of the «sum-of-squares» Higgs potential, especially in models with a higher-rank Higgs representation<sup>6)</sup>. The plan of the paper is as follows. In Section 2 we write the most general SU(5)-invariant Higgs potential with the 75 Higgs representation. In Section 3 we present the stationary points of the potential. Section 4 contains the symmetry-breaking conditions from SU(5) to SU(3) × SU(2) × U(1). In Section 5 we present a full set of renormalization-group equations. In Section 6 we discuss the stability of the symmetry breaking from SU(5) to SU(3) × SU(2) × U(1) and present numerical results with conclusions.

## 2. The SU(5) Higgs potential

The most general SU(5)-invariant Higgs potential with one 75 irreducible representation Higgs field  $\Phi$  is<sup>7)</sup>

$$V(\Phi) = -\mu^2 Q + \gamma C + \frac{1}{2} \lambda_0 Q^2 + \lambda_1 I_1 + \lambda_2 I_2, \quad (1)$$

where

$$Q = \Phi_{\gamma\delta}^{\alpha\beta} \Phi_{\alpha\beta}^{\gamma\delta},$$

$$C = \Phi_{\gamma\delta}^{\alpha\beta} \Phi_{\mu\nu}^{\gamma\delta} \Phi_{\alpha\beta}^{\mu\nu}, \quad (2)$$

$$I_1 = \Phi_{\gamma\delta}^{\alpha\beta} \Phi_{\mu\nu}^{\gamma\delta} \Phi_{\lambda\varrho}^{\mu\nu} \Phi_{\alpha\beta}^{\lambda\varrho},$$

$$I_2 = \Phi_{\gamma\delta}^{\alpha\beta} \Phi_{\mu\beta}^{\gamma\delta} \Phi_{\lambda\varrho}^{\mu\nu} \Phi_{\alpha\nu}^{\lambda\varrho}.$$

Here the indices  $\alpha, \beta, \gamma, \delta, \nu, \lambda, \rho$  run from 1 to 5 and summation over repeated indices is understood. The self-adjoint fourth-rank tensor  $\Phi$ , representing the 75 irreducible representation, satisfies the following relations:

$$\Phi_{\gamma\delta}^{\alpha\beta} = -\Phi_{\gamma\delta}^{\beta\alpha} = -\Phi_{\delta\gamma}^{\alpha\beta},$$

$$\Phi_{\alpha\delta}^{\alpha\beta} = 0 \quad \text{and} \quad \Phi_{\gamma\delta}^{\alpha\beta} = (\Phi_{\alpha\beta}^{\gamma\delta})^*.$$

For the coupling constants  $\mu^2, \gamma, \lambda_0, \lambda_1, \lambda_2$  in the potential (1), we assume that  $\mu^2 > 0$ , and the  $\lambda_0, \lambda_1, \lambda_2$  are contained in such a way that  $V(\Phi)$  is bounded from below. The sufficient (and almost required) conditions for  $V(\Phi)$  to be bounded are<sup>5)</sup>

$$\begin{aligned} \frac{1}{2} \lambda_0 + \frac{1}{10} \lambda_1 + \frac{1}{5} \lambda_2 &> 0, \\ \frac{1}{2} \lambda_0 + \frac{5}{18} \lambda_1 + \frac{7}{27} \lambda_2 &> 0, \\ \frac{1}{2} \lambda_0 + \frac{7}{10} \lambda_1 + \frac{1}{4} \lambda_2 &> 0, \\ \frac{1}{2} \lambda_0 + \frac{1}{6} \lambda_1 + \frac{1}{4} \lambda_2 &> 0, \\ \frac{1}{2} \lambda_0 + \frac{20 - \sqrt{65}}{50} \lambda_1 + \frac{1}{5} \lambda_2 &> 0. \end{aligned} \tag{3}$$

