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SUPERSYMMETRIC PART OF THE HEAVY QUARK-ANTIQUARK POTENTIAL

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We calculate the part of the heavy-quarkonium potential which appears if QCD, responsible for binding of quarks, is extended to SUSY QCD. This involves the calculation of a box graph in which the exchange particles are SUSY partners of quarks and gluons.

1. Introduction

The study of properties of heavy quark-antiquark systems has provided a significant understanding of QCD as a theory of strong interactions[1].

At present there exists a very successful description of ψ- and T-families as quark-antiquark systems made, respectively, of a heavy $c\bar{c}$ and a heavy $b\bar{b}$ pair in terms of a simple non-relativistic potential. The exact form of a potential, in particular its confining part, is not yet known from first principles. As the constituents (c or b) are heavy, a non-relativistic description based on the Schrödinger equation and on a phenomenologically founded flavour-independent central potential is expected to be a good approximation. The spin effects, responsible for a systematic description of quarkonium systems, are usually treated either phenomenologically or as small relativistic corrections.

Other contributions to the bound-state spectra coming from higher-order gluonic degrees of freedom and possibly from the presence of supersymmetric particles as exchange quanta are expected to be small. Nevertheless, their presence and relative importance should be estimated.

In Section 3 of this paper we derive the part of the heavy-quarkonium potential which appears if quantum chromodynamics (QCD), responsible for binding, is extended to SUSY QCD. To lowest order in the coupling constant this involves the calculation of a box graph in which the exchange particles are supersymmetric partners of quarks and gluons.

2. *Supersymmetric extension of QCD*

If supersymmetry (SUSY) is really a symmetry of particle physics, the particle content of standard QCD should be enlarged[2]. The minimal SUSY extension of QCD implies that every particle has a superpartner with spin shifted by a half unit.

The interactions between gluons (gluinos) and (s) quarks are described by the following SUSY-QCD gauge-invariant Lagrangian:

$$\mathscr{L}_{\mathrm{SUSY\ QCD}} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \frac{1}{2}\bar{\lambda}\, \mathrm{i}\gamma_\mu D^\mu \lambda + \bar{q}\mathrm{i}\gamma_\mu D^\mu q + (D_\mu \tilde{q}_L)^\dagger D^\mu q_L +$$

$$+ (D_\mu \tilde{q}_R)^\dagger D^\mu q_R + \mathrm{i}g\sqrt{2}\,\{\bar{\lambda}_R^a \tilde{q}_L^+ T^a q_L + \bar{\lambda}^a \tilde{q}_R^+ T^a q_R + \mathrm{h.c.}\} - \frac{1}{2} g^2 (\tilde{q}_L^+ T^a q_L -$$

$$- \tilde{q}_R^+ T^a q_R)^2, \tag{2.1}$$

where the index $R\,(L)$ denotes that the quark is right (left)-handed, i. e. $q_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\, q$, $\gamma_5^+ = \gamma_5$, $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c$ is the gluon field and $q\,(\tilde{q})$ denotes the quark (squark) field with λ as a gluino field. The covariant derivative D^μ is defined as

$$D^\mu = \partial^\mu - \mathrm{i}g G^{a\mu} T^a,$$

where T^a are the generators of $SU_C(3)$. To this Lagrangian we must add a mass term that breaks supersymmetry. The mass spectrum of the theory thus depends on the way the supersymmetry is broken.

For exact supersymmetry, the gluino is massless and quarks and squarks have the same mass. In the broken SUSY, gluinos and squarks may have different masses. It is argued[3] that squarks and gluinos should be much heavier than the corresponding quarks and gluons, respectively.

3. Construction of a potential

The lowest-order SUSY QCD contributions to the heavy quark-antiquark potential are due to box diagrams shown in Fig. 1a, b. We shall assume that ingoing and outgoing heavy quarks, $Q = c, b$, are on the mass shell. This is justified if the momentum transfer between the quark-antiquark bound-state system is small.

