

## SOLITARY WAVE PROPAGATION IN A SLOWLY VARYING MEDIUM

BALLIAPPADATH V. BABY

*Department of Mathematics, Bharata Mata College, Cochin — 682 021 Kerala State, India*

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The well-known perturbed Korteweg-de Vries equation,  $u_t + 6uu_x + u_{xxx} + f(t)u = 0$  is studied in this paper. It is found that the travelling solitary wave solution is possible only when  $f(t)$  is an arbitrary constant. An exact solution of this system is found when it is integrable. The first three conservation laws and the recursion relation of the remaining are found. Using the recursion operator, the infinite sets of commuting as well as noncommuting symmetries of this equation are also studied in this paper.

### 1. Introduction

The behaviour of water waves on a beach has been the subject of considerable theoretical and experimental studies beginning with Boussinesq in the last century. The celebrated constant coefficient Korteweg-de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0 \quad (1.1)$$

was originally derived as a model for the unidirectional propagation of a long wavelength, plane wave disturbances in a weakly nonlinear and weakly dispersive constant depth shallow water medium.

The propagation of a solitary travelling wave when the medium is inhomogeneous, e. g. the depth of a channel in which water wave travel is not constant, or the unperturbed plasma density in which an ionacoustic wave propagates is a

function of position, or in an LC electric circuit without dissipation the inductance of the linear conductor is varying, is inactive field of research for the last two decades<sup>1-2,5</sup>).

The perturbed Korteweg-de Vries (PKdV) equation,

$$u_t + 6uu_x + u_{xxx} + f(t)u = 0 \tag{1.2}$$

is a standard mathematical model. It describes the propagation of long, weakly non-linear waves in a slowly varying or slightly dissipative media. The inhomogeneity of the medium is expressed in terms of the perturbation term  $f(t)$ . It is shown that if  $f(t) > 0$  the solitary wave's amplitude decreases to zero and does not produce any new solitary waves. Whereas, when  $f(t) < 0$  the wave amplitude grows and disintegrates into number of solitary waves plus an oscillatory tail. In all of the theoretical studies the smallness of the  $f(t)$  is exploited for the approximation of the solution and the conservation laws of the PKdV equation (1.2) in terms of the exact integrability properties of the PKdV equation (Eq. 1.1).

In this paper the similarity transformation of the PKdV equation (1.2) is studied for the various values of  $f(t)$ . Using Darboux's transformation we are to find an exact solution of the PKdV equation, when it is integrable. The conservation laws and their recursive relation are obtained. It is shown that the system has no travelling solitary wave solution when  $f(t)$  is a time dependent function. The recursion operator and infinite set of commuting and noncommuting symmetries are also reported in this paper.

## 2. Similarity transformation of PKdV equation

The family of infinitesimal transformations is given by

$$x^* = x + \varepsilon X(x, t, u) + O(\varepsilon^2) \tag{2.1}$$

$$t^* = t + \varepsilon T(x, t, u) + O(\varepsilon^2) \tag{2.2}$$

$$u^* = u + \varepsilon U(x, t, u) + O(\varepsilon^2) \tag{2.3}$$

$$u^*_{x^*} = u_x + \varepsilon [U_x] + O(\varepsilon^2) \tag{2.4}$$

$$u^*_{x^*x^*x^*} = u_{xxx} + \varepsilon [U_{xxx}] + O(\varepsilon^2) \tag{2.5}$$

$$u^*_{t^*} = U_t + \varepsilon [U_t] + O(\varepsilon^2) \tag{2.6}$$

where  $\varepsilon$  is an infinitesimal parameter,  $X, T, U, \{U_x\}, \{U_t\}$  and  $[U_{xxx}]$  are the infinitesimals of the variables  $x, t, u, u_x, u_t$  and  $u_{xxx}$  respectively. Moreover,

$$[U_t] = U_t + (U_u - T_t)u_t - X_t u_x - T_t u^2 - X_u u_x u_t \tag{2.7}$$

$$[U_x] = U_x + (U_u - X_{t,x}) u_x - T_x u_t - X_u u_x^2 - T_u u_x u_t \quad (2.8)$$