### 3. Extremal points of the most general potential

Let  $H$  be a subgroup of  $SU(5)$  such that the decomposition of  $\Phi$  under  $H$  contains only one  $H$  singlet with the corresponding direction  $\hat{S}$  in a 75-dimensional space spanned by  $\Phi$ . The any potential  $V(\Phi)$  in Eq. (1) has an extremum that lies along the  $\hat{S}$  direction. Let this extremum be at the point  $\langle \Phi \rangle = q \cdot \hat{S}$  with  $q \in R$  and  $Q(\hat{S}) = \hat{S}_{\gamma\delta}^{\alpha\beta} \hat{S}_{\alpha\beta}^{\gamma\delta} = 1$ , so that  $\langle \Phi \rangle^2 = q^2$ . Hence, the form of the potential  $V(\Phi)$  in (1) can be written as

$$V(\langle \Phi \rangle) = -\mu^2 q^2 + \Gamma q^3 + \Lambda q^4, \tag{4}$$

where

$$\Gamma = \gamma C(\hat{S}), \tag{5}$$

$$\Lambda = \frac{1}{2} \lambda_0 + \lambda_1 I_1(\hat{S}) + \lambda_2 I_2(\hat{S}). \tag{6}$$

The stationary equation for  $V(\Phi)$  along the  $\hat{S}$  direction is

$$\frac{\partial V}{\partial q} = q \cdot [-2\mu^2 + 3\Gamma q + 4\Lambda q^2] = 0. \quad (7)$$

The lower of the two extrema will occur at

$$q = -\frac{3\Gamma}{8\Lambda} - (\text{sign } \Gamma) \left( \left( \frac{3\Gamma}{8\Lambda} \right)^2 + \left( \frac{\mu^2}{2\Lambda} \right) \right)^{1/2} \quad (8)$$

with the value of the potential at this point:

$$V(\langle \Phi \rangle) = q^2 [-\mu^2 + \Gamma q + \Lambda q^2] = -\frac{1}{3} q^2 [\mu^2 + \Lambda q^2]. \quad (9)$$

Of course, Eqs. (7)–(9) are valid for an arbitrary direction  $\Phi$  in the 75-dimensional space and correspond to the directional extremum. We point out that these relations imply the extrema of  $V(\Phi)$ , i. e.  $\frac{\partial V}{\partial \Phi} \Big|_{q\hat{S}} = 0$ , for special directions  $\hat{S}$  described at the beginning of this section<sup>6)</sup>.

Hence, the extremal points of the potential (Eq. (1)) appear along the direction  $\hat{S}_1, \hat{S}_2, \hat{S}_3, \hat{S}_4, \hat{S}_5, \hat{S}_6$  which are invariant under the maximal little groups  $U(1) \times U(1) \times U(1) \times U(1), SU(2) \times SU(2) \times U(1), SO(3), SU(2) \times SU(2) \times U(1) \times U(1) \times D_4, SU(3) \times SU(2) \times U(1), Sp(4) \times U(1)$ , respectively. We also note that all  $\hat{S}_i$  ( $i = 1, \dots, 6$ ) satisfy the quadratic eigenvalue equation<sup>6)</sup>. The values  $C(\hat{S}_i), I_1(\hat{S}_i), I_2(\hat{S}_i)$  for  $i = 1, \dots, 6$  are given in Table 1. We note that  $Q(\hat{S}_i) = 1$ .

#### 4. Symmetry breaking from $SU(5)$ to $SU(3) \times SU(2) \times U(1)$

We observe that sufficient conditions for an extremum to be a local minimum are given by the requirement that all squared Higgs masses be positive (i. e. that the Hessian  $\partial^2 V / \partial \Phi_i \partial \Phi_j$  be positive definite) calculated at the extremal point  $\langle \Phi \rangle$ . Especially, sufficient conditions that at the  $SU(3) \times SU(2) \times U(1)$ -invariant extremal point  $\langle \Phi \rangle = q_s \hat{S}_5$  the potential  $V(q_s \hat{S}_5)$  should have a local minimum<sup>7)</sup> are

$$12\Lambda_s q_s^2 + \sqrt{2} \gamma q_s > 0,$$

$$-4(3\lambda_1 + \lambda_2) q_s^2 - 9\sqrt{2} \gamma q_s > 0,$$

$$-4(3\lambda_1 + \lambda_2)q_s^2 - 45\sqrt{2}\gamma q_s > 0, \quad (10)$$

$$-(12\lambda_1 + 5\lambda_2)q_s^2 - 9\sqrt{2}\gamma q_s > 0,$$

$$(12\lambda_1 - \lambda_2)q_s^2 + 9\sqrt{2}\gamma q_s > 0,$$

where

$$q_s = -\frac{3\Gamma_s}{8A_s} - (\text{sign } \Gamma_s) \left[ \left( \frac{3\Gamma_s}{8A_s} \right)^2 + \frac{\mu^2}{2A_s} \right]^{1/2}, \quad (11)$$

with

$$\Gamma_s = \frac{2\sqrt{2}}{9}\gamma, \quad (12)$$

$$A_s = \frac{1}{2}\lambda_0 + \frac{5}{18}\lambda_1 + \frac{7}{27}\lambda_0. \quad (13)$$