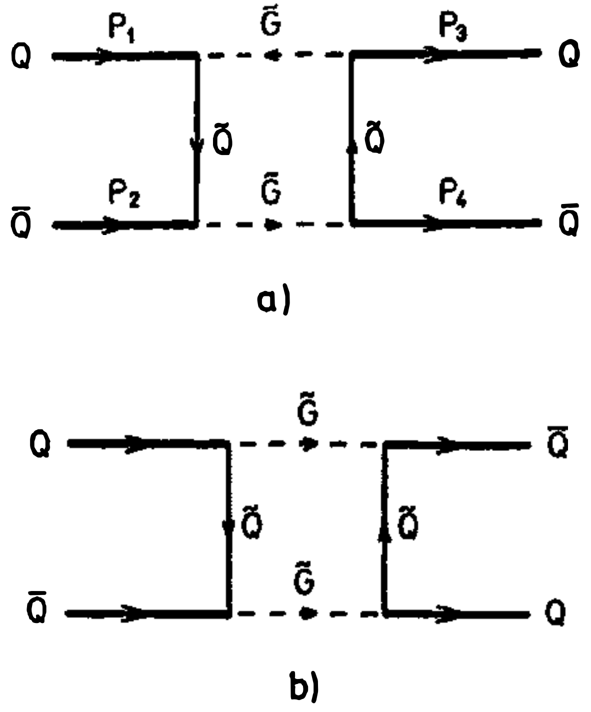

Fig. 1. Two types of SUSY box graphs. They have the same external lines. Here Q ($\bar{Q}$) stands for the heavy quark (antiquark), c or b. $\tilde{G}$ and $\tilde{Q}$ denote the gluino and the squark, respectively.

The exact evaluations of these box diagrams in gauge theories are complicated and lead to expressions which are not very transparent for further analysis of the potental.

In all previous calculations, either the four-momenta of the external quark lines in the box diagrams were set to zero[4] or only their three-momenta were set to zero, in comparison with the masses involved[5]. In both cases, the approximation is too rough and prevents the calculation of the potential which requires that the momentum transfer between the quark and the antiquark in the bound state should be different from zero.

For that reason, our calculation of the SUSY part of the heavy quark-antiquark potential will follow the method proved to be successful in finding the nucleon-nucleon potential: it is based on the Blankenbecler-Sugar-Logunov-Tavkhelidze

(BSLT)[6] equation. It is the three-dimensional integral equation for the off-mass-shell $Q\bar{Q}$ scattering amplitude:

$$\hat{T}(\vec{p}', \vec{p}|W) = \hat{V}(\vec{p}', \vec{p}|W) + \frac{1}{(2\pi)^3}\int \frac{d^3k}{4E_k}\hat{V}(\vec{p}', \vec{k}|W) \times$$

$$\times \frac{\Lambda^1_+(\vec{k})\Lambda^2_+(-\vec{k})}{W^2 - E_k^2 + i\varepsilon}\hat{T}(\vec{k}, \vec{p}|W), \tag{3.1}$$

where $\hat{V}(\vec{p}', \vec{p}|W)$ denotes the matrix of the $Q\bar{Q}$ potential, $\hat{T}(\vec{p}', \vec{p}|W)$ is the BSLT amplitude and

$$\Lambda^{1,2}_+(\vec{k}) = \gamma_0^{1,2}E_k - \vec{\gamma}^{1,2}\cdot\vec{k} - m_{\tilde{G}}, \tag{3.2}$$

with $m_{\tilde{G}}$ being the gluino mass. The variables used in (3.1) are defined in the c. m. s.:

$$\vec{p} = \vec{p}_1 = -\vec{p}_2, \qquad \vec{p}' = \vec{p}_3 = -\vec{p}_4, \qquad W^2 = p^2 + m^2_{\tilde{G}}$$