$$\begin{aligned} [U_{xxx}] = & U_{xxx} + (3U_{xxu} - X_{xxx}) u_x - T_{xxx} u_t + 3T_{xx} u_x u_t + \\ & + 3(U_{xuu} - X_{xxu}) u_x^2 - 3T_{xxu} u_x u_t + (U_{uuu} - 3X_{xuu}) u_x^3 + \\ & + 3(U_{xu} - X_{xx}) u_{xx} - 3T_{xuu} u_x^2 u_t + 3(U_{uu} - 3X_{xu}) u_x u_{xx} - \\ & - 3T_{xu} u_x u_{xx} - 6T_{xu} u_x u_{xt} - 3T_x u_{xxt} - X_{uuu} u_x^4 + (U_u - 3X_{t,x}) u_{xxx} - \\ & - 6X_{uu} u_x^2 u_{xx} - 3T_{uu} u_x^2 u_{xt} - T_{uuu} u_x^3 u_t - 3X_{tu} u_x^2 u_{xx} - 3T_u u_x u_{xxt} - \\ & - 3T_u u_{xx} u_{xt} - 3T_{uu} u_x u_{xt} u_{xx} - 4X_u u_x u_{xxx} - T_u u_t u_{xxx}. \end{aligned} \quad (2.9)$$

A partial differential equation

$$F(x, t, u, u_x, u_{xxx}; \dots) = 0 \quad (2.10)$$

is said to be invariant under the transformations (2.1)–(2.10) if it satisfies the invariant surface condition

$$T \frac{\partial F}{\partial t} + X \frac{\partial F}{\partial x} + U \frac{\partial F}{\partial u} + [U_x] \frac{\partial F}{\partial u_x} + \dots = 0. \quad (2.11)$$

The invariant surface condition associated with the PKdV equation (1.2) is given by,

$$[U_t] + (6u_x + f(t)) U + 6u [U_x] + [U_{xxx}] = 0. \quad (2.12)$$

On substituting the equations (2.7), (2.8) and (2.9) in (2.12) and solving for the infinitesimals  $X$ ,  $T$  and  $U$  we get the following solutions

(1) when  $f(t) = k$ , a constant

$$T = \delta, \quad (2.13)$$

$$X = -6 \frac{a}{k} \exp(-kt) + \beta, \quad (2.14)$$

$$U = a \exp(-kt), \quad (2.15)$$

and the Lie group generators  $G_i$ ,  $i = 1, 2, 3$  and their algebra corresponding to the parameters  $a$ ,  $\beta$  and  $\delta$  are given by

$$G_1 = \frac{\partial}{\partial t} \quad (2.16)$$

$$G_2 = \exp(-kt) \left[ -\frac{6}{k} \frac{\partial}{\partial x} + \frac{\partial}{\partial u} \right] \quad (2.17)$$

$$G_3 = \frac{\partial}{\partial x} \quad (2.18)$$

$$[G_1, G_3] = -kG_2, \quad (2.19)$$

$$[G_1, G_2] = [G_2, G_3] = 0. \quad (2.20)$$

(2) when  $f(t) = kt$ ,  $k$  is a constant

$$T \equiv 0, \quad (2.21)$$

$$X = 6\alpha \int \exp(-kt)^2/2 dt + \beta. \quad (2.22)$$

$$U = \alpha \exp(-kt^2/2). \quad (2.23)$$

(3) when  $f(t) = (C_0 + 2t)^{-1}$ ,  $C_0$  is a constant

$$T = 0, \quad (2.24)$$

$$X = 6\alpha (C_0 + 2t)^{1/2} + \beta, \quad (2.25)$$

$$U = \alpha (C_0 + 2t)^{-1/2}. \quad (2.26)$$

The Lie group generators and the algebra are given by

$$G_1 = \frac{\partial}{\partial u}, \quad (2.27)$$

$$G_2 = 6(C_0 + 2t)^{1/2} \frac{\partial}{\partial x} + (C_0 + 2t)^{-1/2} \frac{\partial}{\partial u}, \quad (2.28)$$

$$[G_1, G_2] = 0. \quad (2.29)$$

(4) when  $f(t)$  is arbitrary

$$T = 0, \quad (2.30)$$

$$X = 6\alpha \int \exp[-\int f(t) dt] dt + \beta, \quad (2.31)$$

$$U = \alpha \exp[-\int f(t) dt]. \quad (2.32)$$

In all the above cases,  $\alpha$ ,  $\beta$  and  $\delta$  are the arbitrary integration constants and they are the continuous parameters of the transformations.