The inequalities (10) correspond to  $m_{(1,1,0)}^2$ ,  $m_{(8,1,0)}^2$ ,  $m_{(8,3,0)}^2$ ,  $m_{(6,2,-5/3)}^2$  and  $m_{(3,2,-5/6)}^2$ , respectively.

Furthermore, the conditions (3) ensure that the potential be bounded from below, implying the absolute minimum. Physically, the true stable vacuum should be identified with the absolute minimum of the potential. Of course, the conditions (10) do not imply the existence of the  $SU(3) \times SU(2) \times U(1)$ -invariant absolute minimum. Therefore, we additionally demand that the value of the  $SU(3) \times SU(2) \times U(1)$  local minimum be lower than the values of the potential at all other extremal points, i. e.

$$V(q_s \hat{S}_i) < V(q_i \hat{S}_i) \quad \text{for } i = 1, 2, 3, 4, 6. \quad (14)$$

TABLE 1.

|   |             | $C(\hat{S}_i)$                     | $I_1(\hat{S}_i)$ | $I_2(\hat{S}_i)$ |
|---|-------------|------------------------------------|------------------|------------------|
| $U(1) \times U(1) \times U(1) \times U(1)$            | $\hat{S}_1$ | 0                                  | $\frac{1}{10}$   | $\frac{1}{5}$    |
| $SU(2) \times SU(2) \times U(1)$                      | $\hat{S}_2$ | 0                                  | $\frac{1}{6}$    | $\frac{1}{4}$    |
| $SO(3)$   | $\hat{S}_3$ | $\left(\frac{8}{105}\right)^{1/2}$ | $\frac{37}{210}$ | $\frac{1}{5}$    |
| $SU(2) \times SU(2) \times U(1) \times U(1) \times D$ | $\hat{S}_4$ | $\left(\frac{1}{12}\right)^{1/2}$  | $\frac{1}{4}$    | $\frac{1}{4}$    |
| $SU(3) \times SU(2) \times U(1)$                      | $\hat{S}_5$ | $\frac{2}{9}\sqrt{2}$              | $\frac{5}{18}$   | $\frac{7}{27}$   |
| $Sp(4) \times U(1)$                                   | $\hat{S}_6$ | $\left(\frac{8}{15}\right)^{1/2}$  | $\frac{7}{10}$   | $\frac{1}{4}$    |

These requirements are necessary but not sufficient for a local minimum to become the absolute minimum. In fact, a given potential may have other extremal points different from those listed in Table 1. However, we have shown that the conditions (14) are much stronger than (3) and (10) and, moreover, by randomly calculating the directional extrema (Eq. (9)) for a few fixed potentials, we have shown that the conditions (14) are practically sufficient.

Finally, we use a special construction of the potential<sup>6)</sup> such that the corresponding absolute minimum has the desired  $SU(3) \times SU(2) \times U(1)$ -invariant minimum. This potential has a sum-of-square form and, for a fixed mass scale  $m$ , depends on two parameters, say  $A$  and  $B$ :

$$V_0(\Phi) = A \left( H - \frac{2\sqrt{2}}{9} m\Phi \right)^2 + B(Q - m^2)^2, \quad (15)$$

where

$$H(\Phi) = \frac{1}{3} \partial C / \partial \Phi \quad (16)$$

transforms as the 75 representation, projected out of the  $75 \times 75$ .

We note that for this sum-of-square-form potential (15), all inequalities (10) are automatically satisfied at the absolute minimum<sup>6)</sup>. We also observe that Eq. (15) is the most general construction of a quartic potential that guarantees the  $SU(3) \times SU(2) \times U(1)$ -invariant absolute minimum in advance. Of course, this form of the potential (15) can be perturbed by the most general additional invariants. Still, it can be found that the absolute minimum exists and that its symmetry is again the same  $SU(3) \times SU(2) \times U(1)$  group (up to conjugation).