and

$$E_k^2 = k^2 + m^2_{\tilde{G}}.$$

We shall assume that Eq. (3.1) is consistent with the perturbation expansion of $\hat{T}$ in SUSY-QCD field theory, i. e. the perturbation series for $\hat{T}$ and $\hat{V}$ satisfy Eq. (3.1) identically in any order of perturbation theory. Therefore, if the perturbation series for $\hat{T}$ is known, the perturbation series for $\hat{V}$ must be considered as defined by Eq. (3.1). Thus, for $\hat{T} = \sum \hat{T}_n$, we have $\hat{V} = \sum \hat{V}_n$ with

$$\hat{V}_1 = \hat{T}_1, \qquad \hat{V}_2 = \hat{T}_2 - \hat{T}_1 G \hat{T}_1, \ldots \text{etc.} \tag{3.3}$$

In our case, $\hat{T}_1 = \hat{T}_{Box\,a} + \hat{T}_{Box\,b}$.

In this paper we shall only calculate $\hat{T}_{Box\,a}$ assuming that intermediate particles are on the energy shell. $\hat{T}_{Box\,a}$ gives a contribution to the direct potential and $\hat{T}_{Box\,b}$ a contribution to the exchange potential. A simple calculation using the Lagrangian (2.1) gives

$$\hat{T}_{Box\,a} = -\frac{g^4}{(2\pi)^3}\cdot\frac{16}{9}\int\frac{d^3k}{E_k}\Lambda^1_+(\vec{k})\Lambda^2_+(-\vec{k})\,[(p^2 - k^2 + m^2_{\tilde{Q}} - m^2_{\tilde{G}} + i\varepsilon)\times$$

$$\times(p^2 + k^2 - 2\vec{k}\cdot\vec{p}' + m^2_{\tilde{Q}})(p^2 + k^2 - 2\vec{k}\cdot\vec{p} + m^2_{\tilde{Q}})]^{-1}, \tag{3.4}$$

where p and k denote the magnitudes of $\vec{p}$ and $\vec{k}$, respectively.

After suitable reorganization of γ matrices and partial integration we obtain (3.4) in the form

$$\hat{T}_{Box\,a} = \sum_{j=1}^{5} \frac{1}{\pi} \int_{4m_{\tilde{Q}}^2}^{\infty} \mathrm{d}t' \frac{\eta_j(p^2, t')}{t' - t} \Lambda_j, \tag{3.5}$$

where explicit forms of the invariant functions $\eta_i(p^2, t')$ are given in Appendix A, and

$$t = -(\vec{p}' - \vec{p})^2 = -\vec{\Delta}^2.$$

Λ_j are operators:

$$\Lambda_1 = 1^{(1)}1^{(2)}, \qquad \Lambda_2 = \gamma_4^{(1)}\gamma_4^{(2)},$$

$$\Lambda_3 = 1^{(1)}\gamma_4^{(2)} + \gamma_4^{(1)}1^{(2)}, \qquad \Lambda_4 = \vec{\gamma}^{(1)} \cdot \vec{\gamma}^{(2)}. \tag{3.6}$$

$$\Lambda_5 = \gamma_5^{(1)}\gamma_5^{(2)}.$$

In the low-energy limit where p^2 can be approximately set to zero we obtain the non-relativistic form of the SUSY-QCD direct potential:

$$V_{Box\,a} = [\overline{u}(\vec{p}')\,\overline{v}(-\vec{p}')\,\hat{T}_{Box\,a}\,u(\vec{p})\,v(-\vec{p})]|_{p^2\sim O}. \tag{3.7}$$