It is interesting to note that the PKdV equation (1.2) has a translational invariance under the variable;  $x$  and  $t$  only when  $f(t)$  is an arbitrary constant  $k$ . In all other cases, the PKdV equation (1.2) is translationally invariant under  $x$  but not in  $t$ . This implies that the travelling wave solution with translational invariance or the travelling solitary wave solution is possible only when the perturbation factor  $f(t)$  is an arbitrary constant. This implies that the dynamical system has a time dependent change and that can be energy dissipation<sup>13,16,19,21</sup>, when  $f(t)$  is a function of time variable  $t$ . The PKdV equation (1.2) with  $f(t)$  as constant is a well-known<sup>3</sup> mathematical model for explaining the damping of ion-sound wave by ion-neutral collisions.

The infinitesimals (2.13), (2.14) and (2.15) give the characteristic equation,

$$\frac{dx}{\beta - 6 \frac{\alpha}{k} \exp(-kt)} = \frac{dt}{\delta} = \frac{du}{\alpha \exp(-kt)} \tag{2.33}$$

This yields the similarity solution

$$u = - \frac{\alpha}{\delta k} \exp(-kt) + g(\tau), \tag{2.34}$$

where  $\tau$  is the similarity variable

$$\tau = \delta x - \beta t - \frac{6\alpha}{k^2} \exp(-kt) \tag{2.35}$$

and  $g(\tau)$  is to be obtained from the similarity reduced equation

$$-\beta g' + 6gg' + kg + \delta^3 g''' = 0 \tag{2.36}$$

where the primes denote the differentiations with respect to the similarity variable  $\tau$ . It is not easy to find an analytic solution for the equation (2.36).

If we put  $\alpha = 0$ , in Eq. (2.33) then we get the similarity solution

$$u = f(\eta) \tag{2.37}$$

where the similarity variable  $\eta$  is given by

$$\eta = (\delta x - \beta t). \tag{2.38}$$

The respective similarity reduced equation is given by

$$-\beta f' + 6\delta f f' + kf + \delta^3 f''' = 0, \tag{2.39}$$

where the prime denotes the differentiation with respect to the similarity variable  $\eta$ . The pure space-time translational invariant solution of the PKdV equation (1.2) can be obtained from Eq. (2.39).

### 3. Lax-pair, Darboux's transformation and exact solutions

Recently<sup>26)</sup>, we reported the Painleve analysis of the PKdV equation (1.2). It is found that the equation satisfies the Painleve property (PP) only when

$$f(t) = (C_0 + 2t)^{-1} \tag{3.1}$$

where  $C_0$  is an arbitrary constant. The auto-Backlund transformation is given by

$$\Phi_{,xt} + 6u_0\Phi_{,xx} + \Phi_{,xxx} + \Phi_{,x}f(t) = 0, \tag{3.2}$$

$$\Phi_{,x}\Phi_{,t} + 6u_0\Phi_{,x}^2 - 3\Phi_{,xx}^2 + 4\Phi_{,x}\Phi_{,xxx} = 0, \tag{3.3}$$

and

$$u(x, t) = \Phi^{-2} (-2\Phi_{,x}^2 + 2\Phi\Phi_{,xx} + u_0\Phi^2) \tag{3.4}$$

where  $u_0$  is a known solution of the PKdV equation (1.2).

Using the Chen's procedure<sup>27)</sup>, we obtained

$$L\psi = \mu\psi \tag{3.5}$$

where

$$|L = g(t)(D^2 + u - \frac{x}{6}f(t)) \tag{3.6}$$

and  $\mu$  is the eigenvalue corresponding to the eigenfunction  $\psi$  and

$$-\psi_{,t} = M\psi \tag{3.7}$$

where

$$M = (4D_{,x}^3 - 6uD_{,x} + 3u_{,x}). \tag{3.8}$$

The compatibility condition,

$$L_{,t} = LM - ML \tag{3.9}$$

gives

$$g(t) = C_1(C_0 + 2t); \quad C_1 \neq 0 \tag{3.10}$$

and

$$f(t) = (C_0 + 2t)^{-1}. \tag{3.11}$$

This implies that the admissible Lax-pair is given by

$$L = C_1 \left( C_0 + 2t \right) \left( D_{t,x} - u - \frac{x}{6(C_0 + 2t)} \right), \tag{3.12}$$

$$M = (4D_x^2 + 6uD_{t,x} + 3u_x) \tag{3.13}$$

where  $D_{t,x} = \frac{\partial}{\partial x}$  and  $C_0$  and  $C_1$  are arbitrary constants. This shows that the PKdV equation (1.2) is Inverse scattering technic (IST) solvable system only when  $f(t)$  is of the form given by Eq. (3.11).