### 5. Renormalization-group equations

There is also another requirement for stability<sup>2-4)</sup>. The effective GU theory with the unbroken  $SU(5)$  gauge group operates between the Planck scale  $M_{Pl} \approx 10^{19}$  GeV and the typical GUT scale  $M_x \approx 10^{16}$  GeV defining the scale of the symmetry breaking from  $SU(5)$  to  $SU(3) \times SU(2) \times U(1)$ . The coupling constants in the potential will evolve between these two scales, with varying momentum scale, according to the RGE's. Hence the absolute minimum of the potential will also vary with momentum scale. Of course, the symmetry of the absolute minimum could also change. It is therefore natural to demand stability<sup>2-4)</sup> of the physical  $SU(3) \times SU(2) \times U(1)$  symmetry of the vacuum between  $M_{Pl}$  and  $M_x$ , i. e. approximately three orders of magnitude in momentum scales:  $p_i/p_f \approx 10^3$ ,  $p_i \approx M_{Pl}$ ,  $p_f \approx M_x$ . In practice, this means that we wish to find a region of parameter space that leads to the same  $SU(3) \times SU(2) \times U(1)$  physical symmetry of the absolute minimum for approximately three orders of magnitude in momentum.

The RGE's for the scale dependence of the parameters in the potential (1) at the one-loop level are

$$\begin{aligned}\frac{d\lambda_0}{dt} &= \frac{1}{4\pi^2} \left[ \frac{83}{4} \lambda_0^2 + \frac{91}{6} \lambda_0 \lambda_1 + \frac{49}{3} \lambda_0 \lambda_2 + \frac{113}{36} \lambda_1^2 + \frac{49}{9} \lambda_1 \lambda_2 + \frac{143}{48} \lambda_2^2 - \right. \\ &\quad \left. - 24g^2 \lambda_0 - 3g^4 \right], \\ \frac{d\lambda_1}{dt} &= \frac{1}{4\pi^2} \left[ 3\lambda_0 \lambda_1 + \frac{13}{2} \lambda_1^2 + 2\lambda_1 \lambda_2 - \frac{1}{144} \lambda_2^2 - 24g^2 \lambda_1 + 9g^4 \right], \\ \frac{d\lambda_2}{dt} &= \frac{1}{4\pi^2} \left[ 3\lambda_0 \lambda_1 - 2\lambda_1^2 + \frac{7}{3} \lambda_1 \lambda_2 + \frac{17}{9} \lambda_2^2 - 24g^2 \lambda_2 + 27g^4 \right], \\ \frac{d\gamma}{dt} &= \frac{1}{16\pi^2} \left[ 6\lambda_0 + 38\lambda_1 + \frac{14}{3} \lambda_2 - 72g^2 \right] \gamma, \\ \frac{d\mu^2}{dt} &= -\frac{7}{8\pi^2} \gamma^2 + \frac{1}{16\pi^2} \mu^2 \left[ 77\lambda_0 + \frac{91}{3} \lambda_1 + \frac{98}{3} \lambda_2 - 48g^2 \right].\end{aligned}\tag{17}$$

From the last two equations for  $\gamma$  and  $\mu^2$  it follows that

$$\frac{d}{dt} \left( \frac{\gamma^2}{\mu^2} \right) = \frac{7}{8\pi^2} \left( \frac{\gamma^2}{\mu^2} \right)^2 - \frac{1}{16\pi^2} \left[ 65\lambda_0 - \frac{137}{3} \lambda_1 + \frac{70}{3} \lambda_2 + 96g^2 \right] \frac{\gamma^2}{\mu^2}.\tag{18}$$

The RG equation for the SU(5) gauge coupling constant is

$$\frac{dg}{dt} = -\frac{b_0}{32\pi^2} g^3, \quad b_0 = \frac{1}{3} (85 - 4N_f),\tag{19}$$

with  $t = \ln q/\Lambda_i$ , the evolution parameter, where  $\Lambda_i$  is the initial momentum and  $q$  is the running momentum. Here  $g$  is the SU(5) gauge coupling and  $N_f$  is the number of fermion generations, which we henceforth take to be three.