In the configuration space we have

$$\langle \vec{r}'|\,V_{Box\,a}\,|\vec{r}\rangle = \delta^{(3)}(\vec{r}' - \vec{r})\,V_{Box\,a}(\vec{r}, \vec{p}; \vec{\sigma}^{(1)}, \vec{\sigma}^{(2)}),$$

where

$$V_{Box\,a}(\vec{r}, \vec{p}; \vec{\sigma}^{(1)}, \vec{\sigma}^{(2)}) = \sum_{\alpha} V_\alpha(r)\,\Omega_\alpha, \tag{3.8}$$

with

$$\Omega_C = 1^{(1)}\,1^{(2)}, \quad \Omega_{SO}^{+} = \frac{1}{2}(\vec{\sigma}^{(1)} + \sigma^{(2)})\,\vec{l},$$

$$\Omega_{SO}^{-} = \frac{1}{2}(\vec{\sigma}^{(1)} - \vec{\sigma}^{(2)})\vec{l}, \tag{3.9}$$

$$\Omega_T = \left[3\,(\vec{\sigma}^{(1)}\vec{r})\,(\vec{\sigma}^{(2)}\vec{r}) - \frac{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})}{r^2}\right]\frac{1}{r^2},$$

$$\Omega_{SS} = \vec{\sigma}^{(1)}\vec{\sigma}^{(2)}, \quad \Omega_{SO_2} = \frac{1}{2}\,[(\vec{\sigma}^{(1)}\vec{l})\,(\vec{\sigma}^{(2)}\vec{l}) + (\vec{\sigma}^{(2)}\vec{l})\,(\vec{\sigma}^{(1)}\vec{l})]$$

and $\vec{l} = \vec{r} \times \vec{p}$ is the orbital angular momentum.

For $V_{\alpha}^{f}(r)$ we obtain

$$V_{\alpha}(r) = -\frac{1}{(2\pi)^2}\int\limits_{4m_{\tilde{Q}}^2}^{\infty} dt'\sqrt{t'}\,\overline{\eta}_{\alpha}(t')\,R_{\alpha}(x)\frac{e^{-x}}{x}, \tag{3.10}$$

where $x = r\sqrt{t}$ and

$$\begin{aligned} R_0(x) &= 1, \\ R_{SO}(x) &= -\frac{t'}{x}\left(1+\frac{1}{x}\right), \\ R_T(x) &= t'\left(1+\frac{3}{x}+\frac{3}{x^2}\right), \\ R_{SS}(x) &= -t', \\ R_{SO_2}(x) &= -\frac{t'}{x^2}\left(1+\frac{3}{x}+\frac{3}{x^2}\right), \end{aligned} \tag{3.11}$$

and

$$\overline{\eta}_{\alpha}(t') = \sum_j \tilde{X}_{\alpha j}\eta_j(p^2 = 0, t'). \tag{3.12}$$

The elements of the matrix $\tilde{X}_{j\alpha}$ are calculated in Appendix B.

We note that $V_{\alpha}(r)$ has the form of a continuous superposition of Yukawa potentials. The range of the potential is given by the mass $m_{\tilde{Q}}$ of the squark.

4. Conclusion

We have presented a detailed calculation of the two-squark exchange potential between the heavy Q-$\overline{Q}$ system. The main result of the paper are the relations (3.8)—(3.12) which give the explicit form of the potential corresponding to the SUSY box-diagram $T_{Box\,\alpha}$.

If $m_{\tilde{Q}} \geq 45$ GeV[8], $V_{\alpha}(r)$ will have a very short range and consequently its contribution to the $Q\overline{Q}$ bound state becomes important at interquark distances $r < 0.004$ fm. Presently, the ψ and T spectroscopies probe distances only between 0.1 fm and 1.0 fm[9]. Therefore, the effects of $V_{\alpha}(r)$ on the heavy $Q\overline{Q}$ bound states are presently outside the experimental detection.

Comparison with the two-gluon van der Waals potential in the Q-$\overline{Q}$ system is presently not possible without additional calculations. The van der Waals QCD-potentials have been discussed only for hadrons[10] and not for the Q-$\overline{Q}$ system.