It is interesting to note that when  $C_0 = 0$ , this model is the well-known cylindrical KdV equation and it is IST solvable system. We will use the Darboux's transformation (DT)<sup>24, 28</sup>, to find the exact solution of the PKdV equation when it is IST solvable system. For that purpose, first we will transform the isospectral eigenvalue problem (Eqs. (3.12) and (3.13)) into a nonisospectral problem using a dependent variable transformation

$$u(x, t) = \frac{x}{6(C_0 + 2t)} - v. \tag{3.14}$$

In terms of the new variable  $v(x, t)$ , we have the new set of nonisospectral Lax-pair corresponding to Eqs. (3.12) and (3.13) as,

$$\Phi_{t,xx} = E\Phi \tag{3.15}$$

$$\Phi_{t,t} = -v_x\Phi + \left( 2v - \frac{x}{(C_0 + 2t)} - \frac{4}{C_1(C_0 + 2t)} \right) \Phi_x \tag{3.16}$$

where

$$E = (v - \lambda) \tag{3.17}$$

and  $\lambda$  is the new time dependent eigenvalue. It is easy to show that

$$\frac{\partial \lambda}{\partial t} = -2(C_0 + 2t)^{-1}. \tag{3.18}$$

Using Eq. (3.14) in Eqs. (1.2) and (3.11) becomes

$$v_{t,t} = (-v_{t,xxx} + 6uv_{t,x}) - (xv_{t,x} + 2v)(C_0 + 2t)^{-1}. \tag{3.19}$$

The DT method is a powerful technique for generating exact solution of an IST solvable equation without the knowledge of any boundary conditions. For a given equation, we can construct a set of new solutions if we know at least one solution, by solving some simple differential equations which are generally linear system.

Let  $v_0(x, t)$  be a known solution of Eq. (3.19), for example  $v_0 = 0$  and  $\Phi(x, t, \lambda)$  be a consistent solution of Eqs. (3.15) and (3.16) for the eigenvalue  $\lambda$ . Then corresponding to the pair  $(\Phi, v_0)$  there exists a new set of variables  $(\Phi, v_1)$  obtained from the following transformation

$$\Phi = \Phi_{,x}(x, t, \lambda) + \frac{f_{0,x}}{f_0} \Phi(x, t, \lambda). \tag{3.20}$$

Then the new solution of Eq. (3.19) is  $v_1(x, t)$

$$v_1(x, t) = v_0(x, t) - 2 \frac{\partial^2}{\partial x^2} (\log(f_0(x, t, \lambda_0))) \tag{3.21}$$

where  $f_0(x, t, \lambda_0)$  is the value of  $\Phi$  at  $\lambda = \lambda_0$ , a particular solution of Eqs. (3.15) and (3.16).

For this case, let us assume

$$v_0(x, t) = 0, \tag{3.22}$$

$$\lambda = -(C_0 + 2t)^{-1}. \tag{3.23}$$

This implies that the equations (3.15) and (3.16) are, respectively,

$$\Phi_{,xx} = (C_0 + 2t)^{-1}, \tag{3.24}$$

$$\Phi_{,t} = -x(C_0 + 2t)^{-1} \Phi_{,x} + 4\lambda \Phi_{,x}. \tag{3.25}$$

A constant solution of the above two equations is given by

$$\Phi(x, t) = A \exp(\Theta) + B \exp(-\Theta) \tag{3.26}$$

where

$$\Theta = (x + 4)(C_0 + 2t)^{-1/2}. \tag{3.27}$$

Let

$$f_0(x, t, \lambda_0) = \Phi(x, t) \tag{3.28}$$

and when  $A = B$  yields the new solution of Eq. (3.19):

$$u(x, t) = \frac{x}{6(C_0 + 2t)} + \frac{2 \operatorname{sech}^2 \Theta}{(C_0 + 2t)}. \tag{3.29}$$

Eq. (3.29) is an exact solution of the PKdV equation (Eq. (1.2)), when  $f(t)$  is given by Eq. (3.11).