## 6. Stability of symmetry breaking from SU(5) to SU(3) $\times$ SU(2) $\times$ U(1)

We start at the scale  $\Lambda_i = M_{PI}$  ( $t = 0$ ), with the sum-of-square form of the Higgs potential (Eq. (15)):

$$V_0(\Phi) = A \left( H - \frac{2\sqrt{2}}{9} m\Phi \right)^2 + B(Q - m^2)^2.$$

If  $A > 0$ ,  $B > 0$ , this potential has an absolute minimum invariant under the  $SU(3) \times SU(2) \times U(1)$  symmetry.

The following relations hold:

$$\begin{aligned}\frac{1}{2} \lambda_0 &= \frac{1}{6} A + B, \\ \lambda_1 &= A, \\ \lambda_2 &= -\frac{4}{3} A, \\ \mu^2 &= \left( 2B - \frac{8}{81} A \right) m^2 > 0, \\ \gamma &= -\frac{4\sqrt{2}}{9} Am.\end{aligned}\tag{20}$$

We solve the RGE's (18) and (19) for  $\lambda_0$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $\gamma^2/\mu^2$  starting at  $t = 0$  with the above sum-of-square potential, i. e. respecting Eq. (20), and taking  $g$  between 0.2 and 0.3 at the scale  $A_t = M_{Pl}$ . For  $M_x < q < M_{Pl}$ , i. e.  $-\ln 10^3 < t < 0$ , we demand three types of condition:

(i) The potential  $V(\lambda(t), \Phi)$  should be bounded from below, i. e. the inequalities (3) should be satisfied.

(ii) Sufficient conditions for the local minimum invariant under the  $SU(3) \times SU(2) \times U(1)$  symmetry, i. e. the inequalities (10) should be satisfied.

(iii) Necessary conditions for the absolute minimum, i. e. the inequalities (14) should be satisfied.

Our aim is to find (compact) regions in the first quadrant of the  $A, B$  plane, defined by  $A > 0$ ,  $B > 0$ , in which conditions (i), (ii) and (iii), determined by Eqs. (3), (10) and (14), respectively, are satisfied at all scales between  $A_t = M_{Pl}$  and  $A_f = M_x$ . The initial sum-of-square-form potential automatically satisfies all the requirements (i), (ii) and (iii).

The RG equations have been investigated numerically using a fourth-order Runge-Kutta-Merson routine. Our results are shown in Figs. 1–5. Fig. 1 shows a selection of those initial  $A, B$  points for which all requirements (i)–(iii) are satisfied between  $M_{Pl}$  and  $A_f \geq 1/500 M_{Pl}$ . Similarly, Figs. 2 and 3 show regions in which all the three requirements are also satisfied, but now between  $M_{Pl}$  and lower final-momentum scales,  $A_f \geq 1/600 M_{Pl}$  and  $A_f \geq 1/700 M_{Pl}$ , respectively. Comparison of Figs. 1–3 shows how rapidly the allowed regions are getting smaller with lowering final-momentum scale. For  $A_t/A_f \geq 700$ , the allowed region in Fig. 3 is quite small.

Furthermore, our analysis has shown that conditions (i), which ensure that the potential be bounded from below, lead to very large regions. For example,