APPENDIX A

$$\eta_1(p^2, t') = -\frac{g^4 2}{9\pi\sqrt{t'}(t'+4p^2)^2}\{T_0[-2(2m_Q^2 + m_{\tilde{G}}^2)^2 +$$

$$+ 2m_{\tilde{G}}^2(m_{\tilde{G}}^2 - \frac{1}{2}t') + 4(2m_Q m_{\tilde{G}} + p^2)^2 +$$

$$+ t'(2m_Q + m_{\tilde{G}})^2 - 8m_{\tilde{Q}}^2 m_Q^2 - 4p^4] - 8m_Q^2 T_1 +$$

$$+ T_2[m_{\tilde{G}}(t' + 4p^2)[m_{\tilde{G}}(t' + 4p^2) - 4m_Q(p^2 + m_Q^2 - m_{\tilde{G}}^2)] +$$

$$+ 8m_Q^2[(p^2 + m_{\tilde{Q}}^2)^2 + (m_Q^2 - m_{\tilde{G}}^2 + m_{\tilde{Q}}^2 - \frac{1}{2}t')(p^2 + m_Q^2 - m_{\tilde{G}}^2)]]\},$$

$$\eta_2(p^2, t') = -\frac{g^4 2}{9\pi\sqrt{t'}(t'+4p^2)^2}\{T_0[(t' + 4p^2)^2 + 8(p^2 + m_{\tilde{G}}^2)\times$$

$$\times(m_Q^2 - m_{\tilde{G}}^2 + m_{\tilde{Q}}^2 - \frac{1}{2}t')] + T_1 8(p^2 + m_{\tilde{G}}^2) +$$

$$+ T_2[(p^2 + m_Q^2)(t' + 4p^2)^2 - 8(p^2 + m_{\tilde{G}}^2)[(p^2 + m_{\tilde{Q}}^2)^2 +$$

$$+ (p^2 + m_Q^2 - m_{\tilde{G}}^2)(m_Q^2 - m_{\tilde{G}}^2 + m_{\tilde{Q}}^2 - 1/2\, t')]]\},$$

$$\eta_3(p^2, t') = -\frac{g^4 2}{9\pi\sqrt{t'}(t'+4p^2)^2}\{S_1 2(t' + 4p^2)[2m_Q(2p^2 + m_{\tilde{Q}}^2 +$$

$$+ m_{\tilde{G}}^2 - m_Q^2) - m_{\tilde{G}}(t' + 4p^2)] + 4m_Q(t' + 4p^2)S_0\}, \qquad (A.1)$$

$$\eta_4(p^2, t') = \frac{g^4 2}{9\pi\sqrt{t'}(t'+4p^2)}\{T_1 + (m_Q^2 - m_{\tilde{G}}^2 - t'^2 - p^2 +$$

$$+ 2m_{\tilde{Q}}^2)T_0 + T_2[p^4 - (m_Q^2 - m_{\tilde{G}}^2)^2 + (t' - 2m_{\tilde{Q}}^2)\times$$

$$\times(p^2 + m_Q^2 - m_{\tilde{G}}^2) - (p^2 - m_{\tilde{Q}}^2)^2]\},$$

$$\eta_5(p^2, t') = 0.$$

The functions T_0, T_1 and T_2 may be expressed in terms of complete elliptic integrals of the first ($\widetilde{F}$), second ($\widetilde{E}$) and third ($\widetilde{\Pi}$) kind, respectively[7]:

$$T_0(p^2, t') = \frac{2}{(k_+^2 + m_{\widetilde{G}}^2)^{1/2}} \widetilde{F}(a),$$

$$T_1(p^2, t') = \frac{2}{(k_+^2 + m_{\widetilde{G}}^2)^{1/2}} [(k_+^2 + m_{\widetilde{G}}^2) \widetilde{E}(a) - m_{\widetilde{G}}^2 \widetilde{F}(a)], \tag{A.2}$$

$$T_2(p^2, t') = -\frac{1}{W} S_1(p^2, t') - \frac{2}{(p^2 + m_{\widetilde{Q}}^2 - m_{\widetilde{G}}^2)(k_+^2 + m_{\widetilde{G}}^2)^{1/2}} \times$$

$$\times \left\{\frac{k_-^2 + m_{\widetilde{G}}^2}{p^2 + m_{\widetilde{G}}^2} \widetilde{\Pi}(\beta; a) - \widetilde{F}(a)\right\},$$