#### 4. Conservation law and symmetries

We will construct the conservation laws from the Lax-pair  $\bar{L}$  (Eq. (3.15) and (3.16)). In fact, those are the conservation laws of the equation (3.19) but when we use the transformation Eq. (3.14) we get the conservation laws of the PKdV equation (1.2).

Substitute

$$F = \Phi'_{,x} \Phi^{-1} \tag{4.1}$$

in equation (3.15) then we get

$$F'_{,x} = (v - \lambda) - F^2. \tag{4.2}$$

We define a power series

$$F = \varepsilon + \varepsilon^{-1} F_1 + \varepsilon^{-2} F_2 + \dots \tag{4.3}$$

where  $\varepsilon^2 = -\lambda$ . Substituting Eq. (4.3) in Eq. (4.2), yields

$$(\varepsilon^{-1} F_1'_{,x} + \varepsilon^{-2} F_2'_{,x} + \dots) = v - (2F_1 + \varepsilon^{-1} 2F_2 + \dots). \tag{4.4}$$

Equating the coefficients of same powers of both sides of the Eq. (4.4), we get

$$F_1 \equiv v/2 \tag{4.5}$$

$$F_0 = -v'_{,x}/4 \tag{4.6}$$

$$F_3 = (v'_{,xx} - v^2)/8 \tag{4.7}$$

$$F_4 = - (v'_{,xxx} - \frac{1}{4} v v'_{,x})/16 \tag{4.8}$$

$$F_5 = (v'_{,xxxx} - 6v v'_{,xx} - 5v'_{,x} + 2v^3)/32 \tag{4.9}$$

etc. Also, (3.6) gives

$$\frac{\Phi'_{,t}}{\Phi} = -v'_{,x} + \left( 2v - \frac{x}{(C_0 + 2t)} + 4\lambda \right) \frac{\Phi'_{,x}}{\Phi}. \tag{4.10}$$

But

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial x} (\Phi'_{,t}/\Phi). \tag{4.11}$$

From equations (4.10) and (4.11) yields

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial x} \left( -v_{,x} + \left( 2v - \frac{x}{(C_0 + 2t)} + 4\lambda \right) F \right). \quad (4.12)$$

Substitute the power series expansion (Eq. (4.3)) of  $F$  in Eq. (4.12) and using Eq. (3.8), yields

$$\frac{\partial}{\partial t} [(C_0 + 2t)^{\frac{2j+1}{2}} F_{2j+1}] = (C_0 + 2t)^{\frac{2j+1}{2}} \frac{\partial}{\partial x} \left[ 2v - \frac{x}{(6+2t)} F_{2j+1} - 4F_{2j+3} \right]. \quad (4.13)$$

Equation (4.13) is the recursive relation for the infinite set of conservation laws of the equation (3.19). The first three conservation laws of the equation are the following:

$$\frac{\partial}{\partial t} ((C_0 + 2t) v) = (C_0 + 2t) \frac{\partial}{\partial x} (3v^2 - \frac{xv}{(C_0 + 2t)} - v_{,xxx}), \quad (4.14)$$

$$\begin{aligned} \frac{\partial}{\partial t} ((C_0 + 2t) (v_{,xx} - v^2)) &= (C_0 + 2t) \frac{\partial}{\partial x} (-v_{,xxxx} + 8vv_{,xx} - 4v^3 + 5v^2_{,x} - \\ &\quad - \frac{x}{(C_0 + 2t)} (v_{,xx} - v^2)), \end{aligned} \quad (4.15)$$

and

$$\begin{aligned} &\frac{\partial}{\partial t} ((C_0 + 2t) (v_{,xxxx} - 6vv_{,xx} - 5v^2_{,x} + 2v^3)) = \\ &= (C_0 + 2t) \frac{\partial}{\partial x} (-v_{,xxxxx} + 12vv_{,xxx} - 42v^2v_{,xx} - 60vv^2_{,x} + 9v^4 + \\ &\quad + 28v_x v_{,xxx} - \frac{x}{(C_0 + 2t)} (v_{,xxxx} - 6vv_{,xx} - 5v^2_{,x} + 2v^3)). \end{aligned} \quad (4.16)$$

The conservation laws of the PKdV equation (1.2) can be obtained from the Eqs. (3.11), (3.14) and (4.13).