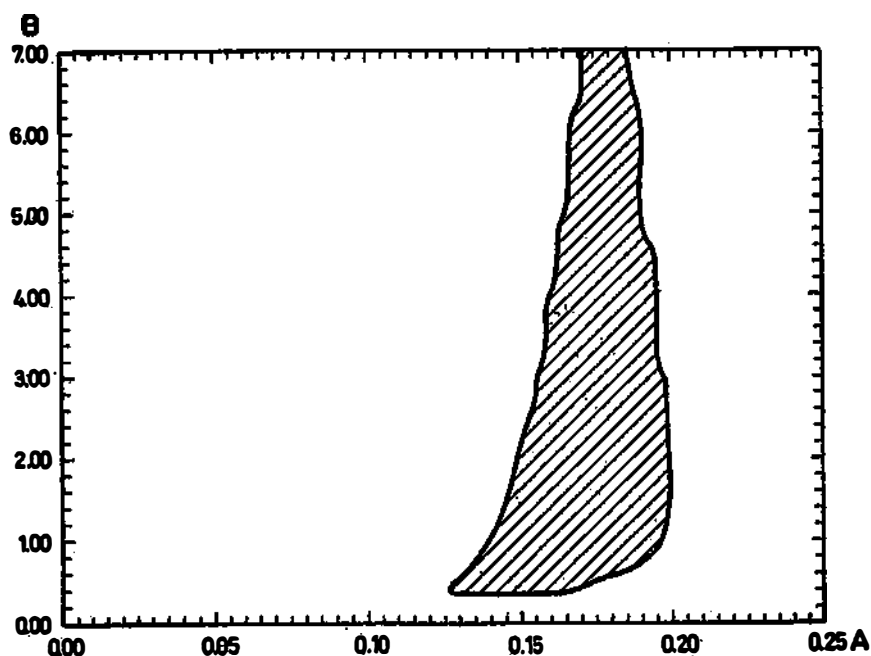


Fig. 1. Region of initial  $A, B$  points for which all requirements (i), (ii) and (iii) are satisfied at all momentum scales between  $\Lambda_t = M_{Pl}$  and  $\Lambda_f \leq 1/500 M_{Pl}$ .

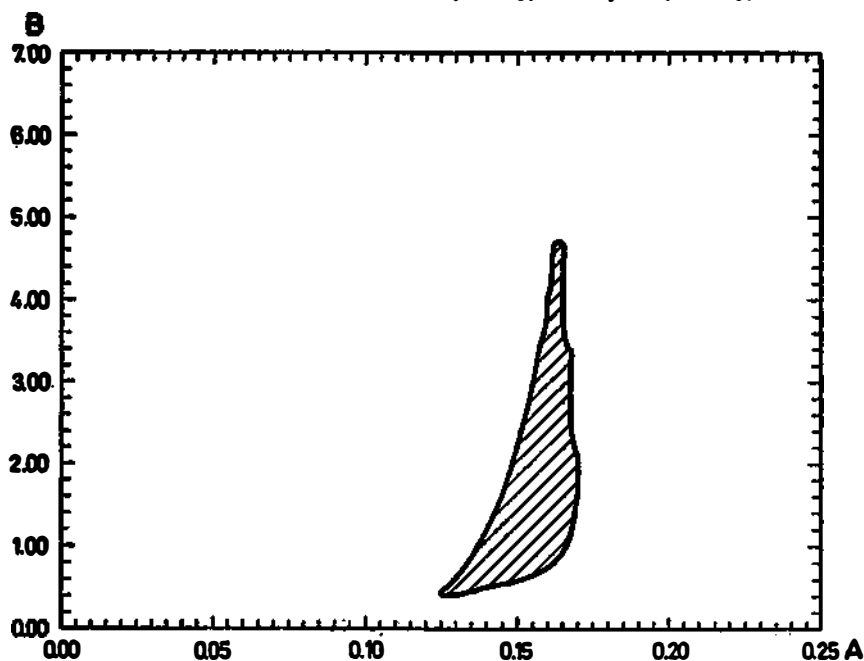


Fig. 2. Region of initial  $A, B$  points for which all requirements (i), (ii) and (iii) are satisfied for all momentum scales between  $M_{Pl}$  and  $\Lambda_f \leq 1/600 M_{Pl}$ .