where

$$\begin{aligned} a &= 1 - \frac{k_-^2 + m_{\widetilde{Q}}^2}{k_+^2 + m_{\widetilde{Q}}^2} \\ \beta &= 1 - \frac{k_-^2 + m_{\widetilde{G}}^2}{p^2 + m_{\widetilde{G}}^2} \end{aligned} ; \quad k_\pm = \left(\frac{1}{4} t' + p^2\right)^{1/2} \pm \left(\frac{1}{4} t' - m_{\widetilde{Q}}^2\right)^{1/2}. \tag{A.3}$$

The functions S_0 and S_1 are defined as

$$S_0(p^2, t') = \pi,$$

$$S_1(p^2, t') = \frac{\pi}{(p^2 + m_{\widetilde{Q}}^2 - m_{\widetilde{G}}^2)^{1/2}} \left[\frac{\Theta(t_C - t')}{(t_C - t')^{1/2}} + \mathrm{i} \frac{\Theta(t' - t_C)}{(t' - t_C)^{1/2}}\right],$$

where

$$t_C = 4 m_{\widetilde{Q}}^2 + \frac{m_{\widetilde{Q}}^4}{p^2 + m_{\widetilde{Q}}^2 - m_{\widetilde{G}}^2}.$$

APPENDIX B

In the Pauli representation, we define $\widetilde{X}_{J\alpha}$ through:

$$\bar{u}(\vec{p}')\, \bar{v}(-\vec{p}')\, \Lambda_J u(\vec{p})\, v(-\vec{p}) = \sum_\alpha \widetilde{X}_{J\alpha} \widetilde{\Omega}_\alpha, \tag{B.1}$$

where

$$\widetilde{\Omega}_C = 1^{(1)} 1^{(2)},$$

$$\tilde{\Omega}^{+}_{SO} = \frac{1}{2}\, i\, (\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)})\, (\vec{p}' \times \vec{p}),$$

$$\tilde{\Omega}^{-}_{SO} = \frac{1}{2}\, i\, (\vec{\sigma}^{(1)} - \vec{\sigma}^{(2)})\, (\vec{p}' \times \vec{p}),$$

$$\vec{\Omega}_{T} = \vec{\Delta}^{2}\, (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) - 3\, (\vec{\sigma}^{(1)} \cdot \vec{\Delta})\, (\vec{\sigma}^{(2)} \cdot \vec{\Delta}), \tag{B.2}$$

$$\tilde{\Omega}_{SS} = \vec{\Delta}^{2}\, (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}),$$

$$\tilde{\Omega}_{SO_2} = \vec{\sigma}^{(1)} \cdot (\vec{p}' \times \vec{p})\, \vec{\sigma}^{(2)} \cdot (\vec{p}' \times \vec{p}),$$

and $\tilde{X}_{j\alpha}$'s in the non-relativistic limit, $p^2 \to 0$, are

	η_1	η_2	η_3	η_4
C	-1	1	0	0
SO^+	0	0	0	0
SO^-	$\frac{1}{2m_{\tilde{G}}^2}$	$\frac{1}{2m_{\tilde{G}}^2}$	0	$-\frac{1}{m_{\tilde{G}}^2}$
T	0	0	0	$-\frac{1}{12m_{\tilde{G}}^2}$
SS	0	0	0	$-\frac{1}{6m_{\tilde{G}}^2}$
SO_2	$-\frac{1}{16m_{\tilde{G}}^4}$	$\frac{1}{16m_{\tilde{G}}^4}$	0	0

(B.3)

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SUPERSIMETRIČNI DIO TEŠKOG KVARK-AKTIKVARK POTENCIJALA

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Računat je dio potencijala teškog kvarkoniuma, koji se pojavljuje ako se QCD, koja je odgovorna za vezanje kvarkova, proširi na SUSY QCD. To uključuje računanje »box« dijagrama u kojem su čestice izmjene supersimetrični partneri kvarkova i gluona.