It is well-known that the conservation laws of constant coefficient PKdV equation are independent of  $x$  and  $t$  variables, whereas that of the PKdV equation are explicitly depended on  $x$  and  $t$  variables. The first three constants of motion  $I_n(x, u, t)$  of the PKdV equation (1.2) are the following:

$$I_0 = (C_0 + 2t)^{1/2} u, \quad (4.17)$$

$$I_1 = (C_0 + 2t)^{3/2} \left( u_{,xx} + u^2 - \frac{xu}{3(C_0 + 2t)} \right), \quad (4.18)$$

$$I_2 = (C_0 + 2t)^{5/2} (-w_{xxx} - 6uw_{xx} - 5u^2_x - 2u^3 + xu_{xx}(C_0 + 2t)^{-1} + \frac{5u_x^1}{3(C_0 + 2t)} - \frac{x^2u}{(C_0 + 2t)^2} + \frac{xu^2}{(C_0 + 2t)} + \frac{x^3}{108(C_0 + 2t)^3} - \frac{5}{36(C_0 + 2t)^2}). \tag{4.19}$$

The conserved covariants  $\gamma_n(u, x, t)$  are given by

$$\gamma_n(u, x, t) = \text{grad}(I_n(u, x, t)). \tag{4.20}$$

The first two nontrivial conserved covariance of the PKdV equation are given by

$$\gamma_1 = (C_0 + 2t)^{1/2} \left( 2u - \frac{x}{3(C_0 + 2t)} \right), \tag{4.21}$$

$$\gamma_2 = (C_0 + 2t)^{3/2} \left( -u_{xx} - 6u^2 + \frac{2xu}{(C_0 + 2t)} - \frac{x^2}{6(C_0 + 2t)^2} \right). \tag{4.22}$$

The set of commuting symmetries assures the Hamiltonian structure of this dynamical system<sup>29)</sup>.

### 5. Discussion

The experimental studies on the propagation of a weakly nonlinear and dispersive wave in an inhomogeneous medium, verified that the wave form after the inhomogeneous medium depends on the ratio of the nonlinear or dispersive term to the perturbation term. It is shown that when the amplitude increases by the inhomogeneity, the wave disintegrate into number of solitary waves after the wave passes through the inhomogeneous region. But in the present study we found that when the perturbation term  $f(t)$  is time dependent then there is no possibility of any travelling solitary wave solution or solitons. Moreover, we found that travelling wave solution is possible only when the  $f(t)$  is an arbitrary constant, but then the system is not of Painleve type and so we are not in a position to derive the Lax-pair using standard procedure. In general absence of PP implies that the system is not IST solvable. If so then we have to conclude that the PKdV equation (1.2) is not integrable or the system has no soliton solution when  $f(t)$  is constant.

In this study we used the well-known definition of soliton as a collisionally stable travelling wave solution of an IST solvable nonlinear partial differential equation with translational symmetry in both time and space variables. It is interesting to note that the linearly nonuniform media with a dissipation can still support soliton type solution for some special cases<sup>31)</sup>. In such cases the solutions are damping in time without radiation and mutually colliding in nonuniform media with a relaxation effect.

So far in all the studies on PKdV equation (1.2) the solution and conservation laws are approximated by using the well known KdV equation (1.1). In the present study, we found the exact solution and conservation laws of this system. These new

results will shed some light on the behaviour of the wave propagation in an inhomogeneous medium and its stability.

Another interesting thing we found the existence of a nonabelian 2-dimensional Lie group (Eqs. (2.16)–(2.20)) associated with the PKdV equation when  $f(z)$  is constant. Recently<sup>32, 33)</sup> it is shown that for a given Ansov flow having one-dimensional strong stable manifolds the foliation by stable manifolds can be parametrised by an action of the affine group.

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PROSTIRANJE SOLITONSKOG VALA U POLAGANO PROMJENLJIVOM  
MEDIJU

BALLIAPPADATH V. BABY

*Department of Mathematics, Bharata Mata College, Cochin — 682021, Kerala State,  
India*

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U ovom članku razmatra se dobro poznata smetana Korteweg-de Vries jednačba  $u_t + 6uu_x + u_{xxx} + f(x)u = 0$ . Nađeno je da je rješenje putujući solitonski val moguć samo onda kada je  $f(x)$  proizvoljna konstanta. Nađeno je točno rješenje sistema, ako je on integrabilan. Nađena su prva tri zakona sačuvanja i rekurzivne relacije za preostale. Koristeći rekurzivne operatore, također se diskutiraju beskonačni skupovi komutirajućih i nekomutirajućih simetrija ove jednačbe.