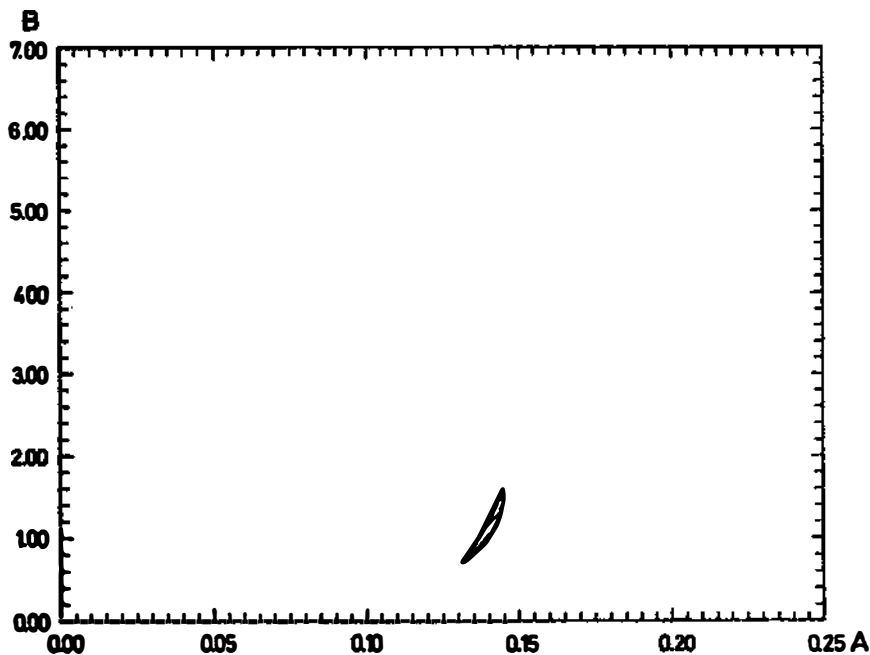


Fig. 3. Region of initial  $A, B$  points for which all requirements (i), (ii) and (iii) are satisfied at all momentum scales between  $M_{Pl}$  and  $A_f \leq 1/700 M_{Pl}$ .

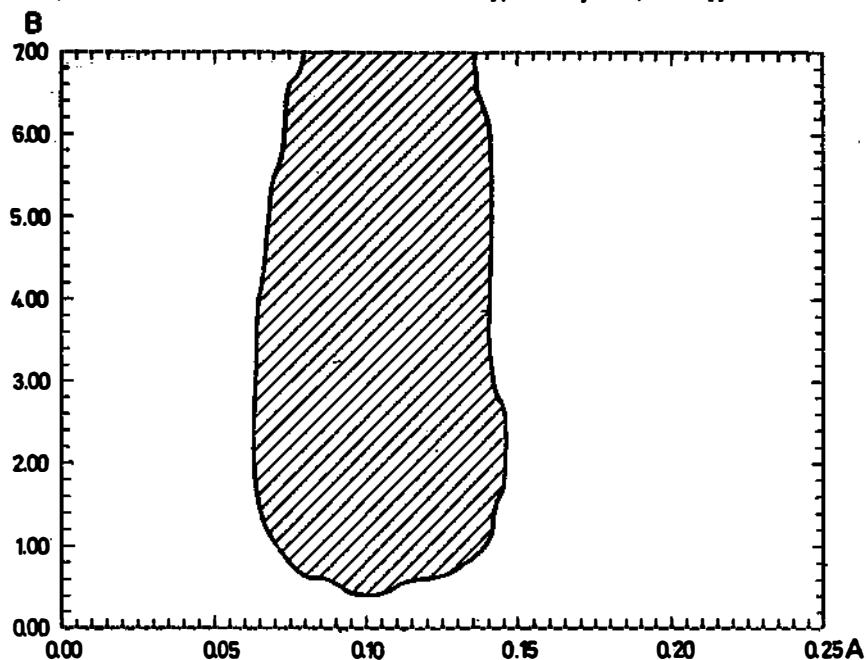


Fig. 4. Region of initial  $A, B$  points for which requirements (i) are satisfied at all momentum scales between  $M_{Pl}$  and  $A_f \leq 1/700 M_{Pl}$ .

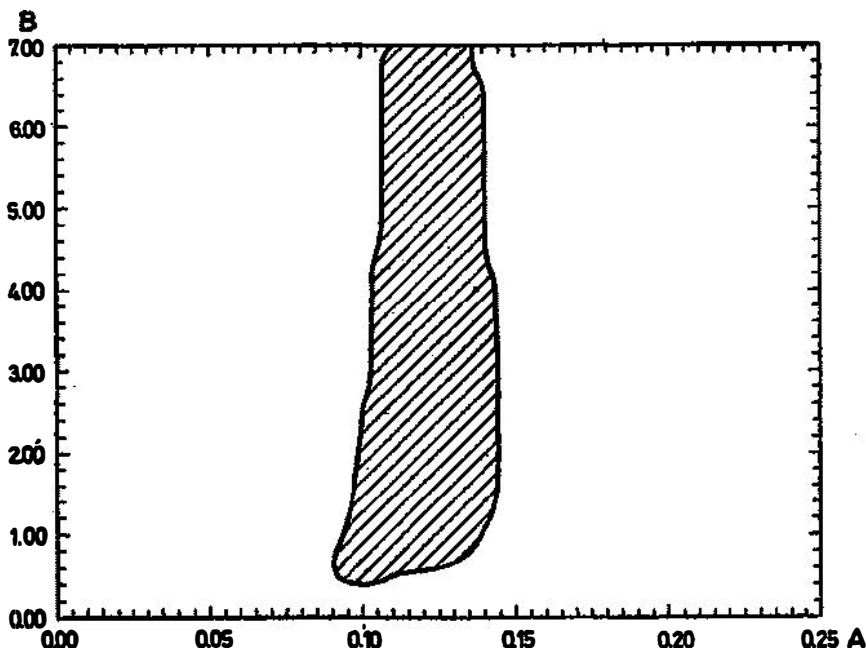


Fig. 5. Region of initial  $A, B$  points for which requirements (i) and (ii) are satisfied at all momentum scales between  $M_{Pl}$  and  $A_f \leq 1/700 M_{Pl}$ .

if  $A_f/A_f \geq 700$ , we show the corresponding allowed region in Fig. 4. Conditions (i) and (ii) ensure the local  $SU(3) \times SU(2) \times U(1)$  minimum and still lead to relatively large regions. This is demonstrated in Fig. 5, where  $A_f/A_f \leq 700$ . Hence, our finding is that conditions (iii), necessary for the absolute  $SU(3) \times SU(2) \times U(1)$  minimum, are much stronger than conditions (i) or conditions (ii) for the local  $SU(3) \times SU(2) \times U(1)$  minimum.

Finally, we can start solving RG equations at  $M_{Pl}$  with any potential (1) in the full parameter space  $\lambda_0, \lambda_1, \lambda_2, \gamma^2/\mu^2$ , whose absolute minimum is  $SU(3) \times SU(2) \times U(1)$  invariant, i. e. which satisfies conditions (i)–(iii). Again, the most convenient way of doing this is to parametrize such potentials starting from the sum-of-square form (15). Then, instead of Eq. (20) we write

$$\begin{aligned} \frac{1}{2} \lambda_0 &= \frac{1}{6} A + B + \frac{1}{2} \lambda'_0, \\ \lambda_1 &= A + \lambda'_1, \\ \lambda_2 &= -\frac{4}{3} A + \lambda'_2, \\ \frac{\gamma^2}{\mu^2} &= \frac{32A^2}{162B - 9A} + \lambda'_3 > 0, \end{aligned} \quad (21)$$

where  $A > 0$ ,  $B > 0$  are fixed. The new origin  $\lambda'_0 = \lambda'_1 = \lambda'_2 = \lambda'_3 = 0$  corresponds to the sum-of-square form (15).

We have investigated regions in the parameter space  $\lambda'_0, \lambda'_1, \lambda'_2, \lambda'_3$  for different choices of  $A, B$  where conditions (i), (ii) and (iii) have been satisfied. Our analysis has shown that solving the RG equations (17)–(19), starting at  $M_P$ , with the potential (21) whose absolute minimum is  $SU(3) \times SU(2) \times U(1)$  invariant, leads to similar conclusions drawn from Figs. 1–5. We have also shown that the maximal ratio of momentum scales is indeed three orders of magnitude,  $\Lambda_i/\Lambda_f = 1000$ , as required.

Hence we may draw the following conclusions. There exist regions in the parameter space where all requirements (i), (ii) and (iii) are simultaneously satisfied between  $M_P$  and  $M_{GUT}$ . Actually, the stability requirements hold between momentum scales with the ratio  $\Lambda_i/\Lambda_f = 10^3$  which is improved by an order of magnitude in comparison with the analysis performed in Ref. 4. In addition, the usefulness of the sum-of-square potential<sup>6)</sup> in the stability analysis is explicitly demonstrated.

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# STABILNOST LOMLJENJA SIMETRIJE U $SU(5)$ MODELU POMOĆU 75-DIMENZIONALNOG HIGGSOVOG POLJA

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Razmotreno je lomljenje  $SU(5)$  simetrije na fizikalnu  $SU(3) \times SU(2) \times U(1)$  simetriju, koje je inducirano efektivnim 75-dimenzijskim Higgsovim poljem. Prikazana je renormalizacijsko-grupna analiza (na nivou jedne petlje) stabilnosti  $SU(3) \times SU(2) \times U(1)$  simetrije apsolutnog minimuma. Nađena su područja u cijelom parametarskom prostoru gdje su svi zahtjevi stabilnosti ispunjeni, između Planck skale  $10^{19}$  GeV i karakteristične GUT skale  $10^{16}$  GeV. Demonstrirana je pogodnost korištenja specijalne konstrukcije potencijala, u obliku zbroya kvadrata, u općoj analizi stabilnosti.